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LIGHT, PHOTOMETRY AND ILLUMINATION

A THOROUGHLY REVISED EDITION
OF
"ELECTRICAL ILLUMINATING ENGINEERING"

BY

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PREFACE

This book is designed as a text in illuminating engineering and a reference for the practicing engineer. It is the outgrowth of sets of notes written and compiled by the author for use in his classes at the Armour Institute of Technology, and of his previous book, "Electrical Illuminating Engineering."

In the construction of this volume the author has freely consulted the volumes of the technical press and particularly the *Transactions* of the Illuminating Engineering Society, the *Illuminating Engineer*, the *Bulletin* of the Bureau of Standards, and the *Electrical World*. It has been the aim of the writer to assemble the best ideas and thoughts available on the subject of artificial illumination and in so doing he has freely drawn from the works of eminent authorities on particular phases of this subject as will be seen from the references cited. In this connection grateful acknowledgment is made to the writings and investigations of Dr. H. E. Ives, Dr. A. S. McAllister, Dr. E. B. Rosa, Mr. M. D. Cooper, Mr. A. J. Sweet, Mr. A. A. Wohlauer and others whose excellent papers and kind permission to use them are greatly appreciated. The author also wishes to express his appreciation of criticisms and suggestions by Dr. H. E. Ives, of the kindness of Mr. F. H. Bernhard in reading proof and criticising copy, and of the efforts of the publishers in editing the book.

W. E. B., Jr.

July, 1912.

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LIGHT, PHOTOMETRY AND ILLUMINATION

CHAPTER I

LIGHT AND ITS PHYSICAL PROPERTIES

The phenomenon of light is inherently the same as that of radiant heat although the vibratory or wave motion is of different frequency. The light waves are those having a periodicity of such value that when they are received by the retina of the eye they produce the sensation of vision in the brain.

Since the velocity of light of all colors is the same or 3×10^{11} millimeters per second (186,000 miles per second) it follows that the wave-length of the light in different parts of the spectrum varies inversely as the frequency of the corresponding wave. Thus if the frequency of light producing highest sensibility is 5.5×10^{14} cycles per second its wave-length will be

$$\lambda=\frac{V}{f}=\frac{3\times 10^{11}}{5\times 10^{14}}=545\times 10^{-6} \text{ millimeters or } 0.545\mu \qquad (1)$$

where μ represents one-thousandth part of a millimeter. The eye responds to radiation between the limits of approximately 0.300 and 1.000μ, but the radiation easily visible to most eyes lies between 0.330μ in the violet end of the spectrum and 0.770μ in the red portion. Good visibility requires radiation between 0.410μ and 0.760μ, or less than one octave. *Highest sensibility* occurs between 0.500μ and 0.600μ. At frequencies lower or at wave-lengths greater than those of red light the energy radiated is in the form of heat and is known as the infra-red or ultra-red rays, also at wave-lengths shorter than those of the violet light the radiation is again invisible and is known as the ultra-violet rays.

Frequency and Wave-length of Energy.—The following table is useful for the comparison of the frequencies and wave-lengths of some of the common forms of energy of scientific interest.

TABLE 1

Frequency and Wave-length of Energy

Form of energy	Cycles per second	Wave-length in air
Alternating current field.............	15	12,500 miles
Alternating current field.............	25	7,500 miles
Alternating current field.............	60	3,100 miles
Alternating current field.............	125	1,500 miles
Wireless telegraph wave..............	10^5 to 10^7	10,000 to 100 ft.
Herzian waves.....................	10^7 to 10^9	100 to 1 ft.
Infra-red rays......................	6×10^{11} to 4×10^{14}	0.2 to .00003 in.
Visible light........................	4×10^{14} to 7.7×10^{14}	.00003 to .000015 in.
Ultra-violet light....................	7.7×10^{14} to 3×10^{15}	.000015 to .000004 in.
Lowest audible sound wave...........	15	66 ft.
Highest audible sound wave..........	8,000	1.5 in.

Luminous radiation may be produced (1) by the influence of temperature; (2) by chemical action or electrical action; or (3) by a combination of temperature influence and chemical or electrical action. In the most numerous luminous sources, luminosity is produced by solid bodies raised to high temperature and the luminous radiation is due to the *incandescence* of the body. When light is produced by other means than by direct temperature effect we have the interesting phenomena known as luminescence, fluorescence, and phosphorescence.

The phenomenon of *luminescence* is produced by chemical or electrical effects and nearly always occurs in gases. The luminosity of the so-called flame and luminous arcs and of the vapor lamps are examples of this phenomenon. With these sources the color of the light and the efficiency of the lamp become functions of the materials or gases involved rather than the temperature.

Fluorescence refers to the property of a substance whereby some of the radiation which is absorbed by it is converted into radiation of a different wave-length. It acts as a frequency converter and usually changes invisible radiation into radiation capable of exciting vision.

Phosphorescence is applied to the property of a body whereby it absorbs energy of radiation such as light in such a way as to give it out again afterward, for example, as a glow when placed in darkness.

Although a very considerable amount of research work has been done in the investigation of fluorescence and phosphorescence, there is as yet little definite knowledge of these phenomena.

In considering now the subject of incandescence where luminosity is due entirely to temperature effects, it will be well to review the laws connecting radiation and temperature.

Black-body Radiation.—A body at any temperature other than absolute zero will radiate energy in all the different wave-lengths from zero upward; the wave-length at which maximum radiation occurs depends on the temperature of the radiator. The amount of energy radiated varies according to certain laws which have been theoretically derived and experimentally verified. First, fundamentally, are the laws based on the principles laid down by Kirchhoff. Briefly these are:

(1) At any given constant temperature every body radiates the same amount of energy that it receives.

(2) Black radiation must exist in every completely enclosed cavity whose walls are opaque and of the same temperature.

Black radiation may be defined as the radiation from a black body, and a *black body*, as one which is not transparent, does not reflect, but absorbs all incident radiation at all temperatures. It is obvious that the total energy radiated from such a body is greater than that from any other dissimilar body at the same temperature.

Probably the most general law is that known as the *Stefan-Boltzmann law of radiation.* Briefly this law is that the total radiation from a black body is proportional to the fourth power of its absolute temperature or,

$$W = kT^4 = k(t+273)^4 \qquad (2)$$

where T is the absolute temperature; t the temperature in centigrade degrees; and k the radiation constant = approximately 5.32×10^{-12}.

From the above equation then the net radiation of energy from a black body to the surrounding air would be

$$W = 5.32 \times 10^{-12}(T^4 - T_o^4) \text{ watts per square centimeter} \qquad (3)$$

where T_o is the absolute temperature of the surrounding air.

It will be seen that this law provides a means whereby a continuous *temperature scale* can be obtained from low to the very highest values of temperature since by transposing

$$T = \sqrt[4]{T_o^4 + W/(5.32 \times 10^{-12})} \qquad (4)$$

where W is the radiation in watts per square centimeter, or

$$T = \sqrt[4]{T_o^4 + J/(1.28 \times 10^{-12})} \qquad (5)$$

if J is the radiation in calories per square centimeter per second.

Since the radiation of a body varies as the fourth power of its temperature, it is evident that the greatest efficiency of radiation will be obtained at the highest temperatures. Although the amount of visible radiation from a luminous source is a small part of the total radiation, it follows from the above that the efficiency of a source as an illuminant will vary greatly with the temperature. The relative values of the radiation at different wavelengths from black bodies at several temperatures are shown in Fig. 1.[1] The total radiation is represented by the areas enclosed by the respective curves. Although the total area representing the radiation increases as the fourth power of the temperature in absolute degrees, it is evident that the rate of increase in visible radiation is still greater due to the shifting of the maxima of the curves as indicated by the dotted line.

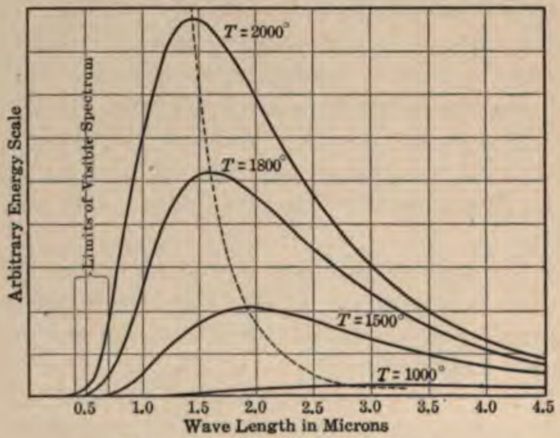

FIG. 1.—Black body radiation.

The Luminous Efficiency.—The relation of the visible radiation to the total radiation is an indication of the luminous efficiency of the source. This efficiency therefore increases with increase in temperature of the source, due to the shifting of the radiation curve toward the visible region. A maximum is reached at about 6000° absolute. At 5000° the maximum of the radiation curve is in the visible region and coincides with the maximum of the sensibility curve of the eye at about the middle of the visible spectrum.

---

[1] Trans. Amer. Inst. Elect. Eng'rs., Vol. 29, p. 1445, Sept.,1909.

Wien's "Displacement Law."[1]—Wien's investigations brought out his "*displacement law*" bearing on the shift of the maxima of the radiation curves. The substance of this law is that the product of the wave-length, (λ_m) in microns, at which the energy radiation is a maximum, and the absolute temperature T is equal to a constant a.

$$\lambda_m T = a = \text{constant} \tag{6}$$

where a is equal to about 2960 for a black body and 2630 for platinum.

By combining this law with that of Stefan and Boltzmann, Wien deduced the relation for black bodies,

$$W_m T^{-5} = b = \text{constant} \tag{7}$$

where W_m is the energy of radiation corresponding to the wave-length λ_m of the maximum energy radiation, and T is the absolute temperature.

It was found that b was equal to about 14,600 for black bodies, For other than black bodies the law is complicated by the absorptive power of the body. For certain metals the relation is closely represented by the expression

$$W_m T^{-c} = \text{constant}. \tag{8}$$

while for other metals this equation will in no way hold. For polished platinum c is equal to about 6.0.

By applying these equations it can be shown that a temperature of 5000° absolute would be required for a black body and 4500° for a body resembling polished platinum in order to obtain the maximum of the energy curve near the center of the visible spectrum or at the wave-length of about 0.545μ.

In the effort to secure high luminous efficiency by raising the temperature of the radiator, a limit, far below those mentioned above, is established by the impossibility of producing a substance capable of withstanding those high temperatures without disintegration. While the sun has been estimated to have a temperature of 6000° absolute, that of the positive crater of a carbon arc is believed to be between 3600 and 4200°; the incandescent gas mantle 1900 to 2400; osmium filament 2250; and a carbon filament about 2050, all in centigrade degrees measured from absolute zero.

Selective Radiation.—A higher luminous efficiency may be obtained for the same temperature, however, by employing a radiator capable of selective radiation in the short wave-lengths. A black body is a "complete radiator," that is, it is said to radiate at any temperature a maximum amount of energy in every wave-length. Such a body will be a complete absorber and will absorb all energy incident upon it. A body which is not a complete radiator will radiate less energy at each wave-length than a black body at the same temperature. If the energy radiated in each wave-length is reduced by a constant proportion, the ratio of the visible energy radiated to the total energy radiated will be the same as that of a black body, and the luminous efficiency will be the same as that of a black body at the same temperature. Such a body is termed a *gray body*, and in place of absorbing all the energy falling upon it, it will reflect a constant percentage of each wave-length. If a body radiates less energy than a black body at the same temperature but radiates a relatively larger proportion of energy at certain wave-lengths than at others compared with a black body, it may be said to *radiate selectively* in the former wave-lengths. Thus if a body radiates selectively in the wave-lengths of the visible region its luminous efficiency will be greater than that of a black body at the same temperature. This accounts in part for the higher efficiency of some of the new types of incandescent lamps, although the principal cause of the higher efficiency is the higher temperature at which they operate.

---

[1] Trans. Illum. Eng. Soc., Vol. 4, p. 67, 1909.

The relation of *visible to total radiation* of incandescent electric lamps is shown graphically by the energy curves derived by Drs. Coblenz and Hyde[1] and shown in Fig. 2. These curves refer to lamps having filaments of untreated carbon (curve *a*), flashed carbon (curve *b*), tungsten (curve *c*), and osmium (curve *d*). The visible radiation is indicated by the shaded portion and extends from 0.4 to 0.7μ (thousandths of a millimeter). The lamps were so operating that the amount of visible radiation was the same for each. It will be seen that the less invisible radiation from the metallic-filament lamps indicates a higher efficiency for these sources. It will also be seen that the maxima of the energy curves shift with these indications of increase in efficiency in the same direction as the maxima of the black body radiation curves with increase of temperature. This is due in part to selective radiation but is largely due to higher temperatures.

The ordinates of the shaded portion merely indicate the amount of radiant energy at those wave-lengths and should not be mistaken for the visual effect of the radiation which will be taken up in the following chapter.

---

[1] Bull. Bur. Stand., Vol. 5, p. 339.

From the preceding discussion it will be seen that the variation of luminous intensity with temperature follows definite laws within certain limits. The color of the light from an incandescent source becomes whiter as the temperature is increased. In the case of luminescent sources there appears to be nothing definite in the relationship of intensity or color of the light to temperature. In some cases the intensity may decrease and the color change toward a blue or violet hue while in other cases, as in the mercury-vapor arc, there is an increase in intensity with increase

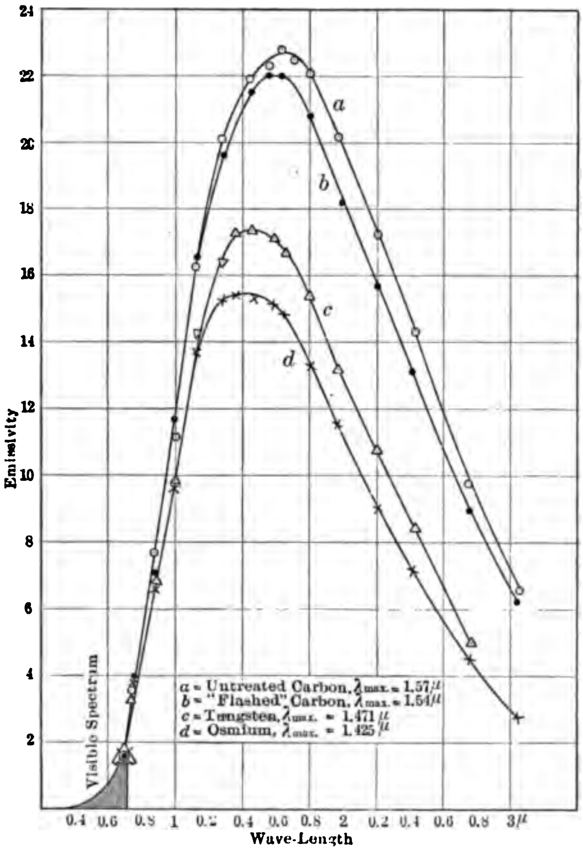

FIG. 2.—Radiation curves for incandescent lamps.

in temperature and the spectrum broadens out becoming more normal and containing more red rays.

It is in luminescent sources that the highest luminous efficiencies of industrial significance are obtainable. In the "flame" and "luminous" arc lamps the light is due to the luminescence of the vapor stream. Its intensity is not a function of the temperature but depends upon the chemical nature of the substances fed into the arc. In the mercury-vapor arc and the Moore tube the color and intensity of light are functions of the chemical composition of the gases which constitute the vapor stream and while the intensity increases with increase in temperature the variation is indefinite and in no way comparable with that of a black body.

Reflection, Absorption and Transmission.—Rays of light falling upon a surface are either *reflected, absorbed* or *transmitted* according to the nature of the substance of which the surface is composed.

Reflection.—The amount of light *reflected* from a particular surface depends (1) upon the molecular condition or color of the surface; (2) the angle of incidence, or the angle which the rays make with a normal to the surface; and (3) the wave-length or color of the incident rays. If the surface is smooth the reflection is *specular* and the rays make an angle with a normal to the surface equal to the angle of incidence. If the surface is rough the reflected rays are *diffused*. By a proper choice and arrangement of the molecules of a substance, rays of a desired frequency may be reflected and all others absorbed thus giving rise to the color of the body. Substances differ greatly in their property of absorption and reflection, and to this is due the variety of colors of objects. In general the per cent. of light reflected by different substances or surfaces is indicated in Tables 2 and 3. In Table 2 the substances represent the materials used for reflectors, interior finishings and in connection with photometric work. Those given in Table 3 are different commercial grades of wall paper. Two sets of data are given in this table, one where sky light is used and the other where the source of light is an incandescent lamp. The effect of the difference in color of the light on the coefficient of reflection is quite marked in some cases. It should be noted that these values are for the light reflected at about 45 degrees from a ray at normal incidence, consequently they represent approximately the values for pure diffusion under average conditions.

TABLE 2

Reflection Coefficients for Various Surfaces[1]

Material	Coefficient of reflection
Highly polished silver	.92
Mirrors silvered on the surface	.70–.85
Highly polished brass	.70–.75
Highly polished copper	.60–.70
Highly polished steel	.60
Speculum metal	.60–.80
Polished gold	.50–.55
Burnished copper	.40–.50
White blotting paper	.82
White cartridge paper	.80
Ordinary foolscap	.70
Chrome yellow paper	.62
Orange paper	.50
Plain deal (clean)	.45
Yellow wall paper	.40
Yellow painted wall	.40
Light pink paper	.36
Yellow cardboard	.30
Light blue cardboard	.25
Brown cardboard	.20
Plain deal (dirty)	.20
Yellow painted wall (dirty)	.20
Emerald green paper	.18
Dark brown paper	.13
Vermilion paper	.12
Blue-green paper	.12
Cobalt blue paper	.12
Black paper	.05
Deep chocolate paper	.04
French ultra-marine blue paper	.035
Black cloth	.012
Black velvet	.004
Additions by the author:	
Tracing cloth	.30
Macadam road	.12
Dead black paint	.01

[1] Dr. Bell, "The Art of Illumination," pp. 47 and 52, 1902.

TABLE 3

Coefficients of Diffuse Reflection [1]

Kind	Color	Cofficient skylight	Coefficient inc. lamp.
Plain ceiling	Faint greenish	.50	.53
	Light ecru	.27	.26
	Very faint gray-cream	.53	.64
	Light gray-green	.26	.23
	Light yellow	.53	.49
	Faint ecru	.47	.55
	Faint pinksh	.41	.43
	Pale bluish-white	.42	.31
Crepe	Medium green	.25	.19
	Deep yellowish-green	.13	.07
	Dark coffee-brown	.08	.06
	Deep green	.05	.06
	Deep yellow-buff	.41	.41
	Full green	.06	.06
	Deep red	.05	.05
	Medium red	.06	.08
Cartridge	Medium green	.15	.11
	Dull green	.11	.07
	Dull yellowish-green	.09	.07
	Light pinkish-brown	.21	.26
	Light green	.23	.18
	Light blue	.21	.20
	Pale gray	.35	.27
	Faint yellowish-green gray	.43	.33
	Salmon buff	.31	.33
	Medium light buff	.44	.44
	Medium full green	.11	.07
	Medium dull red (gray-red)	.06	.07
	Light red	.10	.10
	Very deep ecru	.18	.15
	Pale pink	.25	.19
	Deep yellow-gray	.18	.13
Silky finish	Medium crimson (cross grain)	.08	.12
	Medium gray-green	.17	.12
Stripes	Deep cream	.56	.60
	Deep cream silvery	.56	.57
	Yellow medium	.50	.53

---

[1] Dr. Bell, Trans. Illum. Eng. Soc., Vol. 2, p. 653, 1907.

TABLE 3.—*Continued*

Kind	Color	Coefficient skylight	Coefficient inc. lamp
	Deep buff	.53	.58
	Medium red	.06	.08
	Medium red satin	.07	.11
	Light strawberry pink	.43	.43
	Light strawberry silvery	.51	.49
	Light and dark green (heavily streaked with deep green)	.06	.07
	Silvery light green (heavily streaked with deep green)	.13	.14
	Light green (plain)	.36	.26
	Silvery light green (corded)	.36	.23
Miscellaneous	Dark green and gold (minute figuring much gold)	.24	.19
	Light green and gold (minute figuring much gold)	.31	.28
	Deep and light red	.12	.20
Pique	Light bluish	.46	.47
	Light gray	.38	.38

Effect of Varying the Angle of Incidence upon Reflection.—Investigations by Mr. Gilpin[1] show the effect of varying the incident angle upon the distribution of light, and the amount of light reflected for a number of different kinds of wall paper.

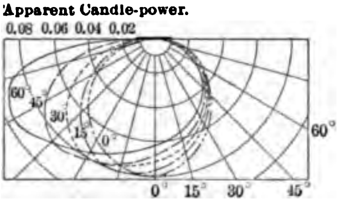

FIG. 3.—Pulp tint, white.

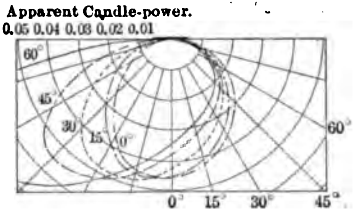

FIG. 4.—Silk fibre, orange-yellow.

The efficiencies of different papers and with different incident angles are given in the following table. The mean value may be considered as corresponding to those values given in the preceding table. The distribution curves of reflected light from the different samples of glossy and semi-gloss paper show a tendency toward specular reflection while the reflection from the matt and rough samples shows the result of the diffusing properties of the surfaces.

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 854, 1910.

In Figs. 3, 4, 5, 6, 7, and 8 are shown the curves representing the distribution of light in a normal plane from small illuminated surfaces composed of some of the samples included in the table, and with the incident light rays making different angles with the normal. These figures refer to numbers 1, 4, 5, 19, 20, and 21, respectively, as indicated under the figures. These represent in general the performance of the different classes of paper and will be found useful in the design of interior installations for illumination and in determining the height and location of lamps when these or similar materials are used on the walls or ceilings.

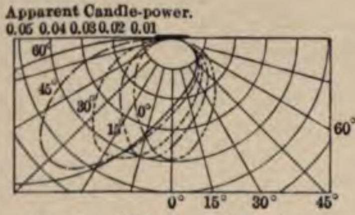

FIG. 5.—Silk fibre, French gray.

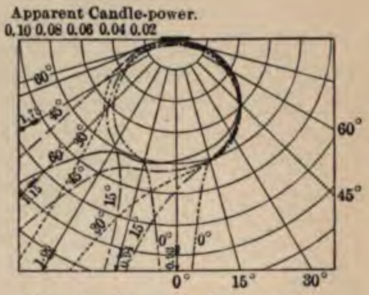

FIG. 6.—Varnished tile, cream.

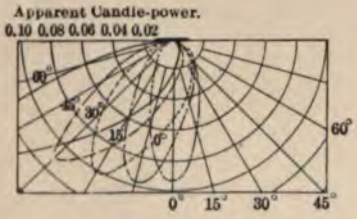

FIG. 7.—Embossed glass, gilt.

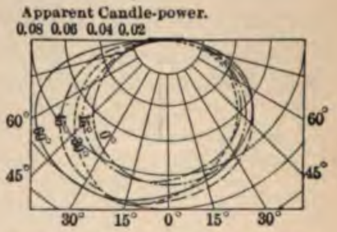

FIG. 8.—White blotting paper.

TABLE 4

Effect of Varying the Incident Angle upon Reflection

Kind of paper	Finish	Color	Efficiency of incident angle with normal of					Mean
			60°	45°	30°	15°	0°	
Pulp tint	Matt (smooth)	White	56.0	61.5	58.2	56.5	54.6	57.3
Pulp tint	Matt (smooth)	Light buff	46.0	47.4	45.7	43.8	42.7	45.1
Pulp tint	Matt (smooth)	Lt. orange yel.	42.5	44.1	43.0	42.5	42.4	42.9
Silk fiber	Semi-gloss	Orange yellow	38.5	36.0	33.3	31.4	30.0	33.8
Silk fiber	Semi-gloss	French gray	37.6	32.2	25.6	22.6	21.0	26.0
Silk fiber	Semi-gloss	Lt. pea green	27.8	25.7	21.4	18.9	18.5	22.2
Silk fiber	Semi-gloss	Dk. pea green	23.5	18.6	14.4	11.2	9.3	15.4
Silk fiber	Semi-gloss	Light brown	16.8	14.6	11.7	10.9	10.3	12.9
Silk fiber	Semi-gloss	Light blue	15.5	13.0	10.0	9.2	8.1	11.2
Silk fiber	Semi-gloss	Cherry red	13.7	11.8	8.9	7.0	6.3	9.5
Imported stock	Fibrous	Tan	19.3	19.0	17.3	16.3	15.7	17.5
Imported stock	Fibrous	Light blue	13.5	13.4	12.0	11.0	10.2	12.0
Duplex	Rough	Light blue	12.7	12.0	10.5	9.1	8.2	10.5
Duplex	Rough	Cherry	6.6	6.4	6.2	6.0	5.7	6.2
Plain	Rough	Yellow buff	31.4	34.3	35.6	35.6	35.0	34.4
Plain	Rough	Lt. pea green	21.5	20.4	19.8	18.8	18.2	19.7
Plain	Rough	Dk. pea green	13.4	12.7	11.3	10.7	8.9	11.6
Plain	Rough	Deep red	7.3	7.0	5.9	5.3	4.9	6.1
Varnished tile	Glossy	Cream	66.3	71.1	73.1	71.8	70.8	70.6
Imported	Embossed gloss	Gilt	54.0	49.3	43.7	37.7	27.0	41.7
White blotter			72.7	80.0	73.9	72.9	71.1	74.0

Influence of Color of Light on Coefficient of Reflection.[1]—The influence of the wave-length of light upon the value of the coefficient of reflection are indicated by experimental data obtained by Mr. Bauder[2], and given in part in the following table:

TABLE 5

Color of Light and Reflection

Type of lamp	Candle-power	Watts per candle	Per cent. reflection of light in the test-room		
			Red walls	Green walls	Blue walls
Treated carbon	20	3.1	53.39	48.57	43.90
Graphitized carbon	20	2.5	50.85	49.35	44.55
Tantalum	20	2.0	49.27	49.60	46.11
Tungsten	20	1.25	47.20	51.08	50.38

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[1] See also results in Table 3.

[2] Trans. Illum. Eng. Soc., Vol. 6, p. 85, Feb., 1911.

These values represent the average reflected light when a room 10 by 10 ft. was equipped with red, green and blue wall materials and with a white ceiling and dark floor, in per cent. of the light reflected when the walls, ceiling and floor were all white. It should be remembered that these luminous sources decrease in the emission of red rays and increase in the amount of light of short wave-length in the order of their decrease in specific consumption. With this point in mind it will be easily understood why the red wall gives a higher efficiency with the carbon lamp and the green or blue wall with the tungsten lamp.

Transmission and Absorption.—It is obvious that the amount of light transmitted by a translucent medium depends upon the quality of the medium and the color of the light transmitted and is equal to the difference between the amount of light received and the amount reflected or absorbed. We are principally concerned with the transmission, or absorption, of the atmosphere and of the different translucent mediums used for illuminating purposes. Daylight illumination is derived exclusively from the sun, and sunlight is greatly modified by the atmosphere. A varying proportion of the sun's rays reach us, after selective reflection and absorption, from minute particles in the atmosphere, from cloud masses and from the surface of the earth. The effect upon sunlight in its transmission through the atmosphere is indicated by the averages of six months' observations (Feb. to Aug., 1903) given in the following table.[1] This table gives the average per cent. transmitted by the atmosphere of light of the different wave-lengths with the sun in zenith.

TABLE 6

Absorption by the Atmosphere

Wave-length in microns	Percentage transmitted
.40	47.5
.45	55.3
.50	62.4
.60	68.2
.70	75.6
.80	80.1

It will be seen that the red end of the sun's spectrum is reduced about 25 per cent. by the influence of the atmosphere, while the violet end loses more than half of its initial intensity.

Of the indirect components of daylight that from the cloudless sky is relatively richer in blue and violet and weaker in red rays than sunlight by greatly varying amounts which depend upon the state of the atmosphere. Daylight composed of the direct rays from the sun and the indirect rays from the sky will differ but little, however, from sunlight since sunlight is by far the dominant factor. Skylight, on the other hand, will be much bluer since the direct rays of the sun are intercepted.

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[1] *Astrophysical Jour.*, Vol. 19, p. 313, 1904.

The increase in absorption of the sun's rays by the atmosphere as the sun passes from the zenith is very considerable. This is shown by the difference in intensity both of heat and of light when the sun is high in the sky and when near the horizon. It is also represented by the decrease in intensity both of light and heat from summer to winter. According to Prof. Basquin[1] the average intensity of daylight in July is ten times as great as in December. It will be well to recall at this time that the visual sensation varies approximately as the logarithm of the stimuli producing it. The intensity of daylight also varies greatly during the day. The average intensity at noon is about 3.5 times as great as at 7.30 a. m. and 4.30 p. m. On a clear day in the middle of July the intensity at 12.30 is 4.4 times that at 6.0 a. m. or 7.0 p. m., 1.44 times as great at 4.0 p. m. as at 8.0 a. m., and 2,5 times as great at 6.0 p. m. as at 6.0 a. m.

The amount of *light absorbed by the globes* and glassware used with lamps to prolong the life of the filament or of the electrode, to diffuse the light and lower the intrinsic brightness of the source, or to transmit light of a desired color, will be seen from the following table to vary from 5 to 95 per cent.:

TABLE 7

Absorption of Glassware

Glassware	Absorption per cent.	Efficiency per cent.
Clear glass	5–12	88–95
Light sand blast	10–20	80–90
Alabaster	10–20	80–90
Canary colored	15–20	80–85
Light blue alabaster	15–25	75–85

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[1] *Illum. Eng.* N. Y., Vol. 1, p. 823, 1906.

TABLE 7.—*Continued*

Glassware	Absorption per cent.	Efficiency per cent.
Heavy blue alabaster	20–30	70–80
Ribbed glass	15–30	70–85
Opaline glass	15–40	60–85
Ground glass	20–30	70–80
Medium opalescent	25–40	60–75
Heavy opalescent	30–60	40–70
Flame glass	30–60	40–70
Signal green	80–90	10–20
Ruby glass	85–90	10–15
Cobalt blue	90–95	5–10

The variations in the values above are obviously due to the difference in thickness and density of the glass.

It is obvious that the percentage of light absorbed by an opaque surface is equal to 100 minus the per cent. reflected. The amount transmitted in the case of translucent material is equal to the total incident light less that reflected and that absorbed in transmission.

Summary and Equations of Chapter I

Light is a form of energy of wave-lengths between 0.3 and 1.0μ. Good visibility requires radiation between 0.41 and 0.76μ, highest sensibility occurring between 0.50 and 0.60μ. Energy of longer wave-lengths assumes the form of heat while that of shorter wave-lengths, as the ultra-violet radiation, is likewise invisible.

The wave-length varies inversely as the frequency.

$$\lambda = V/f \tag{1}$$

Luminous radiation may be due to incandescence, luminescence, fluorescence and phosphorescence, the first involving high temperatures and the second giving the highest luminous efficiencies.

The radiation from a black body is proportional to the fourth power of its absolute temperature.

$$W = kT^4 = k(t+273)^4 \tag{2}$$

$$W = 5.32 \times 10^{-12}(T^4 - T_o^4) \text{ watts per square centimeter} \qquad (3)$$

$$T = T_o^4 + W/(5.32 \times 10^{-12}) \qquad (4)$$

$$T = T_o^4 + J/(1.28 \times 10^{-12}) \qquad (5)$$

The wave-length of maximum radiation shifts toward the visible region as the temperature is increased

$$\lambda_m T = \text{constant} \qquad (6)$$

The greatest luminous efficiency occurs at 6000° C. At a higher or lower temperature the luminous efficiency becomes lower.

The visible radiation is a small part of the total radiation due partly to the practical temperature limits of artificial illuminants and partly to the impossibility of converting the invisible radiation into visible radiation.

The luminous efficacy may be improved by employing radiators capable of selective radiation or by resorting to luminescence.

The *"reduced luminous efficiency"* of

a 4-watt carbon lamp about	0.31 per cent.
a tungsten lamp about	0.95 per cent.
a yellow-flame arc about	4.54 per cent.
a quartz-tube mercury-vapor lamp about	5.15 per cent.

The amount of light reflected from a surface depends upon:

1. The molecular condition or color of the surface.
2. The angle which the light rays make with the surface.
3. The wave-length or color of the incident light rays.

Reflection may be specular or diffuse.

Light striking an opaque surface is partly reflected and partly absorbed. Light striking a translucent body is partly reflected, partly absorbed and partly transmitted.

Of the sun's rays about 47.5 per cent. at 0.40μ,
and 80.1 per cent. at 0.80μ.
are transmitted by the atmosphere.

CHAPTER II

COLOR VALUES OF ILLUMINANTS AND COLOR EFFECTS

Color plays an important part in illuminating engineering through the ultimate physiological and psychological effects which light of various hues produce. Not only is the acuteness of vision dependent to a certain extent upon the color of the light, but the effect of selective absorption, which so deceives the eye when colored objects are viewed by colored light, shows the necessity of a careful consideration of the color values of artificial light. Again, in the comparison of illuminants we have the Purkinje and color effects introducing difficulties in determining their relative candle-power values. With these in mind the question of a standard of color value, in terms of which the color of an illuminant may be expressed, naturally arises.

Average Daylight.—That average daylight should be accepted as such a standard in terms of which to compare the color values of the existing sources of artificial illumination seems based on a sound philosophical basis. The maximum of the luminosity curve of the normal eye is in the same region of the spectrum as the maximum of the radiant energy of sunlight. The organs of vision have for ages been developed, for the most part, in daylight and they are accustomed to light possessing the spectrum of natural illumination. It only seems reasonable and rational that the color values of illuminants should be expressed in terms of the components of average daylight.

If we define daylight as the natural light illuminating a shaded surface we shall by no means have a light of constant spectral relations. Its components will be different in different parts of the world, at different times of the year, at different times of the day, and under different atmospheric conditions. There is also a great difference in sunlight at different times of the year and at different times of the day. Of the different kinds of daylight the two most definite are the light from a *clear blue zenith sky* and *noon sunlight* with a clear atmosphere.

By reducing the available spectrophotometric determinations of sunlight and skylight of radiant energy, Dr. Ives[1] found that:

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 189, 1910.

(1) Clear noon sunlight in summer corresponds closely to the energy determinations of a black body at 5000° absolute.

(2) Clear blue skylight averages twice as much energy in the blue (.450μ) as does sunlight for equal intensity in the yellow (.590μ).

(3) Light from cloudy skies varies from near blue to sunlight.

(4) The light from the sun at low altitudes shows a deficiency in blue similar to the excess in blue of the clear sky.

Because of the above variations in daylight Dr. Ives suggests that the best way to arrive at an acceptable value of average daylight is to take advantage of the coincidence above stated between the maximum of sensibility of the eye and the maximum of the sun's energy as it reaches us through the atmosphere. Moreover, if this is not merely a coincidence but the result of adaptation, we have a criterion of average daylight which is more exact than could be obtained by any number of series of daylight measurements. Since this adaptation exists, the inference is that the most probable average daylight is the light whose spectral maximum falls in the region of maximum sensibility. This corresponds to *clear noon summer sunlight* which is the most efficient daylight and which agrees closely with the visible radiation of a black body at 5000° absolute. Since the eye permits considerable variation in the hue of light and still gives the sensation of "white" light, we may use the terms "average daylight" and "white light" interchangeably.

With this value of daylight as a standard Dr. Ives proceeded to investigate the color values of the artificial illuminants with reference to it. An excellent idea of the color values of an illuminant may be obtained by determining the proportion of each of the three primary colors (red, green and blue), which must be mixed to match the visible color of the illuminant. To carry out this investigation recourse was made to the Ives colorimeter, the construction and manipulation of which is given below.

The Ives Colorimeter.—The Ives colorimeter[1] is an instrument designed by Mr. F. E. Ives for the measurement of *all colors in terms of three primary colors.* By means of the colorimeter it is possible to describe a color accurately in terms of the red, green and blue components of a standard white light. If in the white light we assume red 100, green 100, blue 100, then the color pink may be designated as red 100, green 50 and blue 80.6. This means that by mixing red, green and blue in the proportion here given the sensation produced in the eye is that of pink. Two colors alike to the eye measure alike in the colorimeter. Thus, we have a means of comparing numerically the visual effect of such dissimilar sources of light as a gas flame and a mercury-vapor arc. In this it differs from the spectro-photometer which gives the intensity at every point in the spectrum but only an approximate indication of how the eye will compare the color in question with another.

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[1] **Trans. Illum. Eng. Soc., Vol. 3, p. 625, 1908; *Illum. Eng.*, N. Y., Vol. 4, p. 344, 1909.**

The instrument consists essentially of an oblong box, at one end of which are placed four slits—one clear, the three others equipped respectively with red, green and blue screens. By means of levers the openings of the three colored slits can be altered to read by scales from zero to one hundred. Within the instrument is a wheel of lenses which when rotated rapidly by a small motor causes the three colors to pass across the field of vision viewed through an optical device, thus mixing them by persistence of vision. The optical arrangements are such that one observes a divided field, one part consisting of the mixture of the three primary colors, the other the color to be matched as viewed through the clear slit.

To make a measurement, the three colored slits are opened by moving the levers until white is matched, and the scales are adjusted to read 100 for each color. Then any color matched by moving the three levers can be read off the scales in terms of the per cent. of red, green or blue necessary to match white.

The colorimeter was designed to measure the color of fabrics by reflected light and when used for that purpose is manipulated as indicated above. But for measuring the color of light from a lamp the apparatus is modified somewhat. Since there are two sources involved the region in front of the instrument is divided by a partition and the light to be studied is allowed to fall on a flat surface of magnesium oxide placed before the clear slit. It is obvious that summer noon sunlight is not suitable for a working standard, consequently a constant comparison lamp is placed on the other side of the partition and both daylight and the other lights are measured against it, by the substitution method. For this purpose a tungsten lamp with a frosted bulb and operating a little above normal voltage proves very satisfactory. It may be placed near the colored slits and a strip of flashed opal glass will serve to diffuse and equalize the illumination of the slits.

The details of construction and operation can be better understood from Figs. 9 and 10. Fig. 9 shows a side sectional view of the instrument. The eye-piece is at *H* through which the observer sees a divided field, one-half of which is illuminated by a mixture of the three primary colors, and the other side by the light to be investigated. The rotating lenses are at *A* and are driven by a small motor. Focusing lenses are shown at *F* and *I*. The optical arrangement is shown by the horizontal section through the instrument in Fig. 10. The test light is admitted through the opening *L*, and the colored screens and variable diaphragms are located at *G*, *R*, and *B*.

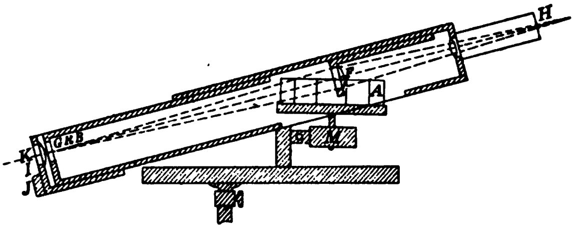

FIG. 9.—Vertical section of colorimeter.

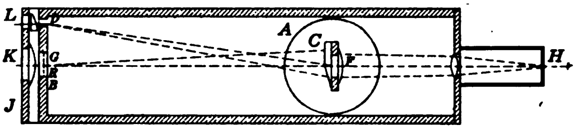

FIG. 10.—Horizontal section of colorimeter.

By means of the colorimeter and with the value of average daylight equal to that of noon summer sunlight, Dr. Ives proceeded to investigate the color relationship of artificial illuminants to daylight. The values given in Table 8[1] express this relation in terms of the three primary color sensations,—red, green and blue.

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[1] Trans. Illum. Eng. Soc, Vol. 5, p. 208, 1910.

TABLE 8

Color Values of Light Sources

Source	Sensation values		
	Red	Green	Blue
1. Black body at 5000° absolute	33.3	33.3	33.3
2. Overcast sky	34.6	33.9	31.5
3. Blue sky	32.0	32.2	35.8
4. Moore carbon dioxide tube	31.3	31.0	37.7
5. Afternoon sunlight	37.7	37.3	25.0
6. Mercury-vapor arc	29.0	30.3	40.7
7. Direct-current arc	41.0	36.3	22.7
8. Welsbach mantle, 1/4 per cent. cerium	42.5	40.8	16.7
9. Welsbach mantle, 3/4 per cent. cerium	45.5	42.0	12.5
10. Welsbach mantle, 1 1/4 per cent. cerium	47.2	41.8	11.0
11. Tungsten 1.25 watts per candle	48.7	40.5	10.9
12. Nernst	49.2	40.7	10.1
13. Acetylene	49.1	40.5	10.5
14. Yellow flame arc	52.0	37.5	10.5
15. Carbon lamp, 3.1 watts per candle	51.3	40.4	8.3
16. Hefner	55.0	38.8	6.2

It will be seen from this table that nearly all illuminants give more red and green rays and less blue rays than are found in average daylight or, what is approximately the same, in light from a black body at 5000° absolute. In other words, the light is yellow. The exceptions to this are the carbon dioxide tube and the mercury-vapor arc. In regard to these figures and especially in the case of the mercury lamp, it should be remembered that the values refer to the color of the light itself when illuminating a white surface. The fact that some of these sources are deficient in certain of the primary colors is only apparent when viewing colored objects in their respective lights. Of all these illuminants the Moore carbon dioxide tube gives a light most closely resembling daylight. It is somewhat bluer than sunlight and less blue than skylight. Its light falls well within the visual region of white light.

The location of these illuminants and of black bodies at different temperatures with reference to daylight are shown in the color triangle of Fig. 11.[1] The dotted line indicates the color of light from black bodies or incandescent solids at different temperatures up to 7000° absolute. It will be seen that nearly all of the artificial illuminants lie near this curve. The reason for the position of the carbon arc may be attributed to the blue or violet in the arc itself. Those illuminants differing from black bodies are obviously located farthest from this line. The Welsbach mantles are above the curve because of the greater amount of green due to selective radiation of energy in the visible spectrum; the flame arc, little of the light from which is due to incandescence of a solid body, lies a considerable distance from this curve; the light from the carbon dioxide tube agrees closely with that from a black body at 6000° absolute.

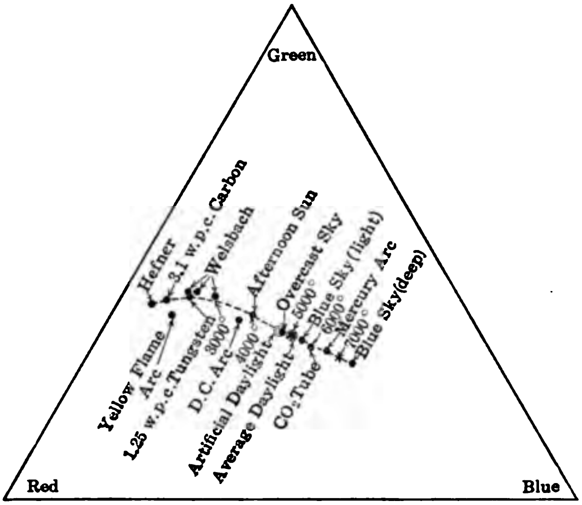

FIG. 11.—Color triangle and location of light sources.

The question of color is of importance in interior lighting and especially so where colored merchandise is to be viewed by artificial light. In service of the latter nature it is obvious that the color of the light should approximate as nearly as possible the color values of diffused daylight. In the lighting of a ballroom or an assembly hall and even in residential lighting, a soft light of a yellowish hue is usually preferable. The cause of this may be habitual or it may be attributed as a natural sequence to the gradual increase of red in daylight as the period of artificial lighting approaches.

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 205, 1910; *Elect. Wld.*, Vol. 57, p. 1092, May 4, 1911.

Color Effects.—The effect of *selective absorption*, which so deceives the eye when colored objects are viewed by light other than white in color, shows the necessity of a careful consideration of the color value of artificial light for that class of service. We are all familiar with the effect of light from the mercury-vapor lamp upon an object which would ordinarily be of reddish hue, as the human skin for example. When viewed in the rays of light from a green lantern, greens, browns, yellows and grays are nearly of the same color but of different shades. In the same light pink looks red and red looks black. Practical illuminants, however, do not often cause serious deceptions, although some of the sources rich in red rays will change the hue of delicate colors containing greenish-blue, blue or violet. Moreover, shades of the same color are often differently affected by artificial light. Colors matched in the light from an arc lamp may stand the test in daylight but prove faulty when viewed in the light from a gas or incandescent lamp. Colors seem to change when changing the intensity of light from a high to a low value, but this is a purely physiological matter, the eye differing in its sensibility to different colors at different intensities, as will be explained later.

An idea of the effect of colored light on objects of different colors can be obtained from Table 9 (reproduced from the Standard Hand-book for Electrical Engineers). As a whole, this table will be applicable to designs of scenic effects to be produced by transmitting light through different colored media. The orange, yellow and green light will represent the performance of some of the commercial illuminants, but most lamps will not give so decided results because of the continuity of their spectra.

Artificial Daylight.—In order to obtain artificial daylight Ives and Luckiesh employed the subtractive method.[1] They used a *tungsten lamp enclosed by absorbing media* which absorbed the excess of yellow and red radiation. The adaptation of the eye allows for a considerable variation from the different daylight values, and still give the daylight effects. However, the average daylight value previously described was assumed and colored glass obtained having selective absorption sufficient to imitate the daylight spectrum when used with a tungsten lamp. The

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[1] *Elect, Wld.*, Vol. 57, p. 1092, May 4, 1911.

TABLE 9

COLOR OF INCIDENT LIGHT RESULTS

Original color	Red light	Orange light	Yellow light	Green light	Blue light	Violet light
Black	Purplish-black	Deep maroon	Olive-yellow	Greenish-brown	Blue-black	Faint violet-black
White	Red	Orange	Light yellow	Green	Blue	Violet
Red	Intense red	Scarlet	Orange	Brown	Violet	Reddish-violet
Orange	Orange-red	Intense orange	Yellow-orange	Faint greenish-yellow	Violet-brown	Light red
Yellow	Orange	Yellow-orange	Orange-yellow	Yellowish-green	Green	Brown, faintly red
Light green	Reddish-gray	Yellow-green	Greenish-yellow	Intense green	Blue-green	Light purple
Deep green	Reddish-black	Rusty green	Yellowish-green	Deep intense green	Greenish-blue	Bluish-gray
Light blue	Violet	Orange-gray	Yellowish-green	Green-blue	Vivid blue	Violet-blue
Deep blue	Violet-purple	Orange-gray	Green-slate	Blue-green	Intenser blue	Bright blue-violet
Indigo blue	Purple, slightly violet	Orange-maroon	Dull orange-yellow	Dull green	Dark indigo blue	Deep blue-violet
Violet	Purple	Red-maroon	Yellow-maroon	Bluish-green-brown	Deep bluish-violet	Deep violet

curves showing the performance of such a combination are reproduced in Fig. 12. Curve (a) shows the transmission of an ideal absorbing medium, while (b) represents the luminosity of the light from a tungsten lamp and (c) the luminosity curve of the light after passing through the absorbing medium. It will be seen that the spectrum of this transmitted light is continuous and suitable for color matching, and work of that nature. In this respect it possesses an advantage over a "white" light made up by combining red, green and blue or yellow and blue. While a good daylight reproduction may be obtained by this latter method there is a liability of the light possessing certain excess spectral bands, while other certain colors may be entirely missing.

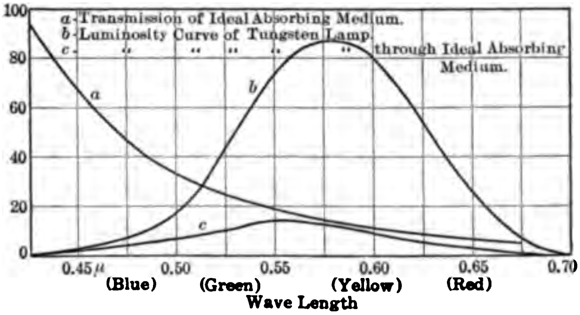

FIG. 12.—Transmission and luminosity curves for white light from a tungsten lamp.

For the absorbing media cobalt blue and signal green seemed the best suited. The blue alone gave a purple light due to the transmitted red. Signal green alone gave too green a light, but reduced the red. A combination of the two gave a pronounced band of yellow-green transmission of the cobalt. This defect was overcome by the use of a dye—rozazeine—which reduced the yellow-green band.

Comparison of curves (b) and (c) shows that the resultant candle-power is about 15 per cent. of the original. This is the daylight efficiency of the tungsten lamp and corresponds to about 10 watts per mean spherical candle, or 1.25 lumens per watt.

The *Moore tube fed with carbon dioxide gas* has been proposed as a standard of color values and its performance in this respect as indicated in the preceding table certainly justifies its choice so far as other artificial illuminants are concerned. It has been found entirely suitable and satisfactory for matching all colors. Its spectrum consists of many lines and bands so located as to give the same practical effect as a continuous spectrum. Objects illuminated by it are claimed to have the identical shades of color they would possess in average diffused daylight.

The lumichromoscope[1] was designed for making practical comparisons of the color effects of light from different illuminants. It consists essentially of a number of small booths with like colored objects displayed in each booth, and each booth illuminated by a different type of lamp. The construction of such a device is shown in Fig. 13. The results obtained with this apparatus as given by Mr. Moore are reproduced in Table 10. There were three ribbons of the three primary colors displayed as shown in the figure and samples of brown, yellow and lavender fabrics included in each booth. It will be seen that some of the color changes are very marked, and go to emphasize the importance of a careful consideration of the color values of illuminants for the illumination of merchandise of delicate colors.

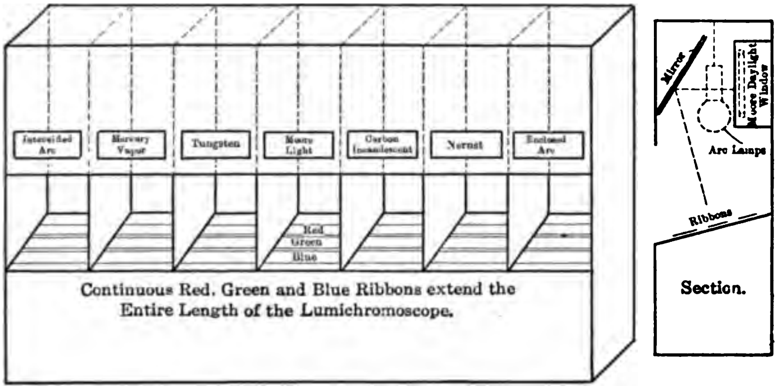

FIG. 13.—The lumichromoscope.

Summary of Chapter II

Noonday summer sunlight appears to be the most consistent standard of average daylight and the most generally acceptable as a standard of color values.

The color values of artificial illuminants with average daylight

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 225, 1910.

TABLE 10. Color of Incident Light and Results

Kind of light	Appearance	Primary colors			Brown	Yellow	Lavender
		Red	Green	Blue			
Average diffused daylight.	Bluish-white or pure white.	A normal red as standard.	A normal green as standard.	A normal blue as standard.	Brown..........	Yellow..........	Lavender.
The white Moore light.	Pure white......	A normal red as standard.	A normal green as standard.	A normal blue as standard.	Brown..........	Yellow..........	Lavender.
Intensified arc....	Bluish-white	Changed to a lighter red.	Green...........	Changed to a lighter blue.	Brown..........	Changed to a lighter yellow.	Changed to a lighter lavender.
Enclosed arc......	Bluish-white.....	Changed to a considerably lighter red.	Green...........	Changed to a much lighter blue.	Changed to a lighter brown.	Changed to a lighter yellow.	Changed to a lighter lavender.
Tungsten.........	Yellow-white.....	Changed to a good many shades lighter.	Changed to a yellowish-green.	Changed to a dark purple.	Changed to a reddish-brown.	Changed to an orange.	Changed to a pink.
Nernst...........	Deep lemon-yellow.	Changed to yellowish-red and a little lighter.	Changed to a yellowish-green.	Changed to a little darker blue.	Changed to a golden brown and many shades lighter.	Changed to a golden yellow and lighter.	Changed to a light pink.
Carbon incandescent.	Orange-yellow...	Changed to a good many shades lighter.	Changed to a yellowish-green.	Changed to a dark purple.	Changed to a reddish-brown.	Changed to a deep orange.	Changed to a pink.
Hewitt mercury vapor.	Blue-green.......	Changed to a very dark red—nearly black.	Changed to a little lighter green.	Changed to a good many shades lighter blue.	Changed to a distinct green.	Changed to a greenish-yellow.	Changed to a bluish-gray.
Welsbach........	Greenish-yellow..	Changed to a darker red.	Changed to a slightly yellowish-green.	Changed to a navy blue.	Changed to a little darker brown.	Changed to a light orange.	Changed to a faded lavender.
Fish-tail gas......	Pale yellow......	Changed to a little lighter red.	Changed to a yellowish-green.	Changed to a deeper navy blue.	Changed to many shades darker brown.	Changed to orange.	Changed to old rose.

as a standard may be determined by means of the colorimeter. Tests of this nature show that the Moore CO_2 tube gives light more closely resembling average daylight than any of the other artificial illuminants. All of the other illuminants except the mercury-vapor lamp emit more red and green rays than noonday summer sunlight.

Coincidence is noted between artificial illuminants and black bodies at different temperatures except where luminescence and selective radiation are involved.

Artificial daylight may be obtained by inclosing an illuminant in an envelope of translucent selective absorbing material.

The effect of selective absorption when viewing colored goods makes a study of the color values of illuminants imperative where the light is to be used in a place where colored fabrics of delicate hues are displayed or matched.

CHAPTER III

THE LUMINOUS EQUIVALENT OF RADIATION

A definition of light in terms of its physiological influence is as yet more or less arbitrary and depends primarily upon the conception of the luminous equivalent of radiation. The problem is to determine the quantity and quality of light from a physiological basis having given a certain spectral distribution of radiant energy. The luminous equivalent of radiation varies greatly with the wave-length and varies to some extent with the intensity of the radiation concerned. It is a complex function of quality, quantity and duration. It may be considered with reference to the *subjective sensation produced* or to the *objective stimulus causing* or capable of causing the sensation. The principle of the conservation of energy cannot be applied to light in either a subjective or objective sense.

Problems which have arisen in connection with progress in affairs pertaining to illuminating engineering have insistently demanded the establishment of some definite relation between light and its luminous equivalent. While no complete and precise solution of the subject has yet been developed, certain essential interrelations between functions which will form the basis of such a solution have been established and will now be considered. These may include:

1. The spectral distribution of radiant energy from different sources of light.
2. The sensibility and visibility of the eye to radiation from different regions of the spectrum.
3. The sensibility and visibility of the eye to intensity and variations in intensity of light.

Spectral Curves for Luminous Sources.—The spectral distributions of energy from some of the artificial sources of light have been shown in the preceding chapters. Other curves bearing more particularly on the distribution of energy in the visible region were determined by Dr. Ives[1] and are reproduced in Fig.

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 204, 1910.

14. These results are qualitative rather than quantitative and are plotted to cross at 0.59μ as is the usual convention where qualitative results are concerned. These curves represent merely the distribution of the radiation in the spectral region and, while they represent to a considerable extent the color values of the light, they give no clue as to the physiological effect upon the eye.

In order to express *light in terms of radiation* we must know the visibility of the human eye for the same amount of radiant energy at each wave-length of the visible spectrum and the variation of sensibility with intensity at constant wave-length. Then spectral energy multiplied by the relative sensibility at each wave-length is luminosity or objective light and the effect of this objective light upon the eye with respect to its variation of sensibility with intensity will give the subjective light or visual sensation.

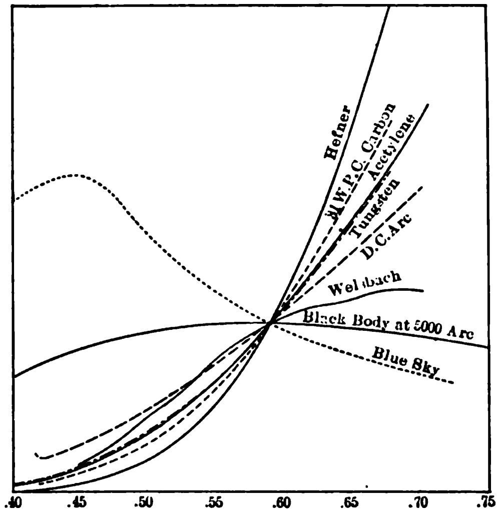

FIG. 14.—Spectral curves for luminous sources.

The Luminosity of Monochromatic Light.—The relative visual sensibility of the eye to light from different parts of the spectrum or the luminosity of monochromatic light, as investigated by Dr. Ives,[1] is shown in Fig. 15. Curve *a* shows the luminosity of different wave-length light reduced to a normal equal energy spectrum. Curve *b* shows approximately the distribution of energy in the visible region from the ordinary incandescent lamp. While the curves in the preceding chapter, referred to above, show that a small part of the total radiation is in the visible region, these curves show that much of the visible radiation is of low effective value. In this figure it will be seen that for a constant energy distribution in the spectrum maximum luminosity occurs at 0.545μ.

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 711, Nov., 1910.

Knowing the relative luminosity for the same amount of energy in different parts of the spectrum and the spectral energy curves for the various illuminants, one can easily obtain the luminosity curves for the different illuminants by multiplying the value of the energy at the various wave-lengths by the relative luminosity of the energy of the respective wave-lengths. Such curves will show the relative intensities of sensation produced by different parts of the spectrum for the particular illuminant. While maximum sensibility occurs at 0.545μ with uniform energy distribution in the spectrum, the wave-length of maximum luminosity for most of the artificial sources of light shifts toward the red end of the spectrum due to the predominance of energy of the red wave-lengths. Such curves are shown in Figs. 44 and 45, which show the actual luminosity curves found

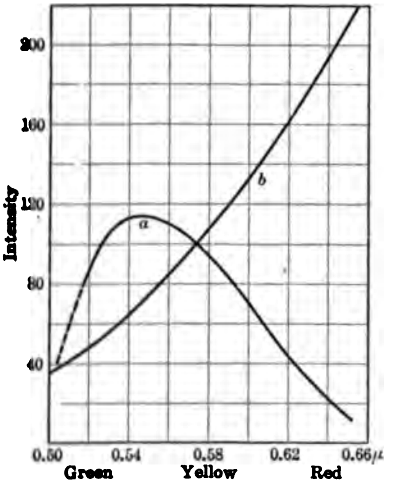

Fig. 15. Luminosity curve for an equal energy, spectrum and spectral distribution of visible radiation from a typical incandescent source.

for the source whose energy distribution is represented by curve *b* of Fig. 15. The luminosity curves for different types of illuminants will differ in shape somewhat due to the difference in the distribution of energy in the spectrum.

The Reduced Luminous Efficiency.—That the amount of visible radiation from luminous sources is a small part of the total radiation may be inferred from the curves of Fig. 2 and is verified by the results obtained by various investigators in efforts to determine the mechanical equivalent of light. These results from different sources, although differing in many instances by several hundred per cent. for the same illuminant, all indicate a low percentage of visible radiation.

Results of this nature derived by Dr. Ives,[1] giving what is termed the "reduced luminous efficiency" are shown in Table 11. This table gives data for some of the various commercial illuminants in terms of a theoretically ideal source, that is, one which radiates its entire energy in the most efficient region of the visible spectrum. Since there is a direct relation between the amount of light flux emitted by an illuminant and the reduced luminous efficiency, one may easily obtain the relative equivalent for any other light source from the ratio of the lumens per watt for the lamps in question.

TABLE 11

The Reduced Luminous Efficiency of Light Sources

Source	Spherical candles per watt applied	Lumens per watt applied	Reduced luminous efficiency in per cent.
Ideal yellow-green source, wave-length 0.545 μ	65	800	100
Fire-fly	?	?	96.5
Black body at 6000°	10	125	15.
Black body at 6000° between .400 and .700 μ.	26.	330	40.
Carbon lamp, 4 watt	0.21	2.6	0.31
Tungsten lamp	0.63	7.9	0.95
D. C. arc lamp	1.1	13.8	1.66
Yellow flame arc lamp	3.0	37.8	4.54
Quartz mercury-vapor lamp	3.4	42.8	5.15

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 113, 1910; *Elect. World*, Vol. 57, p. 1565, June 15, 1911.

Visual sensibility to variations in intensity decreases continually with increasing intensity. Compared with other physical instruments the eye has a most extraordinary range. It may be used with ease at intensities a million times the minimum perceptible intensity. This great power of accommodation is due chiefly to the decrease of sensibility with increase in intensity. The part which the eye plays in protecting itself against high intensities and adapting itself to low intensities can be best understood from a brief description of its construction.

The Human Eye.—The eye consists essentially of three parts—*the iris*, *the focusing lens*, and *the retina*. It resembles a camera in general structure. The *iris* is a diaphragm which expands or contracts to regulate to a certain extent the amount of light which, passing through the focusing lens, falls upon the retina at the back of the eyeball. The *retina* contains the optical nerves which communicate with the brain. At the center of the retina is the region of greatest sensitiveness known as the "fovea." It is at this spot that vision is most acute and where the image is formed when the eye is directed at the object of vision. The impression formed on the remainder of the retina is used merely for orientation.

The peculiarities of the optical system with reference to different intensities of light have given rise to the theory of a double nerve system in the retina. These nerves are known as the "rods" and "cones" and have decidedly different sensibilities both in respect to intensity of light and to color of stimuli.

The *cones* are supposed to be the form-receiving and color-perceiving elements of the retina; in other words, they are the visual cells.

The supposition is that the *rods* are the special apparatus for vision in dim light (night vision). They contain a pigment, known as visual purple, which is very sensitive to light. This visual purple is found only in the external segments of the rods; the cones do not contain it; therefore the fovea, which has only cones, does not contain it. It has been shown that a photograph may be made upon the surface of the retina by the bleaching of the visual purple where it is exposed to light. In the visual purple there is, therefore, an unstable substance readily decomposed by the mechanical or chemical effect of the ether waves. Some radiations are very much more active than others in bleaching this substance, greenish-yellow being most active, yellow next, blue next, violet next and red least active in this process. It has been shown that provision exists in the retina for the constant regeneration of the visual purple, the restoration process taking place rapidly in dim light or darkness. The external segments of the rods impinge upon a layer of heavily pigmented cells—the pigmentary layer mentioned above. When the eye is exposed to light the black pigment of these cells migrates so as to be in position to restore the color to the bleached or used up visual purple. At low intensities, below 0.02 foot-candle, vision appears to be accomplished by the retinal rods alone, while vision at higher intensities brings into play the less sensitive but more efficient retinal cones. The two functions overlap widely so that the transition from rod to cone vision is very gradual.

At intensities greater than one foot-candle the cones exert the predominating influence and show maximum sensibility to light of a yellowish-green hue, while at low intensities, where the rods only respond, the eye is most sensitive to light from the bluish-green part of the spectrum. Thus the eye accommodates itself to high or low intensities by closing or opening the iris and by employing sets of nerves of different sensibility.

The human eye is not achromatic and is not corrected for chromatic aberration. Its dispersive power is slightly greater than that of water. It is known that for near vision the normal eye will generally focus itself for blue rays, while difficulty is experienced in attempting to focus the eye for red rays. The reverse is true for distant vision. While this is true for large and small distances of vision, differences in the ease of accommodation for red and blue rays become less marked for vision at intermediate distances. At a distance of 1 meter, according to M. Luckiesh,[1] no noticeable difference in ease of accommodation for red and blue rays appeared to three observers who had experienced no abnormality in their power of accommodation.

As a person increases in age the elasticity of the lens diminishes thus reducing the accommodation of the eye. This change begins at an early age and continues through life. According to Godinez[2] the amplitude of accommodation at different ages is as indicated in the following table:

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[1] *Elect. World,* Vol. 58, p. 1255, Nov. 18, 1911.

[2] Trans. Illum. Eng. Soc., Vol. 6, p. 802, Nov., 1911.

TABLE 12

Accommodation of the Eye

Age in years	Amplitude of accommodation diopters
10	14
15	12
20	10
30	7
40	4.5
50	2.5
60	1.0
75	0.0

Experimental data obtained by Cravath[1] show that the intensity of illumination required by individuals of different ages for good visibility is in general proportionally identified with the age of the individual as might be inferred from the above table.

The Intensity of Natural Illumination.—The values of the natural incident illumination of the earth vary from approximately zero for a dark moonless night with the sky heavily overcast, to something like 12,000 foot-candles for the normal intensity and 9000 foot-candles for the horizontal illumination due to direct rays from a noonday sun with the sky clear. These latter figures are approximately those obtained by Lewinson in Sept., 1908.[2]

With a full moon, the maximum normal and horizontal intensities during the night were 0.0388 and 0.0216 foot-candle, respectively. The rate of increase during the hour preceding sunrise is surprising. The intensity at about seven minutes before sunrise was approximately 10,000 times that of one hour previous. At about 25 minutes before sunrise and 25 minutes after sunset the illumination was approximately 2 foot-candles. It was also found that certain cloud formations have the effect of increasing the intensity of illumination by diffusion, while other clouds act as absorbing media and reduce the illumination intensity. Variations in illumination intensity due to clouds are often of a large order and sometimes occur suddenly, but in the absence of clouds the rate of change of intensity between the hours of 8 a. m. and 4 p. m. is regular. During the hours of dawn and twilight the rate of change of intensity is very high.

[1] Trans. Illum. Eng. Soc., Vol. 6, p. 793, Nov., 1911.

[2] Trans. Illum. Eng. Soc., Vol. 3, p. 482, 1908.

The intensity of illumination at night due to the stars and sky is about 0.001 foot-candle. The average intensity during the night due to a full moon is about 0.014 foot-candle. Sunlight illumination varies from 2000 to 9000 foot-candles between the hours of 8 a. m. and 4 p. m. (See also daylight values on page 15 and brightness of the sky on page 38.)

The intensity of *natural interior illumination* also varies over a considerable range. Mr. Marks,[1] in a series of tests of this nature, found men working by intensities of illumination varyingfrom 1.5 to 400 foot-candles with equal ease and satisfaction.

The unconsciousness which we have of these great variations in the amount of natural light by which we see and the lack of annoyance or discomfort under even extreme cases merely corroborates the great range of adaptability and accommodation of the human eye.

The maximum intensity of light in the direct range of vision which the eye can endure without annoyance and possible harm is about 4 or 5 candles per square inch. The greater the brightness or the larger the area of the source the more will the eye be affected. Only paraffin candles, oil lamps, and certain gas and kerosene lamps give a flame of intensity below this limit. Other sources of light having an intrinsic brightness above this value should be surrounded with a globe of diffusing material of such size and diffractiveness as to present an intensity not greater than 4 or 5 candles per square inch.

Luminous sources should be so placed that the rays will not pass directly into the eye. The result of sources thus wrongly located is that objects back of the source, in the case of direct rays, and the reflecting medium, if the rays are reflected, are more or less indistinct. Light coming from an unusual angle should be avoided. We receive our light usually from above and the retina becomes accustomed to light from that direction. Light reflected into the eye from below, as from snow or the direct rays as from foot-lights, may not only cause fatigue but sometimes temporary blindness. *Streaks of light* and *sharp contrasts* are injurious to the eye, the effect being similar to that of a flickering light.

During all work with *ultra-violet rays* it is necessary to protect the eyes with suitable glasses, since it has been found that exposure of the unprotected eye to light of short wave-lengths extending far into the ultra-violet produces very serious and often lasting injuries to this member. Much of the existing eye trouble is attributed to this cause.

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[1] Trans. Illum. Eng. Soc., Vol. 4, p. 828, Nov., 1909.

Brightness of Light Sources.—The intensities of some of the common sources of light in candle-power per square inch are given in the following table, composed of data compiled from various sources; chiefly from results by Dr. Ives and Mr. Luckiesh.[1]

TABLE 13

Brightness of Light Sources

Source		Candle-power per square inch
The Moore tube		0.6
Dark overcast sky		1.0
Blue sky		2.2
Overcast sky		3.3
Frosted incandescent lamp (25 watt tungsten)	Tip	1.67
	Side	6
Candle flame		2–4
Gas flame (fish tail)		2.7
Oil lamp		3–5
Cement pavement illuminated by sunlight		6
Kerosene lamp		9
Culumus sky		10.4
Cooper Hewitt lamp		14.9–16.7
Welsbach gas mantle		20–35
Welsbach mesh		53
Acetylene flame 1/4 ft. burner		33
Acetylene flame 1 ft. burner		53
Enclosed a. c. arc (depending on globe)		75–200
Enclosed d. c. arc (depending on globe)		100–500
Incandescent lamp (4 watts per candle)		325
Incandescent lamp (3.5 watts per candle)		400
Incandescent lamp 3.1 watts per candle)		480
Tantalum lamp (2 watts per candle)		580
Gem lamp (2.5 watts per candle)		750
Tungsten filament (1.25 watts per candle)		1060
Sun on the horizon		2000
Nernst glower (115 volts, 6 amps. d. c.)		3010
Magnetite arc		4000
Flaming arc		5000
Open arc lamp		10,000–50,000
Open arc crater		200,000
Sun 30 degrees above the horizon		500,000
Sun at zenith		600,000

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[1] *Elect. World*, Vol. 57, p. 438, Feb. 16, 1911; Trans. Illum. Eng. Soc., Vol. 6, p. 687, Oct., 1911.

While the spectrum of all illuminants extends into the region of the ultra-violet rays, the harmful effect of the ultra-violet radiation is made negligible due to the absorption of these rays by the enclosing glassware usually employed. When quartz glass is used as the medium for enclosing the luminous source, as in the case of the high temperature vapor lamps, an auxiliary globe of ordinary glass should be used since quartz glass is translucent to the ultra-violet radiation. None of the existing artificial illuminants thus enclosed with ordinary glass radiate ultra-violet rays of a value near so great as the amount of similar radiation from the sun for the same intensity of illumination.

Experience shows that at *one or two foot-candles* the eye is working so near its normal condition that any further increase in illumination is of relatively small value. The values here specified are those affecting the eye and not those by which the objects are illuminated. This must necessarily be reflected light and, as the coefficients of reflection and diffusion vary over a wide range, the calculation of the amount of light required from the primary source is not a simple problem. However, if the class of service, for which artificial light is to be used, be known, the approximate intensity of the source of light required can be calculated from general physiological and physical data. The lowest permissible illumination is, of course, for work on light colored objects or where only the general outlines of the objects are required while the highest illumination would be used for close discrimination of details on dark objects.

The subject of light and its physiological effect thus far has been considered in a general sense. A deeper study of it involves a distinction between terms often used indiscriminately. Moreover, with an understanding of the general features of the subject and the construction and operation of the eye we are in a position to more easily understand the essentials tending toward the establishment of a light scale for the average human eye in terms of radiation.

In the following discussion distinction is made between sensation, sensibility and visibility, as well as between radiation, illumination, luminosity, and visual brightness.

Sensation is the subjective effect produced by light while *visibility* refers in the objective sense to the acuteness of vision or the ability to distinguish detail. *Sensibility* may refer to the sensitiveness of the eye to changes in either subjective or objective light.

By means of visible radiation an object is illuminated, objective light, and the illumination is proportional to the radiation. *Luminosity* is the subjective equivalent of objective illumination. *Visual brightness* is the sensation produced by the illumination and is a logarithmic function of the luminosity.

Fechner's Law.—About 1830 Weber observed that the least perceptible increment to a stimulus affecting several of the sense organs, including the organs of vision, under fixed conditions of fatigue, attention, and expectation, bore a definite relation to the amount of that stimulus. Fechner later extended these investigations and formulated his results in the mathematical expression

$$dI/I = \text{constant} \tag{9}$$

where I is the value of the stimulus and dI the least perceptible increment.

This law appears to hold over a wide range of intensities, failing only at very low and exceedingly high values. Fechner proceeded further to assume that the above constant was proportional to the corresponding increment to the sensation or physiological conception of light and derived the expression

$$B = c(\log I/I_o) \tag{10}$$

where I is the intensity as above, and I_o is the threshold value or the least perceptible value of intensity, and c is a proportionality constant.

The values of I_o are given in the following table:

TABLE 14

Least Perceptible Illumination

Wave-lengths................	.430	.470	.505	.535	.575	.605	.670μ
Color of light................	violet	blue	green	greenish-yellow	yellow	orange	red
Least perceptible illumination (I_o.)	.00012	.00012	.00017	.00072	.0029	.0056	.06

Although these expressions have been extended[1] to apply to all values of stimuli and are rather lengthy, the fact remains that for ordinary intensities Fechner's equations will give very approximate results. Expressing Fechner's equations in words we have what is sometimes termed "*Fechner's law of sensation*," which, briefly stated, is this: The same percentage change in intensity of illumination, calculated from the least amount perceptible to the eye, gives the same change in sensation. In other words, the sensations produced by the optical nerves vary approximately as the logarithm of the value of the stimuli producing those sensations. This law, for light of the color of that from the most common illuminants, is represented graphically by the curve in Fig. 16 plotted with intensities in foot-candles as abscissæ and the values of the logarithms of intensities as ordinates. Referring to this curve it will be seen that the same percentage change in intensity produces the same change in sensation. Thus by increasing the intensity from 2 to 4 foot-candles the same change in sensation will be obtained as if the intensity were increased from 4 to 8 or from 20 to 40 foot-candles, the percentage increase being the same in all cases.

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[1] Bull. Bur. Stand., Vol. 5, p. 291, 1908.

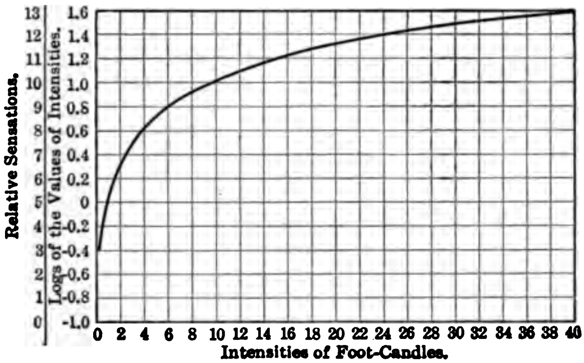

FIG. 16.—Relative sensations of brightness for different illumination intensities.

A study of this law will reveal the reason for the statement often seen in the technical press, that the effects produced by the use of additional lamps do not warrant the additional expenditure of energy. Thus it is obvious that this law is of much practical importance and should be understood by every man who pretends to handle illumination problems with engineering intelligence. It might be stated that this law fails to hold at illumination intensities of exceedingly high or low values but since these values are far removed from those used in practice the fact becomes of little consequence.

The visual sensibility of the eye to the same amount of radiation of various wave-lengths varies considerably with variations in the intensity of radiation. The region of maximum sensibility shifts with increasing intensity from the bluish-green over to the yellowish-green. This is shown by the curves of Fig. 17 by Dr. Bell, plotted from Konig's corrected data.[1] These curves show the sensibility of the eye as a function of the wave-length of light with a uniform distribution of energy in the spectrum. It will be seen that with increasing intensity the visibility curve broadens and the region of maximum visibility shifts toward the red end of the spectrum. An understanding of this variation of the visibility of radiation is of the utmost importance. It shows that the assumption of luminosity proportional to intensity can be true only under special conditions and with certain limitations. Fortunately, the greatest variations do not occur within the range of illumination intensities most common in practice.

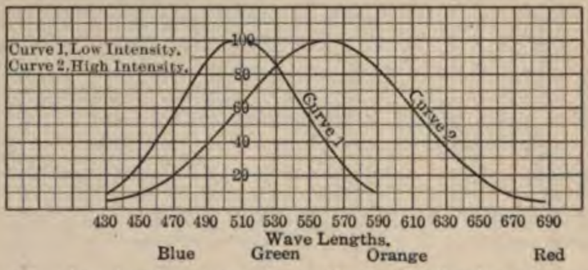

FIG. 17.—Luminosity curves for uniform distribution of energy.

Visual Acuity and Wave-length of Light.—The ability of the eye to distinguish detail depends very largely upon the character of the light which enters the eye, according to M. Luckiesh.[2]

This is attributed to the fact that the eye is not achromatic. It has been shown by Dr. Bell[3] and M. Luckiesh[4] that monochromatic light has greater defining power than light having an extended spectrum. There appear to have been differences in opinion as to the region of the spectrum having the greatest defining power. The work of Mr. Luckiesh using monochromatic light of equal intensity throughout the spectrum appears to fully establish this relation. The results of this investigation are shown in Fig. 18. Curves I, II, and III were obtained by the investigator and represent the average of a large number of readings.

[1] *Elect. World*, Vol. 57, p. 1163, May 11, 1911.

[2] *Elect. World*, Vol. 58, p. 1252, Nov. 18, 1911.

[3] *Elect. World*, Vol. 57, p. 1103, May 11, 1911.

[4] *Elect. World*, Vol. 58, p. 450, Aug. 19, 1911.

The readings for curves I and II were obtained in the morning and afternoon, respectively, of the same day, and illustrate the fact that visual acuity is quite a variable function. Curve III represents the mean for the observer's eye. Curves IV and V were obtained by two other observers and represent the mean of a less number of readings. The illumination maintained throughout the experiment was 4.2 foot-candles on a surface of magnesia. This was viewed through a 2.5 mm. pupil opening and eye-piece lens thus reducing its resultant brightness somewhat. The focusing distance of the observer's eye was about 14 in.

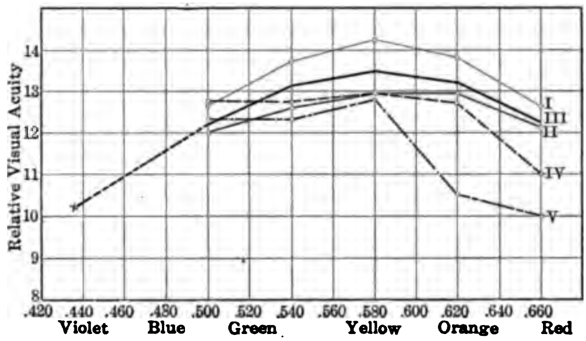

FIG. 18.—Variation of visual acuity with wave-length of light.

While it may seem to some that the acuity method is the only valid one for comparing the illuminating values of light, it must be remembered that in the case of general illumination the results obtained by this method would not give a correct measure of the luminosity of the radiation.

In the case of general illumination, the eye, as a rule, observes comparatively large objects which present comparatively small brightness differences, and not minimal-sized details presenting the highest practical contrast as typical print. The fact that monochromatic light produces a more distinct retinal image does not establish the conclusion that it is the better light for constant use in distinguishing fine detail. This advantage may be more than offset by the loss of the ability to discriminate colors. Moreover, a thorough study of the physiological effect of monochromatic light may show it to be more disturbing to the other functions of the eye than light which more clearly approaches in spectral character the natural light under which the eye has evolved.

It is only fair to state, at this point, that while the mercury-vapor lamp is the nearest approach to a monochromatic source of any of our artificial illuminants and while it is undesirable where color values are concerned all investigations of its physiological effects which have come to the notice of the writer indicate that its light is no more harmful than that from any other illuminant.

Relative Variation of Acuity and Brightness.—In the same investigation it was found that throughout a wide range of illumination visual acuity varied much less than visual brightness. The relative changes of the two is shown in Table 15 where the unit of relative brightness is 4.2 foot-candles as used above. It would appear from these data that for equal variations in intensity or brightness visual acuity varies more toward the extremities of the spectrum than in the central portion.

TABLE 15

Variation of Acuity with Brightness at Different Wave-lengths

Wave-length in $\mu\mu$	Relative brightness	Relative acuity[1]
660	1.00	1.00
	.25	.85
620	4.00	1.05
	1.00	1.00
	.25	.93
580	4.00	1.05
	1.00	1.00
	.25	.95
540	4.00	1.08
	1.00	1.00
	.25	.88
500	4.00	1.08
	1.00	1.00
	.25	.82
	1.00	1.00
	.25	.83

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[1] The values of relative acuity in the last column are comparable only with those in the same group. Groups cannot be compared with each other.

It has been shown by Luckiesh,[1] that brightness values of monochromatic light obtained by the visual acuity method cannot be added. A red light added to a blue light will give less acuity than either source alone. This is obvious because of the want of achromatism of the eye. This point is well explained by Dow.[2] Assuming the yellow rays to be brought to a focus on the retina, the violet rays, see Fig. 19, will be brought to a focus in front of it and the red rays behind it. It is obvious then that, if the eye cannot focus itself for a certain color of light, no amount of that light will make the observer see distinctly, and since it cannot focus itself for two different colors, it will doubtless focus on the resultant color of the mixture which will not be monochromatic and, consequently, will produce less acuity.

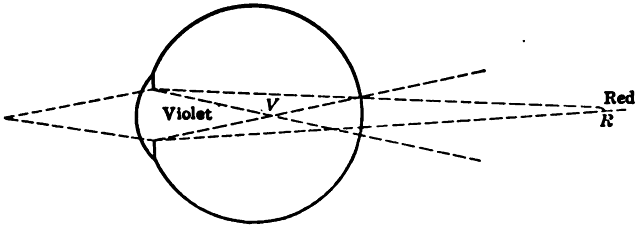

Fig. 19.—Diagram showing want of achromatism of the eye.

Psychological Effects.—The impressions received of light and illumination are liable to be distorted by influences of a psychological nature. The result of habitude is likely to influence the choice or preference of a particular illuminant or a particular type of installation. Public comment and personal opinions may prejudice the mind of an individual. The laws of association, suggestion, attention and expectation should be understood by the student of illuminating engineering. Any or all of these psychological peculiarities may operate to the annoyance of the practicing engineer or may be used by him to his advantage.

Since artificial light is used for the purpose of observing objects its effectiveness is judged from the impressions which it produces. The impression is usually the result of attention, stimulated perhaps by association, suggestion, or expectation. Attention will not attach itself firmly to uninteresting objects. It will decline if some change or some new attribute is not discovered in the object. Fatigue of the optical system results from the strain of continuous attention toward any one object and the impression becomes less intense. This may easily be proven by continuing one's gaze intently upon one particular object. When attention to one object becomes exhausted we obtain rest either by giving ourselves over to reflex attention or by directing our attention to other subjects.

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[1] *Elect. World,* Vol. 58, p. 450, Aug. 19, 1911.

[2] *Elect. World,* Vol. 58, p. 955, Oct. 14, 1911.

Attention continuously centered upon the same subject or upon an unchanging object has been proved by experiment to tend toward either a hypnotic state or a comatose condition. Thus it will be seen that in the installation of illumination systems as well as in the disposal of the areas illuminated the arrangement should be such that there will be contrast in the intensity of objects illuminated and in the intensity of the shadows produced.

A peculiar psychological phenomenon occurs, after looking at a brightly illuminated body, due to fatigue of the eye. If, for instance, after looking intently for some time at a red surface we turn our eyes to a white surface it will appear greenish-blue, or after looking through a green glass for a few moments white will appear a reddish-white. Another peculiarity of the optical system is that if alternate strips of black and white are looked at closely the white will appear much whiter by contrast with the black and a black body with a red background can be distinguished as to detail, at a greater distance than if the background were green. Moreover, as *illumination grows very bright*, all colored objects incline toward a whitish-yellow tint, which must gradually modify the quality of the sensation appropriate to that particular color of light. A red surface appears brighter than a blue one in daylight while the reverse occurs if these surfaces are viewed in weak daylight, when red may appear black while the blue will still be visible in its proper color. In a bright light, red, orange, and yellow surfaces are relatively more brightly illuminated than blue or violet surfaces, while just the opposite relations occur in a weak light.

Flicker Effect.—In all classes of illumination a *flickering light* should be avoided, since the iris cannot keep pace with the rapid fluctuations and too little and too much light falling alternately upon the retina causes fatigue. This is familiar to all who have endeavored to read by means of the light from an open gas flame or a 25 cycle arc lamp. It has been found that if repeated stimuli succeed each other within their period of persistence the sensation will be that of continuous light, but when the interval between the stimuli is nearly equal to the time of dying away of a sensation the light will appear to flicker. The frequency at which the flicker from an alternating source cannot be detected varies with the temperature of the luminous source and with the intensity of illumination of the object viewed.

To obtain the flicker-vanishing frequency Dr. Kennelly and Mr. Whiting[1] illuminated a white stationary target to different intensities and produced the flicker by means of a sectored disk placed between the illuminating source and the target. The lamp was placed at different distances from the target to produce the different intensities and the flicker frequency varied by changing the speed of the motor driving the disk. The average values of this investigation are given in Table 16.

TABLE 16

Intensity and Flicker

Meter-candles on target	2.065	3.67	5.29	8.26	14.69	33.04	132.2
Flicker-vanishing frequency	25.6	27.4	59.3	32.0	35.2	36.8	44.1

With such an arrangement the difference between maxima and minima will be 100 per cent. of maxima. It is obvious that flicker will disappear at a lower frequency with the same intensity as the difference between the maxima and minima decreases, and that the flicker will cease to exist when the difference in visual impression becomes less than the least perceptible difference in illumination. Hence the above data, which are for 100 per cent. flicker, will be somewhat high for light sources supplied with alternating current where the difference between maxima and minima is less.

Stroboscopic measurements of light fluctuations of incandescent lamps on alternating current circuits show that the variation of light throughout a half cycle is greater when the lamp operates at higher temperatures and also when the filament is of metal instead of carbon. While these merely corroborate the laws of radiation and temperature and possibly illustrate the influence of the smaller thermal capacity and higher thermal conductivity of the metal filament, they indicate a higher flicker-vanishing frequency than those given above and a somewhat higher vanishing frequency for carbon lamps than when lamps with metallic filaments are employed.

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[1] Nat. Elect. Light Assoc., Vol. 1, p. 327, 1907.

The results of tests of this nature by Dr. Sharp[1] showing the variation of light in per cent. of maxima for carbon, metallized and tungsten filament lamps are given in the following table:

TABLE 17

Stroboscopic Measurements of Light Fluctuations.

Frequency in cycles per second	15	20	25	30	35	40	45	50
16 candle-power carbon, 3.1 watts per candle	42.5	32.5	22	13	12	11.5	11	10.5
16 candle-power carbon, 2.5 watts per candle	47.5	36.5	25	18	17	15.5	14	12.5
Metallized filament, 2.5 watts per candle	52	41.5	31	24	22	19.5	17.5	15.5
Tungsten filament, 1.25 watts per candle	54	42	37	32	26.5	22.	....	....
8 candle-power carbon, 3.1 watts per candle	54	39	24	21	19.5	18.5	17.5	16.5
8 candle-power carbon, 2.5 watts per candle	65	52	37	25	23	21	19	17

Summary of Chapter III

It is shown by Fig. 15 that much of the visible radiation from our artificial illuminants is of low effective value. The radiation most efficient for illuminating purposes is between 0.5 and 0.6μ while by far the greater part of the visible radiation from incandescent sources lies between 0.6 and 0.8μ.

At low intensities highest visibility occurs around 0.5μ. The visibility curve assumes the general shape of the probability curve. As the intensity is increased the position of maximum visibility shifts into the region of longer wave-lengths and occurs at about $.545\mu$ at ordinary intensity for an equal energy spectrum. This peculiarity is explained by the theory of rod and cone vision. The rods being the predominating organs at low intensities are more sensitive to light of a bluish or greenish hue. The cones are the form-receiving and color-perceiving elements of the retina and are active at ordinary and high intensities.

The sensation produced by illumination is a logarithmic function of the luminosity. Hence sufficient light is enough.

A flickering light is objectionable. The vanishing-flicker frequency is a function of the intensity of illumination.

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[1] Trans. Amer. Inst. Elec. Engrs., Vol. 25, p. 839, Dec., 1906.

CHAPTER IV

STANDARDS OF LUMINOUS INTENSITY

Standards of luminous intensity or candle-power are devices for producing visible radiation of constant and measurable value. Those existing or proposed secure this constancy of luminous radiation by three methods.

1. By the first method we have the flame standards. In the construction and use of this class certain specifications are closely followed, and their accuracy is based on the assumption that if these specifications are followed the standard intensity will be reproduced. These specifications refer to such details as the size, material and weave of the wick, the size and construction of the burner and chimney, the composition and rate of combustion of the fuel, and the pressure and humidity of the atmosphere. All of the primary standards used to-day belong to this class.

2. By the second method the intensity of radiation is controlled by the observation or measurement of some accompanying physical variable, such as temperature. The Violle platinum standard, the arc standard, and the black body standard belong to this class.

3. By the third method it is proposed that the intensity of visible radiation be controlled by the direct measurement of the radiation, as radiation. To this class belong the three spectral line standard proposed by Steinmetz and the "definite quantity of the most efficient possible radiation" standard proposed by Ives.

The standard of luminous intensity was, for a long time, the light of a candle made to certain specifications and consumed at a given rate; and it has been the almost universal custom to refer the intensity of a light source to that of the candle and to give the value of its luminous intensity in terms of the candle or candle-power.

The demand for accurate and reproducible standards of light arose with the development of the two great industries of gas lighting and electric lighting. Gas photometricians and electrical engineers have worked at the problem quite independently and have been long in reaching a common understanding. Real progress in this field began only with the definite abandoning of the candle and the search for new sources of light.

A practical standard of luminous intensity should be simple in construction and operation, durable in all its parts, constant in operation, and capable of being reproduced with accuracy. All flame standards are subject to errors other than those due to the constituents involved. In 1859 Frankland and Tyndall made some interesting experiments concerning the effect of atmospheric pressure on the luminosity of a flame and the amount of combustible consumed. They found that the rate of combustion was unaffected by atmospheric pressure, but at low atmospheric pressures, such as occur at the top of Mt. Blanc, the candle became as non-luminous as the Bunsen burner, while by increasing the pressure an alcoholic flame becomes white and luminous. The conclusion was that the variation in the illuminating value of a flame depends chiefly on the access of atmospheric oxygen as regards the interior of the flame.

When a chemically inert gas or vapor is introduced into a luminous flame there usually results a decrease of its illuminating power. In this way non-combustible constituents of the atmosphere, such as aqueous vapor and carbon dioxide, may be expected to influence the light value of all flames, and as the proportions of these substances are all the time changing and especially so in the average photometer room, they must introduce errors in flame measurements. Methven found that a standard candle, burning at a constant rate of 120 grains per hour, gave an intensity of 1.196 units when burning in dry air while the intensity was only 1.104 units, or a decrease of 8 per cent., when burning in moist air under the same conditions of flame height and consumption.

The flame standards of luminous intensity which have been used to quite an extent in the past and some of which are still being used to some extent, especially in the gas industry, are the English and German candles, the Carcel lamp, the Methven screen, the kerosene standard, the pentane standard, and the Hefner lamp.

The English candle.—The English candle was made of spermaceti, carefully prepared from the oil of the sperm whale. It was subjected to various investigations, and extended series of measurements performed with great accuracy and care led to its almost universal condemnation as a standard of illumination. Its faults appeared to be the uncertainty in the composition of the spermaceti and in the weaving of the wick, the variation in the length and shape of the wick while burning, the filling and emptying of the cup of the candle, and the adventitious circumstances attending its use, which, as a whole, it is not possible to regulate or define.

The German Candle.—The German candle was constructed under unusually minute specifications as to both wick and combustible. The combustible was paraffin highly purified and having a melting point of 55° C. But paraffin being a mixture of indeterminate composition is unsuited for a standard of candle-power; moreover, the general features detrimental to the English candle apply to this candle as well.

In addition to the English and German candles there are a number of less widely used standard candles but they possess no essential differences from the two described. The unreliability of the candle lead to the development of more constant and trustworthy standards.

The Carcel Lamp.—The Carcel lamp having a central draft type of burner, known as the Argand, and burning oil furnishes a flame of remarkable steadiness and of marked whiteness. It was developed in 1802 by Carcel and attained great favor in France. It was fully investigated as a light standard, and its constants and dimensions carefully determined and defined. Until recently it was the recognized standard of that nation, where the illuminating power of light sources has been invariably expressed in terms of the Carcel unit. The combustible generally employed was Colza oil. This lamp was not, however, received with favor by the other countries.

The Methven Screen.—The Methven screen as a standard consists of an Argand burner gas lamp provided with a screen in which is an opening so placed as to be opposite the center of the flame. On this screen is mounted a thin slide with two rectangular openings, one for plain gas and the other for carburetted gas. The theory of this type of standard is that the light received from a definite part of a flame only would be more constant than if received from the whole flame. The height of the flame is gaged by two pins one on either side of the chimney. Careful investigations of the Methven screen proved it to be insufficiently constant and reproducible to serve as a primary standard of luminous intensity.

The Kerosene Standard.—The kerosene standard, of which the Elliot lamp is the most reliable, is worthy of consideration as a working standard. The Elliot lamp is of the student type having a flat wick, and equipped with a screen having a rectangular opening. Some little difficulty is experienced, however, in keeping the chimney clean and some skill and patience are necessary in trimming the wick to get a good flame. If these two points are given careful attention and good light oil is used the lamp will operate for seven or eight hours without changing more than a fraction of 1 per cent. in candle-power. When heavy oil is used the initial intensity is approximately the same but a greater decrease in candle-power may be expected during its use. Mr. Crittenden[1] is of the opinion that as a standard its intensity can be reproduced from day to day with less variation than that of the standard candle and states that the reproducibility of intensity can be improved by a slight modification of the regular lamp. This change consists in making the top of the slot in the screen somewhat lower than that of the standard lamp, or by adjusting the flame so that its top is more than the specified quarter inch above the top of the slot. In either case a certain height of flame can be found where the candle-power is a maximum and small variation in the height of the flame does not make so much difference in the intensity.

This standard is affected by atmospheric changes similar to the other flame standards, but it possesses certain advantages over them. Its intensity does not fluctuate; it is not affected by drafts; and its color is in its favor.

There are two uses for which this standard is well suited, since its intensity is approximately constant for several hours when once adjusted. One use is in the photometry of sources of light where the highest degree of accuracy is not required, since it has the advantage of requiring no attention during a period of actual photometric work and being previously calibrated will be sufficiently accurate where the results are not likely to form the basis of controversy. The other application is in the substitution method as a secondary standard where it may be placed at one end of the photometer bar and the standard lamp at the other end of the bar where it may be replaced by the lamps to be measured after checking the intensity of the kerosene lamp.

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[1] Trans. Illum. Eng. Soc., Vol., 6 p. 417, 1911.

The Hefner Lamp.—The two most successful primary flame standards are the Hefner amyl acetate lamp and the Harcourt pentane lamp. The Hefner lamp is especially favored in Germany and is the official standard recognized by the Physikalisch-Technische Reichsanstalt. It has been subjected to thorough and accurate investigations and its faults as well as its merits are clearly understood.

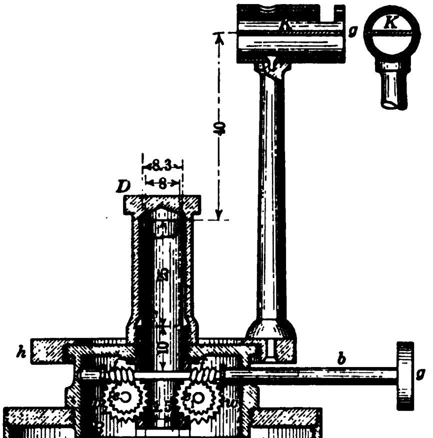

Fig. 20.—The Hefner lamp.

The Reichsanstalt lamp has been universally adopted as the standard for the use of amyl acetate. The material of construction is of brass except the wick-tube which is of German silver to prevent corrosion by the combustible. The wick is moved by means of a worm gear which actuates two spur wheels shown in Fig. 20. All of the fittings of the lamp are attached to the wick tube which unscrews from the cup for filling. The character of the wick has practically no influence on the candle-power of the lamp, since amyl acetate vaporizes at such a low temperature that the wick does not project into the flame and burn. A metal cap is kept screwed down over this wick when the lamp is not in use.

A test gage is furnished with each lamp for the purpose of adjusting the height of the flame, which burns with greatest constancy and steadiness when about 40 millimeters in height. Accuracy in setting the flame is essential since a variation of 1 millimeter in height will produce about 3 per cent. difference in the illuminating power. Under the above conditions and with normal atmospheric pressure and humidity the horizontal intensity of the flame is 0.9 candle-power.

The luminous intensity of the Hefner lamp, according to Liebenthal,[1] can be expressed as a linear function of the flame height, the equations for which are:

$$I = 1 + 0.025(h - 40) \tag{11}$$

and

$$I = 1 - 0.034(40 - h) \tag{12}$$

for heights of flames greater than 40 millimeters and less than 40 millimeters respectively.

It was also found that an increase or decrease of 1 millimeter in the diameter of the wick tube caused a variation of 1 per cent. in the light intensity.

Herr von Hefner-Alteneck, the inventor of this lamp, pointed out and insisted on the necessity of using a combustible of known chemical composition. Amyl acetate, because of its chemical simplicity and definite composition, is excellently fitted for the combustible of a standard. It is a colorless liquid and has the chemical constitution of $C_7H_{14}O_2$ and burns with a clear flame of not great brilliancy and of a somewhat reddish color; it is obtained from the distillation of amyl alcohol obtained from fusil oil, with a mixture of acetic and sulphuric acids, or by distilling a mixture of ethyl alcohol, sulphuric acid, and potassium acetate. The presence of impurities, which are most common in amyl acetate, has but very little effect upon the value of the light from this lamp. The results of tests to ascertain the influence of these impurities are given in the following table·

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[1] *Elek. Zeit.*, Vol. 9, p. 975, 1888.

TABLE 18

Effect of Impurities in Amyl Acetate

Amyl acetate diluted with	Specific gravity	Hourly consumption in grains	Deviation from normal	
			consumption in per cent.	intensity in per cent.
20 per cent. fusil oil	0.8645	9.96	+6.9	−2.0
2 per cent. diamylen	0.8725	9.24	−0.8	0.0
5 per cent. alcohol and 4 per cent. castor oil	0.8745	9.88	+6.0	Impossible to determine.
10 per cent. isobutyl acetate and 10 per cent. amyl alcohol	0.869	9.28	−0.4	+0.4
50 per cent. alcohol	0.8408	12.92	+39.0	+40.0
Pure amyl acetate	0.8735	9.318		

Amyl acetate for photometric purposes should be chemically pure; the tests to determine this condition as prescribed by the Reichsanstalt are as follows:

1. The specific gravity at 15° C. should be from 0.872 to 0.876.

2. Ninety per cent. should distill between the temperature limits of 137 and 143° C.

3. The reaction should be practically neutral.

4. It should mix with equal volumes of ether, benzine, or carbon-bisulphide, without becoming milky.

5. A clear solution should result upon shaking in a test tube, 1 cu. cm. amyl acetate with 10 cu. cm. ethyl alcohol, 90 per cent. Trallar, and 10 cu. cm. of water.

6. A drop placed on white filter paper should evaporate without leaving a grease spot.

Amyl acetate should be kept in a glass-stoppered bottle and, as it has a tendency to decompose in strong light, it should be stored in a dark place, or in opaque vessels.

The effects of temperature due to the lamp itself are rendered negligible by the thinness of the walls of the tube, while atmospheric temperatures have no discernible influences.

Atmospheric moisture, however, is not without its effect. Data showing the monthly averages for the light of the Hefner lamp for one year, vary from 101.9 per cent. normal candle-power for February to 97 per cent. for July. During March, April, May, October, and November the average intensity was normal. For June, July, August, and September the average was about

2 per cent. below normal and for December, January, and February the average was about 2 per cent. high. The results of tests extending through a period of two years showed a maximum difference of 8.5 per cent. or 103.3 per cent. in January and February, and 94.8 per cent. in May and July. Hence this lamp can be relied upon to give values within 4 per cent. of the normal standard candle-power. The equation correcting these results for humidity is

$$y = 1.049 - 0.0055x \tag{13}$$

where y is the illuminating power of the lamp with x liters of moisture to the cubic meter of air, free from carbon dioxide and at normal pressure. The intensity decreases uniformly by an amount equal to 0.55 per cent. per liter.

Carbon dioxide has an effect similar to humidity. The expression, correcting for its effect on the candle-power between the limits of 0.6 and 13.7 liters per cubic meter is

$$y' = 1.012 - 0.0072x \tag{14}$$

y' being the candle-power, and x' the number of liters of carbon dioxide to the cubic meter of dry air, hence a variation of one liter of carbon dioxide to the cubic meter of air causes a change of 0.7 per cent. in the candle-power.

By comparing the equations for moisture and carbon dioxide corrections it will be seen that the effect is not the same but as 1 to 1.3 for the same amount of each in the air. Yet in reality, the effect of carbon dioxide is slight compared with the influence of moisture because of the smaller quantity present. These equations show that the lamp has unit candle-power when there are 9 liters of moisture and 1.7 liters of carbon dioxide to the cubic meter of air. In well ventilated photometer rooms the amount of carbon dioxide present will not cause a change in intensity of more than 0.2 per cent. but in small rooms and especially in all enclosed photometric apparatus there may arise considerable error, since the air in them is not only soon vitiated to a marked degree by the addition of water vapor and carbon dioxide but by the removal of the oxygen.

The effect of atmospheric pressure has been investigated by Butterfield, Haldane, and Trotter.[1] Their results are represented graphically in Fig. 21. The candle-power values are shown in per cent. of normal at a pressure of 760 mm. Variations for both the Hefner and the pentane (which will be referred to later) lamps are given in this figure. It will be seen that these curves are nearly straight lines between 700 and 850 mm.; within these limits 1 per cent. increase or decrease in intensity is produced by an increase or decrease of 25 mm. in the case of the Hefner and by 12.5 mm. in the case of the pentane lamp.

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[1] *Electrician* (Lond.), Aug. 11, 1911.

Tests as to the accuracy and constancy of the Hefner lamp have given very satisfactory results. The chief objection to this lamp as a standard is that the flame is too red in color and this is liable to introduce errors in photometry.

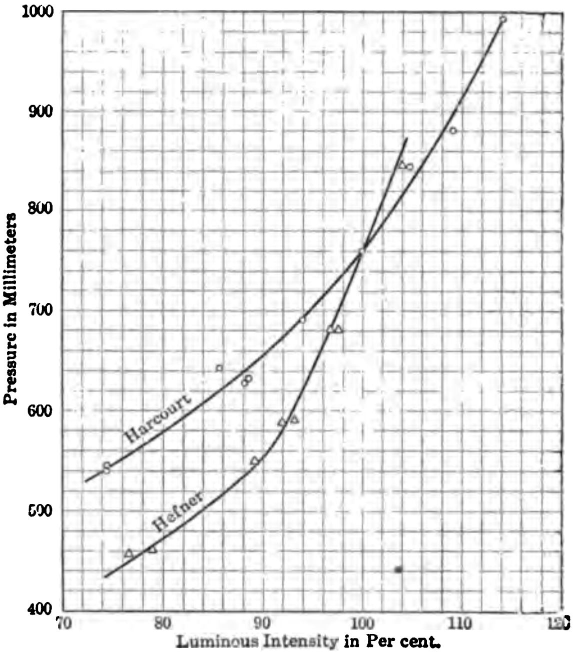

Fig. 21.—Effect of variation of pressure.

The Harcourt Pentane Lamp.—The Harcourt 10 candle-power pentane standard[1] is used extensively in England and is the official standard of the London Gas Referees and the National Physical Laboratory. This lamp is less convenient than the Hefner, relatively complicated in construction, and requires a much better ventilated photometer room. On the other hand, its greater candle-power (100/9), steadier flame and better color are decided advantages. Moreover, the candle-power of a particular pentane lamp is more accurately reproducible than that of the Hefner, when both are operated under proper conditions, but there is a greater difference among different pentane lamps than among different Hefners.

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[1] Abstracted from the Trans. of the Illum. Eng. Soc., Vol. 5, p. 753, 1910.

No wick is used in the pentane lamp. The pentane (C_5H_{12}) is contained in a reservoir or saturator. This saturator, Fig. 22,[1] is divided by vertical vanes into a maze through which air passes and becomes mixed with the pentane which vaporizes very readily. This mixture passes through the supply pipe to the burner. In hot weather the pentane may evaporate so rapidly that air will not enter the inlet to the saturator and only pentane vapor will be fed into the flame. Air, heated by passing through

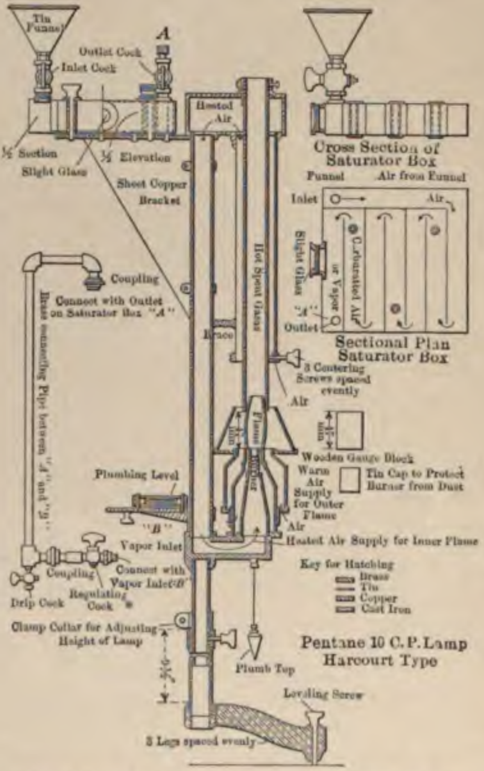
Tin Funnel
A
Outlet Cock
Inlet Cock
Heated Air
½ Section
½ Elevation
Slight Glass
Sheet Copper Bracket
Cross Section of Saturator Box
Funnel
Air from Funnel
Inlet
Air
Slight Glass
Carburated Air or Vapor
"A"
Outlet
Sectional Plan Saturator Box
Hot Spent Gases
Coupling
Connect with Outlet on Saturator Box "A"
Brass connecting Pipe between "A" and "B"
Brace
3 Centering Screws spaced evenly
Air
Flame
Wooden Gauge Block
Warm Air Supply for Outer Flame
Tin Cap to Protect Burner from Dust
Plumbing Level
"B"
Vapor Inlet
Air
Heated Air Supply for Inner Flame
Coupling
Connect with Vapor Inlet "B"
Regulating Cock
Drip Cock
Key for Hatching
Brass
Tin
Copper
Cast Iron
Clamp Collar for Adjusting Height of Lamp
Plumb Top
Pentane 10 C.P. Lamp Harcourt Type
Leveling Screw
3 Legs spaced evenly

FIG. 22.—Sectional view of pentane lamp.

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[1] Trans. Illum. Eng. Soc., Vol. 3, p. 133, 1908.

the annular space between the inner and outer chimneys, passes on down through the hollow standard and into the central chamber below the burner. This air so heated rises feeding the interior of the flame resulting from the combustion of the vapor of pentane as it issues from the ring of 30 holes in the steatite burner.

The height of the flame is determined by the distance between the burner and the chimney, and by the rate of supply of the combustible which is regulated by means of a stop-cock. Adjustments are made for a flame 47 mm. in height which is the condition of maximum intensity. The position of the tip of the flame indicates the proper intensity of the source and it must be watched and frequently adjusted in work where precision is desired.

Like all flame sources the intensity of the flame depends upon the dimensions of the lamp, the composition of the combustible, the condition of the atmosphere in which it burns, and the manipulation of the lamp, especially in regard to the flame height and the screening of the flame.

The specifications of the standard lamp were carefully drawn but were not sufficiently exact and complete to produce lamps that do not differ appreciably in candle-power, or within 2 or 3 per cent. The candle-power of this lamp built to the general specifications will usually fall a little short of 10 candles or from 9.6 to 9.8 candles, even when the humidity, atmospheric pressure, combustible and all external conditions are right.

The pentane lamp increases in candle-power for a short time after lighting and then decreases for a time necessitating a wait of from 15 to 30 minutes before recording measurements. The maximum is from 1.5 to 3.0 per cent. above the final value. The height of the chimney is a very important dimension but this differs little among different lamps. The cooling of the flame due to the conduction of heat, varies with the amount of metal in the region of the burner and with the details of construction.

The correction for humidity for the pentane lamp is 0.57 per cent. per liter of water vapor per cubic meter of air. The equation for the candle-power of a pentane lamp is, according to Rosa and Crittenden:

$$I = I_n[1 - 0.00567(e - 8) + 0.0006(p - 760)] \qquad (15)$$

where I is the candle-power with e liters of water vapor per cubic meter of air and a reading of p millimeters on the barometer, and

I_n is the normal candle-power with 8 liters of water vapor per cubic liter of air and an atmospheric pressure of 760 millimeters of mercury. For variations in intensity with atmospheric changes in pressure see Fig. 21 and discussion of same for the Hefner lamp.

Color Values of Flame Standards.—The color values of the various flame standards thus far described are given in the following table[1] together with the voltage of a carbon filament lamp, consuming 4 watts per candle at 107 volts, when matching these in color.

TABLE 19

Color Values of Flame Standards

	Ives colorimeter			Comparison voltage
	Red	Green	Blue	
4-watt carbon lamp	100	100	100	107
Flat gas flame (coal gas)	100	100	100	107
Kerosene standard, light oil	100	101	102	108
Kerosene stanard, heavy oil	100	96	85	100
Carcel lamp	100	94	76	96
Pentane lamp	100	91	68	91
Candles	100	88	61	87
Hefner lamp	100	87.5	59	86

The Violle Platinum Standard.—The Violle standard consisted of the luminous radiation from one square centimeter of incandescent platinum at the temperature of liquification. In some of the earlier investigations of this standard a lower temperature was used but under these conditions it proved unsatisfactory. The rapid disintegration of the platinum gave considerable change in temperature for a given value of current. This unit, however, has much to recommend it. The temperature of the melting point of a pure metal can be accurately defined and reproduced. The composition of the light from melting platinum corresponds more nearly with that of modern light sources than does the light from the other standards previously mentioned.

[1] Crittenden, Trans. Illum. Eng. Soc., Vol. 6, p. 417, 1911.

The larger value (20 bougies or candles) is another advantage. Experiments by Petanel[1] in which he used hydrogen instead of coal gas in the blast lamp for melting the platinum, indicate that Violle's standard is neither impractical nor inaccurate.

The *Bougie candle* is equal to 0.05 of the Violle standard or to the light from 5 square millimeters of incandescent platinum at the temperature of liquification.

The relation of these various standards to one another and to the international candle are given below:

TABLE 20

Relation of the Various Standards

Standard	International candle	Hefner unit	Bougie decim.	Violle platinum standard	Harcourt 10 c.-p. pentane lamp	Carcel lamp	English candle	German candle
International candle.	1.0	1.11	1.0	0.05	0.1	0.104	0.96	0.95
Hefner unit..	0.9	1.0	0.9	0.045	0.09	0.0936	0.864	0.855
Bougie decimale.	1.0	1.11	1.0	0.05	0.1	0.104	0.96	0.95
Violle platinum standard.	20	22.2	20	1.0	2.0	2.08	19.2	19.0
Harcourt 10 c.-p. pentane lamp.	10	11.1	10	0.5	1.0	1.04	9.6	9.5
Carcel lamp..	9.61	10.66	9.6	0.48	0.96	1.0	9.24	9.19
English candle.	1.04	1.154	1.04	0.052	0.104	0.1	1.0	0.98
German candle.	1.055	1.17	1.055	0.053	0.105	0.109	1.02	1.0

The Arc Standard.—The arc standard is the name applied to a standard in which it is proposed to use the light from the positive crater of a carbon arc. It has been found that the intrinsic brightness of the positive crater between pure carbon electrodes possesses remarkable constancy with varying specific consumptions of energy.[2] The question of reproducibility appears to be dependent only on the location of the crater and the use of a standard quality of carbon electrode.

The Black Body Standard.—It has been proposed to define the unit of light as the physiological effect of the equivalent of one watt of visible radiation. Since this requires a definition of the distribution of energy throughout the visible region, it was further suggested to accept the visible radiation from a black body at such a temperature that the ratio of the radiation in two definite regions of the spectrum have a specified relation.

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[1] Proceedings of the Royal Society, Vol. 65, p. 469.

Proc. of the Royal Soc., Vol. 27, p. 157; *Bull. Bur. Stand.*, Vol. 1, p. 109.

It is obvious that such a source possesses unfavorable characteristics, such as the deviation of the radiator from a black body; unequal absorption of the glass envelope to energy of different wave-lengths; liability of inaccuracy in locating the proper limit in the red portion which would seriously affect the results because of the magnitude of the radiation in this region compared with the physiological effect; and the color of the light which must be yellow instead of white because of the temperature limitations imposed by the radiator.

Object of a Primary Standard.—The object of the primary standard is obviously not to obtain a working standard for laboratory purposes but rather a reproducible unit to which reference may be made from time to time to prevent any drifting in one way or another which is likely to occur in the status of the arbitrary standard of luminous intensity in vogue at the present time. None of the primary standards already mentioned are by any means satisfactory, both from the scientific and the practical sides. They are in no way connected with the fundamental units of space, mass, and time. Until thus coördinated they can be considered as little more than convenient arbitrary references to be used in practice, pending the determinations of their real values.

The sensation of light is caused by the impinging of radiant energy upon the retina, consequently it seems only logical that the scientific specifications for the standard of light should be based on the measurement of radiation.

The complexities and difficulties arising in connection with the development of such a standard are obvious. It was shown in the preceding chapter that mere intensity of radiation is not an indication of luminous intensity, for not only is a large part of the radiation from most artificial light sources invisible, but even the visible part varies greatly in effectiveness in producing the sensation of brightness. Moreover, the mode of action of the eye is such that the luminous equivalent of radiation of a certain spectral wave-length varies with the absolute intensity.

The Proposed Radiation Standard.—A primary standard based on the measurement of radiation was proposed by Dr. Steinmetz[1] in 1908. He suggested the selection and mixture of three primary colors of the visible spectrum, namely blue, green, and red in definite proportions, so chosen as to give a white or yellowish-white light. A comprehensive treatment of possible standards of this type is given in a paper by Dr. Ives,[2] to which the reader is referred for greater details. In this paper it is stated, in reference to a possible three-line standard, that from the standpoint of purity in respect to the primary sensations, the colors possessing the nearest claims to primary colors are a red of greater wave-length than 0.64μ, a green of 0.506μ, and a blue of 0.48μ. These wave-lengths are almost exactly furnished by an arc between cadmium electrodes. Moreover, it is these three cadmium lines, red (0.6439μ), green (0.5086μ) and blue (0.4800μ), that are the standards of wave-length in terms of which the standard meter bar at Paris has been calibrated by Michelson. It is suggested that if a three-line standard ever be considered seriously, it would be well to consider the advisability of having the same source which furnishes our ultimate standard of length also furnish our ultimate standard of luminous intensity.

While the three-line standard permits the making of a standard of any desired color, the specification of the radiation-light relationship is complicated and spectroscopic, radiometric, and photometric difficulties render it probably impractical.

Dr. Ives suggests as a standard a *definite quantity of the most efficient possible radiation.* This could be achieved practically by measuring both as radiation and light a monochromatic radiation whose luminous efficiency is known. The green radiation of mercury actually corresponds, within the limits of present measurements, to the most efficient possible radiation and suggests itself as a practical means of obtaining this standard. It may be obtained from the quartz mercury arc in sufficient quantity for both radiometric and photometric purposes and may be easily isolated by color screens. No spectroscopic device is necessary for separating this line from the others. A sheet of neodymium glass will obstruct the neighboring yellow lines. A solution of potassium bichromate in a glass trough will obstruct the blue, violet, and ultra-violet radiation.

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[1] Trans. Amer. Inst. Elect. Engs., Vol. 27, p. 1319, 1908.

[2] Trans. Illum. Eng. Soc., Vol. 6, p. 258, April, 1911.

Thus an absorption cell can be easily prepared through which green radiation will pass in great intensity and purity.

Such a standard has much in its favor. Compared with a three- or two-line standard its theoretical simplicity is evident. It possesses the simplest possible relationship between radiation and light. Its establishment as a primary standard would make the unit of light the watt per square centimeter of the most efficient possible radiation. The lumens emitted by any given source would be obtained at once by multiplying its consumption in watts by the "reduced luminous efficiency" of the source.

Recently also Houstoun[1] and Strache[2] have suggested the specification of a standard in terms of radiation weighted as light. Houstoun would do this by a special absorbing screen. Strache by a template over the spectrum, which is then recombined. Both of these suggestions when analysed, consist in specifying the light standard by the least quantity of radiated energy which can produce the standard intensity.

The Incandescent Lamp as a Comparison Standard.—The incandescent lamp, aside from its illuminating value, is of inestimable value as a secondary or comparison standard of light. The incandescent lamp has probably been of more service in photometry than any other source of light, because of the constancy of the luminous intensity of a particular filament under proper conditions. This has made possible concordant data and a quantitative knowledge of the variations in flame standards. The color of the light from an incandescent lamp, especially if the lamp has a metallic filament, can be varied by changing the voltage so as to resemble the compared light in nearly all cases. They may be moved along the photometer bar at the pleasure of the operator and are very convenient as to portability and for use in a portable photometer where a low voltage lamp may be used in conjunction with a storage cell.

The illuminating power of a lamp to be used as a standard should be determined by comparison with the standards maintained at the Bureau of Standards when possible. If such a course is not feasible it may be calibrated by means of a Hefner lamp and corrections made for atmospheric conditions. It is well to keep one or more carefully standardized lamps for occasionally checking the working standards. Care should be exercised to avoid the temporary set or hysteresis in the resistance of the filament and especially the permanent set due to abnormally high voltages. A lamp should not have voltages impressed upon it much in excess of that at which it is aged and calibrated.

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[1] Proc. Royal Soc., Vol. 85, p. 275, 1911.

[2] *Zeit. f. Beleucht.*, 1911.

It is necessary to know the position of the lamp relative to the photometric axis when being standardized, and the voltage, current or wattage at which it was standardized in order to be able to reproduce known conditions. Incandescent lamps are usually operated as standards at a constant voltage, the current being used as a check. Sometimes they are operated at constant current and the voltage varied slightly if necessary. The method would be immaterial provided there was no change in the resistance of the filament during its period of operation. But since the resistance of a carbon filament decreases during the first part of its life and increases during the later periods it becomes a question of prime importance whether the best performance of a lamp can be secured during its useful life as a standard by operating it at constant voltage, constant current or at constant wattage. It is obvious that the most constant candle-power would be obtained by operating the lamp at constant wattage if there occurred no change in the surface of the filament or blackening of the bulb, since at a constant rate of energy supply there would result a constant light flux. But it is well known that there is a change in the surface of the filament and a greater or less increase in the absorption of the bulb due to blackening, which cause a decrease in candle-power during the life of a lamp.

At constant voltage the candle-power will increase during the period of decreasing resistance and decrease after the resistance has become a minimum. At constant current the candle-power decreases till the resistance becomes a minimum and then increases, but the blackening of the bulb and the decrease in emissivity of the filament decrease the candle-power and counterbalance to some extent the increase in candle-power at constant current due to the increase in the resistance of the filament.

From the above it may be expected that the life performance with respect to intensity will be more constant at constant wattage, and such a decision is corroborated by the results of experimental tests which show that the best performance of a standard will be obtained by operating it at such voltage or current as to maintain the watts constant. In the case of a carbon filament lamp there is slightly less liability of error due to voltage fluctuations by maintaining the watts constant by means of the current because of the negative temperature coefficient of resistivity of the carbon filament.

The carbon lamp has been used the most extensively as a secondary standard but the tungsten lamp, possesses properties more favorable and characteristics more desirable for a secondary standard of luminous intensity. The advantages of the tungsten lamp are (1) the greater constancy of candle-power through its useful life even when operating at its normal efficacy; (2) the greater range in color values which it is possible to obtain without subjecting the lamp to abnormal conditions, and (3) less variations in candle-power with changes in voltage because of the positive temperature coefficient of resistivity. Reports of cases where this lamp has been used as a working standard are most favorable and indicate an extensive adoption for this class of work. Unlike the carbon lamp, it is obvious that greater constancy of intensity at constant wattage may be maintained with fluctuating voltage through the agency of the electromotive force than by means of the current because of the positive temperature coefficient.

It can be seen from the life curves of incandescent lamps that a lamp is not suitable for a standard until it has been in operation for 50 or 100 hours. After burning about this length of time its performance becomes less variable, and it is at this period of its existence that it should be standardized.

In photometry it often becomes desirable where exact voltage regulation is impossible, to correct the values of intensity obtained for variations in voltage. In order to do this a knowledge of the performance of the unit under those conditions becomes desirable. It has been found that for incandescent lamps the intensity can be represented over a small range by the equation

$$I = aV^k \qquad (16)$$

where I is the candle-power, V the voltage, and a and k are constants. If I_o be the candle-power at a voltage V_o then the candle-power I_x at some other voltage E_x may be found by the expression

$$I_x = \left(\frac{V_x}{V_o}\right)^k I_o \qquad (17)$$

In order to use this equation it becomes necessary to know the value of the exponent k which is a function of the specific consumption of the lamp. Determinations of this nature as obtained by Cady[1] are given in Table 21.

TABLE 21

Values of k for Different Filaments at Ordinary Specific Consumptions

Type of filament	Watts per horizontal candle-power	Watts per mean spherical candle-power	k
Untreated carbon	4.0	4.85	6.6
Treated carbon	3.1	3.76	5.4
Graphitized carbon	2.5	3.12	4.8
Tantalum	2.0	2.5	4.0
Osmium	1.5	1.85	3.8
Tungsten	1.25	1.56	3.6

Summary of Chapter IV

A standard of luminous intensity should be simple in construction and operation, durable in all its parts, constant in operation, and capable of being reproduced with accuracy.

The various flame standards of greater or less importance are:

1. The English candle made of spermaceti.
2. The German candle made of paraffin.
3. The Carcel lamp having a central draft burner and burning colza oil.
4. The Methven screen, an Argand burner gas lamp with the outer portions of the flame screened.
5. The kerosene standard having a flat flame, "student" feed, and the outer portions of the flame screened.
6. The Hefner lamp burning amyl acetate.
7. The pentane lamp consuming pentane vapor.

All flame standards are affected by atmospheric changes. The Hefner and the pentane lamps are the most reliable and their variations have been investigated and determined.

The Violle platinum standard, the arc standard and the proposed radiation standards possess possibilities as primary standards but will scarcely be practical as working standards.

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[1] Trans. Illum. Eng. Soc., Vol. 3, p. 549, 1908.

The incandescent lamp is of inestimable value as a secondary standard. Its intensity is kept most constant during its life by maintaining it at constant wattage. Sensibility of the instruments being the same, a carbon lamp can be maintained at constant wattage more accurately by means of an ammeter and the tungsten lamp by means of a voltmeter.

CHAPTER V

PHOTOMETRIC UNITS AND NOMENCLATURE

The units of illumination and photometry in vogue in this country are based on a system of units adopted at the International Congress of Electricians at Geneva in 1896.

This assembly accepted the bougie (candle) as the unit of luminous intensity, which unit should be equal to the luminous intensity of five square millimeters of incandescent platinum at the point of solidification. This was intended to be one-twentieth of the Violle standard, which consisted of 1 square centimeter of platinum under the same conditions.

The unit of illumination intensity was termed the lux and was that intensity of light on a normal plane 1 meter away from a source of one bougie.

The lumen or unit of flux was that light on one square meter having a uniform intensity of one lux.

The lumen-hour was the product of unit flux by unit time.

These units have been modified somewhat and new terms and definitions added to clarify the relations between the various photometric quantities.[1] A general résumé of the efforts and recommendations of those organizations most directly interested is given in the following discussion:

Due to an international agreement between the national laboratories and many of the interested engineering organizations of England, Germany, France and the Uuited States, a common unit of candle-power known as the international candle came into existence in 1909.

Previous to this time the value of the unit at the Bureau of Standards, maintained through the medium of incandescent lamps, was intended to be in agreement with the British unit or 100/88 of the Hefner unit of Germany.

The unit of light at the National Physical Laboratory of

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[1] The American Institute of Electrical Engineers, Transactions, Vol. 30, p. 1923, July, 1911; The Illuminating Engineering Society, Transactions, Vol. 5, p. 473, June, 1910; and The Bureau of Standards, Bulletin, Vol. 6, p. 543, Nov., 1910.

London was 0.1 that given by the 10 candle-power Harcourt pentane lamp burning at normal barometric pressure (760 mm.) in an atmosphere containing 8 liters of water vapor per cubic meter.

The unit of light at the Physikalisch Technische Reichanstalt was that given by the Hefner lamp burning at normal barometric pressure in an atmosphere containing 8.8 liters of water vapor per cubic meter.

The unit of light at the Laboratoire Central of Paris was the bougie decimale or 0.05 of the Violle standard and was taken equal to 0.104 of the Carcel lamp. The bougie decimale had the same value within the limits of experimental error as the pentane unit, was 1.6 per cent. less than the unit of the United States and 11 per cent. greater than the Hefner unit.

In order to establish a common unit the Bureau of Standards proposed to reduce the value of its candle 1.6 per cent. provided Great Britain and France would unite with the United States in keeping the common value constant and with the approval of the other nations call it *"the international candle."*

Consequently, with the acceptance of this proposal the unit of candle-power in this country became of the same value as the unit of England, the bougie candle of France and 1.11 times the Hefner unit of Germany. Therefore the Hefner unit is equal to 0.90 of the candle.

The luminous intensity of a source of light (I) is expressed in *candle-power* or *candles*, and refers to the intensity of light in a certain direction. The candle-power of a source in a given direction is equal to the normal incident illumination (E_n) in foot-candles on a surface normal to that direction multiplied by the square of the distance in feet.

$$I = E_n r^2 = F/\omega \qquad (18)$$

The *unit* of *candle-power* should be derived from the standards maintained at the Bureau of Standards at Washington, D. C.

The *mean horizontal candle-power* (I_h) of a lamp refers to the mean of the candle-power values around a lamp in a horizontal plane through its luminous center.

The *mean spherical candle-power* (I_{ms}) is the mean of the intensities radiating in all directions from the source as a center. It is equal to the light flux in lumens divided by 4π

$$I_{ms} = F_s/4\pi \qquad (19)$$

Since there are 4π solid angles emanating from a point the mean spherical candle-power equals the average flux in lumens per unit solid angle.

The *mean lower hemispherical candle-power* (I_{lh}) and the *mean upper hemispherical candle-power* (I_{uh}) refer to the mean of the intensities in all directions below and above a horizontal plane through the source. These values are also equal to the flux in the respective hemispheres divided by 2π.

$$I_{lh} = F_{lh}/2\pi, \text{ and } I_{uh} = F_{uh}/2\pi \qquad (20)$$

The *spherical reduction-factor* is the ratio of the mean spherical candle-power to the mean horizontal intensity. Hence the mean spherical candle-power is equal to the product of the mean horizontal candle-power, and the spherical reduction-factor. The total flux of light in lumens is equal to 4π multiplied by the product of the mean horizontal candle-power and the spherical reduction-factor.

$$I_{ms} = fI_h \text{ and } F = 4\pi f I_h \qquad (21)$$

The spherical reduction-factor should only be used when properly determined for the particular type and characteristics of each lamp. The spherical reduction-factor permits of substantially accurate comparisons being made between the mean spherical candle-power of different types of lamps, and may be used in absence of proper facilities for direct measurements of mean spherical intensity.

The *intrinsic brightness* (b) is the candle-power radiating normally per unit area of the source. It is usually given in candle-power per square inch

$$b = I/s. = I/s' \cos e \qquad (22)$$

The unit of light flux is called the *lumen* and may be defined as that flux of light radiating with unit intensity through a unit solid angle or steradian. Hence the flux of light in lumens radiating from a lamp is equal to the mean spherical candle-power multiplied by 4π, and the light flux on a given surface is equal to the average intensity in lumens per unit area multiplied by the area of the surface.

$$F = 4\pi I_{ms} = E_h S \qquad (23)$$

If the source of light is not a point but a small sphere of radius r, the flux $4\pi I_{ms}$ passes out from a surface $4\pi r^2$. The flux density of radiation or the specific radiation is

$$E' = F/s = 4\pi I_{ms}/4\pi r^2 = \frac{I_{ms}}{r^2} \qquad (24)$$

Thus we may speak of the luminous radiation at any point in space and of the flux density of such radiation.

The term *"quantity of light"* (Q) is expressed in candles. It is equal to the brightness, multiplied by the area of the source or

$$Q = bS \tag{25}$$

Lighting refers to the product of the quantity of light and the time and may be given in lumen-hours or candle-hours, preferably the former, thus:

$$LH = FT \quad \text{or} \tag{26}$$

$$CH = I_{ms}T \tag{27}$$

The *foot-candle* or *lumen per square foot* refer to the intensity of illumination.

The *intensity of illumination* is measured in foot-candles or lumens per square foot.

A *foot-candle* is the normal incident illumination produced by unit candle-power at a distance of 1 ft. When the average illumination on a square foot of surface is 1 foot-candle the light flux thereupon will be one lumen and the intensity one lumen per square foot. Hence it follows that the average illumination of a surface in foot-candles is equal to the flux of light in lumens falling on the surface divided by the area of the surface in square feet.

$$E = I/r^2 = F/S \tag{28}$$

The intensity I in candle-power is the angular density of the flux and the illumination intensity E is surface density. Therefore both I and E are flux ratios, lumens per unit solid angle and lumens per unit area. One lumen per unit solid angle is the candle and one lumen per square foot is the foot-candle.

The *meter-candle* or *lux* is the normal incident illumination intensity produced by one unit of candle-power at a distance of 1 meter.

The *foot-hefner* and the *meter-hefner* are the normal incident illuminations produced by 0.9 unit candle-power at the distances of 1 foot and 1 meter respectively.

The relations of these different units of illumination intensity are given in the following table.

TABLE 22

Relative Values of the Units of Illumination Intensity

	Foot-candle	Meter-candle	Foot-hefner	Meter-hefner
Foot-candle	1.0	10.76	1.11	11.94
Meter-candle Meter-bougie International Lux	0.0929	1.0	0.103	1.11
Foot-hefner	0.9	9.68	1.0	10.76
Meter-hefner German Lux	0.0836	0.9	0.0929	1.0

The constant 10.76 is the square of the number of feet in a meter.

The *efficiency* of an electric lamp should be expressed in terms of the *lumens radiated per watt consumed.* When applied to gas lamps it should give the number of lumens obtained per cubic foot of gas consumed per hour.

Since the flux of light in lumens is equal to 4π multiplied by the mean spherical candle-power, 2π multiplied by the hemispherical candle-power when all of the light is below the horizontal, and to 4π multiplied by the spherical reduction-factor multiplied by the mean horizontal candle-power, making a uniformly fair basis of comparison, there seems no excuse for the existence of efficiencies other than those in terms of the lumens radiated per watt consumed. In illuminants requiring auxiliary power-consuming devices outside of the luminous body, such as steadying resistances in constant-potential arc lamps, a distinction should be made between the net efficiency of the luminous source and the gross efficiency of the lamp. This distinction should always be stated. The gross efficiency should include the power consumed in the auxiliary resistance, etc. The net efficiency should, however, include the power consumed in the controlling mechanism of the lamp itself.

Comparison between such sources of light should be made on the basis of gross efficiency, since the power consumed in the auxiliary device is essential to the operation.

A standard circuit voltage of 110 volts, or a multiple thereof, may be assumed, except where expressly stated otherwise.

The specific consumption of a lamp should be given as the power supplied per lumen developed. It is the practice, however, to compare lamps in terms of the number of watts per mean spherical candle-power. Watts per mean hemispherical candle-power is often applied to lamps having nearly all of their light in the lower hemisphere; and watts per candle is used commercially in connection with incandescent lamps and refers to the watts per mean horizontal candle-power.

Photometric tests, in which the results are stated in candle-power should always be made at such a distance from the source of light that the latter may be regarded as practically a point. Where tests are made at shorter distances, as for example in the measurement of tubular sources of light or lamps with reflectors, the results should always be given as "apparent candle-power" at the distance employed, which distance should always be specifically stated.

Either the total flux of light in lumens or the mean spherical candle-power and preferably the former, should always be used as the basis for comparing various luminous sources with each other, unless there is a clear understanding or statement to the contrary.

It is customary to rate incandescent lamps on the basis of their mean horizontal candle-power; but in comparing incandescent lamps in which the relative distribution of luminous intensity differs, the comparison should be based on their total flux of light measured in lumens, or on their mean spherical candle-power.

Where standard photometric measurements are impracticable, approximate comparative candle-power measurements of illuminants such as street lamps may be made by comparing their *"reading distances", i.e.,* by determining alternately the distances at which an ordinary size of reading print can just be read, by the same person or persons, when all other light is screened. The angle below the horizontal at which the measurement is made should be specified when it exceeds 15 degrees.

In *comparing different luminous* sources not only should their candle-power be compared, but also their relative form, intrinsic brightness, distribution of illumination, and character of the light. Moreover, the life performances should be considered and no little attention given thereto, since the decrease in intensity of some illuminants during their useful life is several times as great as that of others. Also it often happens among the same class of lamps that those giving the highest initial efficiency often decrease at a faster rate and soon become less efficient and of less value from an economical standpoint than those of poorer initial performance.

Summary and Equations of Chapter V

The luminous intensity (I) is measured in candle-power or candles,

$$I = E_n r^2 = F/\omega \qquad (18)$$

The mean spherical reduction factor is the ratio of the mean spherical to the mean horizontal intensities.

The intrinsic brightness is the candle-power radiating normally per square inch (or square centimeter) of the source.

$$b = I/s = I/s' \cos e \qquad (22)$$

The lumen is the light flux radiating with unit intensity through a unit solid angle.

$$F = 4\pi I_{ms} = E_h S \qquad (23)$$

Lighting is the product of the quantity of light and the time, and is expressed in lumen-hours or candle-hours.

$$LH = FT \qquad (26)$$

$$CH = I_{ms} T \qquad (27)$$

The foot-candle or lumen per square foot is the unit of illumination intensity. It is equal to the incident illumination on a normal plane 1 ft. from a source of unit candle-power.

$$E = I/r^2 = F/S \qquad (28)$$

The meter-candle or lux is the incident illumination at a meter's distance from a unit source and equals 0.0929 (1/10.76) foot-candle.

The efficacy of a lamp should be expressed in lumens radiated per watt consumed, and lamps should be compared on this basis.

The results of photometric tests of tubular and surface sources should be given as "apparent candle-power" and the distance from the screen at which the lamp is photometered should be stated.

Lamps should be compared on the basis of their efficiency, brilliancy, distribution of light, character of light, and life performance.

CHAPTER VI

THE PRINCIPLES OF PHOTOMETRY AND TYPES OF PHOTOMETERS

In measuring the luminous intensity of a source of light the standard of candle-power is the fundamental feature and its reliability is of great importance. Having a reliable light standard the next important feature is the photometer. A photometer is an apparatus for comparing illumination intensities.

All elementary photometry is based on the assumption of a point source of light and that the intensity of illumination due to the light from a point source varies inversely as the square of the distance from the source. Although the *law of inverse squares* is familiar to all students of science and the simple methods of photometry are elements of common knowledge, a brief review at this time, may not be out of place. A graphical presentation of this law is shown in Fig. 23, where the source of light is assumed to be at 0 and the rays emitted radially in all directions. It will be seen from this figure that the area of the surfaces normal to the point, enclosed by the radial lines, vary as the square of the distance from the source and since the light rays originate at the source the amount of light flux on each surface must be the same. Hence it is obvious that the intensity must vary inversely as the square of the distance from the source.

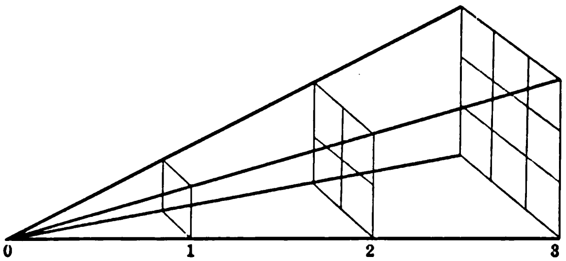

FIG. 23.—Law of inverse squares.

The question naturally arises as to *how extensively the assumption of a point soucre will apply*. This subject has been investigated by Dr. Hyde[1] for a tubular source and a flat circular disk. The results tabulated below give the per cent. error obtained by assuming the source to be a point source, for different distances from the source. The values in the first column are the ratios of the distance from the source at which the measurements are made to the length of the tube or the diameter of the disk.

TABLE 23

Errors due to the Assumption of the Inverse Square Law

Ratio of distance from source to the greatest dimension of the source	Per cent. deviation from inverse square law	
	Tubular source, diam. 0.05 of the length	Surface source circular disk
100	±0.00 per cent.	± 0.00 per cent.
50	+0.05 per cent.	− 0.03 per cent.
30	....	− 0.10 per cent.
25	+0.11 per cent.	....
10	+0.20 per cent.	− 0.98 per cent.
5	+0.09 per cent.	− 3.8 per cent.
4	−0.09 per cent.	....
3	....	−10.0 per cent.
2.5	−1.13 per cent.	....
1.0	....	−50.0 per cent.

From these data it will be seen that at a distance of five times the length of a tubular source or ten times the diameter of a circular surface source, the light may be considered as emanating from a point as far as appreciable error from the assumption of the inverse square law is concerned.

In photometering a lamp, the illumination on one part of the photometer, varying inversely as the square of the distance from the source, is made equal to that on another part received from some source of known candle-power. Then, since the illuminations are equal, it is obvious that the candle-power values vary directly as the square of the distances or,

$$I_x = I_s \frac{d^2_x}{d^2_s} \qquad (29)$$

where I_x and I_s are the candle-power values, and d_x and d_s are the distances from the screen of the unknown and known sources, respectively.

If the *distance between the standard and the screen* be fixed and the candle-power be maintained constant, then I_s/d^2_s becomes constant and

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[1] *Bull. Bur. Stand.,* Vol. 3, p. 81, 1907.

$$I_x = Kd^2_x \tag{30}$$

If the *scale of the distances* be graduated according to the square of the distances the above equation becomes,

$$I_x = Kr \tag{31}$$

where r is the reading on the photometer scale.

From the equation $I_x = Kd^2_x$, the value of I_x may be easily *calculated by means of a slide-rule* in the following manner: Place the zero of the slide on the value of K on the upper or A scale, place the cross-hair over the value of d on the C scale, then under the cross-hair there will be the value of d^2 on the B scale and the product $Kd^2 = I_x$ on the A scale.

The Photometer.—The photometer in all its modifications consists of a screen and its accessories. The screen either reflects or diffuses the illuminations under comparison and may be observed directly by the eye unaided, or through the agency of some optical train. The light reflected by the screen is always less than the illumination which it receives and may or may not be of the same quality. *Selective absorption* is often utilized whereby reflected light from an appropriately colored surface agrees in color with the compared light. The *sensitiveness of the apparatus* increases with the reflecting power of the screen.

Diffusing screens scatter the light in its transmission through them. They consist of some translucent substance and like the reflecting screens reduce the intensity of the light. They may be designed to change the quality of the light by selective absorption. The *sensitiveness of a photometer screen* depends upon its luminous efficiency, which is proportional to the reflecting or transmitting power, as the case may be, of the material of which it is composed. The sensitiveness of the photometer, distinguished from the above, depends upon the least amount of change in light which the observer is able to detect. This amount may vary from one part in 60 for a weak illumination to one part in 120 for an intense illumination.

Types of Photometers.—There are two general types of photometers:

(1) The *stationary photometer* which is usually installed in the dark room and is used for making accurate measurements of candle-power values, and

(2) The *portable photometer*, which is built in a compact form of light weight and may be easily employed for making candle-power measurements under actual commercial conditions or for making illumination measurements or surveys.

Photometric Devices.—The different photometric devices may conveniently be divided into four classes:

(1) Those dependent on *visual acuity*, which measure the light by the ability of the eye to detect objects illuminated by it.

(2) Those in which a photometrical balance is obtained by the "*equality of brightness*" of two surfaces, each of which receives light from a different source.

(3) Those where the balance is obtained by contrasting the illuminated surfaces and which are known as "*contrast*" photometric devices.

(4) The "*flicker photometer*" wherein surfaces lighted by either source are presented to the eye in rapid succession.

The relative merits of these different types of photometrical devices still remain a debatable subject. The choice of any one will many times depend upon conditions under which it is to be used.

Methods of Obtaining a Photometrical Balance.—Various methods are employed to obtain the photometrical balance; *i.e.*, to produce equal illumination on both parts of the screen. These methods may be enumerated as given below.

(1) *By varying the distances between the sources and the screen:* This may be accomplished by moving the comparison lamp, keeping the position of the screen and test lamp fixed; by varying the position of the test lamp; or by changing the position of the photometric device relative to the two lamps. Reference to the equations at the beginning of the chapter will indicate that calculations are simplified by having the distance between the standard lamp and the screen fixed. This may be conveniently accomplished in dark room photometry by rigidly coupling the supports of the standard lamp and the photometric device together and by varying their position relative to the test lamp placed at zero on the photometer bars. Of the different methods of varying the intensity of illumination on the photometer screen this method is the most universally applicable; it possesses accuracy, reliability, simplicity, flexibility, and can be easily verified.

(2) *Absorbing medium* offers a very satisfactory and practical means of decreasing the illumination intensity if it be free from

selective absorption. It may be used in connection with the variations of distances and in this way increase the range of the apparatus. It may be placed between the screen and the test lamp if large candle-power values are involved, or between the screen and the standard lamp if low illumination intensity on the screen is desired or if the standard is of too high candle-power. In this case the equation mentioned above involves another constant which will be the transmission coefficient of the absorbing media. If the photometer scale is graduated proportional to the square of the distances, equation 31 will become

$$I_x = Kr/k' = K'r \tag{32}$$

if the absorbing medium is between the test lamp and the screen, and

$$I_x = k'Kr = K''r \tag{33}$$

if the absorbing material is between the standard lamp and the screen; k' being the transmission coefficient of the absorbing material.

(3) The *rotating sectored disk* is one of the most valuable adjuncts in photometrical measurements. The principle of this device as applied to photometry is based on Talbot's law. This law as stated by Helmholtz[1] is as follows: "If any part of the retina is excited by intermittent light, recurring periodically and regularly in the same way, and if the periods are sufficiently short, a continuous impression will result, which is the same as that which would result if the total amount of light received during each period were uniformly distributed throughout the whole period."

The application of this law to the rotating sectored disk was investigated by Dr. Hyde[2] using different colored light and various size openings in the disk. These investigations verified the law within the limits of experimental error. Dr. Hyde's conclusions were as follows: "Talbot's law, in its application to a rotating sectored disk, is verified for white light for all total angular openings between 288 and 10 degrees, to within a possible error of 0.3 per cent., which probably expresses the limit of accuracy of the experiments. The observed deviations from the law for red, green, and blue light are of the same order of magnitude as those for white light, and hence Talbot's law is verified for red, green, and blue light, though not to such a high accuracy as for white light. Moreover, a difference in color on the two sides of the photometer screen produces no appreciable change in the observed deviation from the law."

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[1] *Phil. Mag.*, Ser. 3, Vol. 5, p. 321, 1834.

[2] *Bull. Bur. Stand.*, Vol. 2, p. 1, 1906.

In construction the disk can be made in several different forms. The simplest method of obtaining various ranges by means of the disk is obviously to make a set of disks from some thin material as sheet metal or fiber board having different size openings in each. In this way the intensity can be reduced by definite steps from nearly normal value to zero. These different disks may be mounted one at a time on the shaft of a small motor.

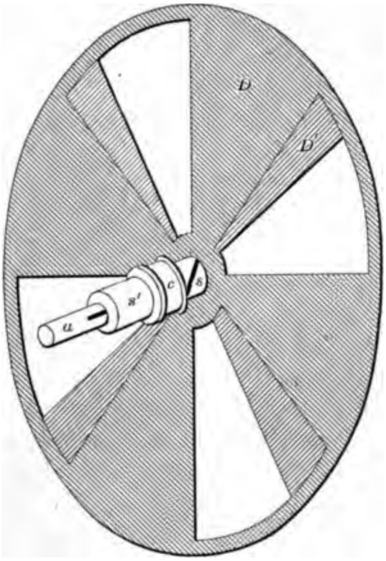

Fig. 24.—The sectored disk.

It is obvious that a disk, the openings of which could be varied without stopping the motor and changing disks, would be desirable. Such a device is shown in Fig. 24. There are two disks, *D* and *D'*, which may be moved relative to each other about the same axis. Both are mounted on the same shaft but on separate sleeves, one sleeve over or outside of the other. By making a spiral slot in one sleeve and a longitudinal slot in the other and by placing a pin through them into the shaft, the size of the openings between the sections of the disks may be easily increased or decreased by sliding the sleeves along the shaft in the proper direction. This change in adjustment may be made while the disk is in operation by placing the end of a lever in the groove of the collar *C* which is attached to the sleeves. The other end of the lever may be placed above a scale calibrated to indicate by the position of the lever the value of the openings in the disk. With this arrangement the intensity can only be varied through a range of about 50 per cent.

The minimum speed of rotation is that at which no flicker effect is experienced. Higher speeds give the same results.

It is obvious that where the flicker is produced by a rotating sectored disk, it would be more pronounced than if due to alternating current supplied to the illuminant. Investigations of the critical speed of the disk were conducted by Ferry[1] and Porter[2] where light of known intensity and quality were passed through a rotating sectored disk. The critical frequency or the frequency at which a flicker was just imperceptible is indicated by the curve in Fig. 25.[3] It was found that the vanishing flicker frequency appeared to be a function of optical intensity alone and independent of the color value of the light.

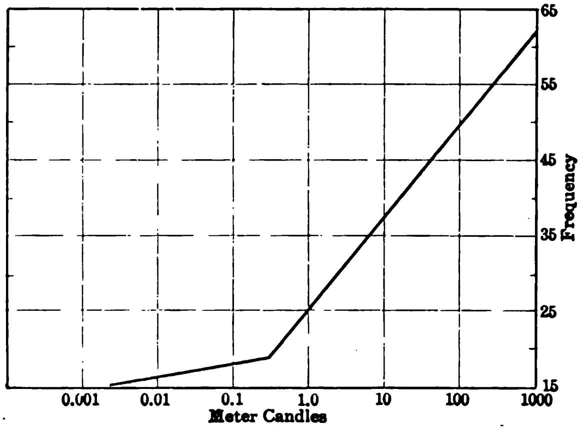

FIG. 25.—Frequency at which flicker becomes perceptible for different intensities of illumination. (Sectored disk.)

Porter also found that the critical frequency was a linear function of the logarithm of the intensity. For intensities greater than 0.2 meter-candles,

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[1] *American Jour. of Science*, Vol. 44, p. 192, 1892.

[2] Proc. Roy. Soc., Vol. 70, p. 313, 1902.

[3] *Bull. Bur. Stand.*, Vol. 5, p. 282, 1908.

$$f_1 = 12.4 \log E + 24.9 \qquad (34)$$

and for intensities less than this value,

$$f_2 = 1.56 \log E + 19.6 \qquad (35)$$

for meter-candles and common logarithms.

From this curve it appears that rod vision changes or begins to change to cone vision at about 0.2 meter-candles.

On the following pages are described some of the different types of photometrical devices in use in this country and abroad, together with the prevailing practice concerning their use and manipulation. A careful study of this portion of the subject together with the preceding part of the chapter should give the reader a good working knowledge of the fundamentals of the subject.

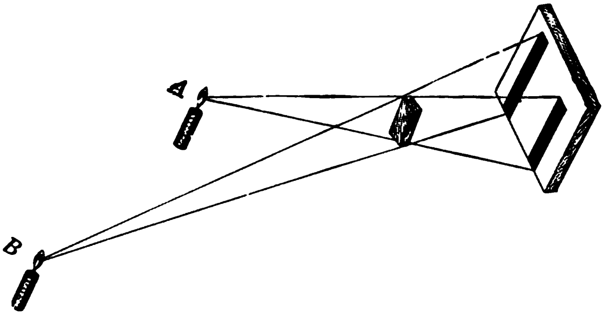

FIG. 26.—The Rumford photometer.

Of photometric devices the **Bouguer photometer** is said to be the oldest form of apparatus for comparing illuminating sources.

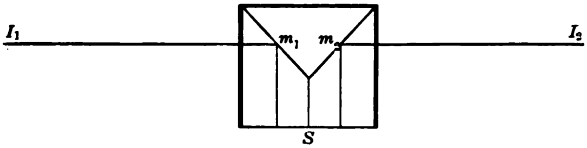

FIG. 27.—The Ritchie mirror photometer.

It consists of an opaque reflecting screen with a blackened partition normal to its plane at its center. The two sources of light are placed one on either side of the partition and their distance fromthe screen adjusted until the illumination of the screen is the same on both sides of the partition.

Later **Potter**, substituted a translucent screen in place of the opaque one, which enabled the observer to place the screen between himself and the sources of light.

The **Rumford** or **shadow photometer** (Fig. 26) consists of a white screen with a small rod in front of it. The sources are then placed at distances from the screen such that the shadows due to each lamp are of the same density.

The **Ritchie mirror photometer** has its light sources placed in fixed positions at opposite ends of the bar. Mirrors placed 45 degrees to the common axis of the sources (Fig. 27) throw the light from each lamp on a comparing screen supplied with a blackened partition separating the rays from the two sources. The screen and mirrors are movable and since the screen is viewed from a point normal to the photometric axis the photometer can be enclosed in a compact sight box.

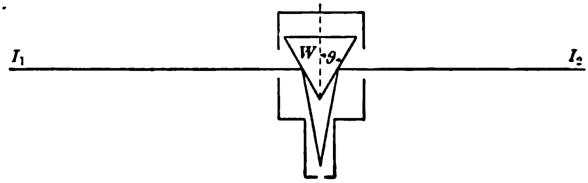

FIG. 28.—The Ritchie wedge photometer.

In the **Ritchie wedge photometer** (Fig. 28) mirrors are replaced by pieces of white opaque material having a diffusing surface. These surfaces need not make an angle of 45 degrees with a normal to the rays of light. In fact the sensibility of this instrument is greater if the sides of the wedge make an angle of 20 or 30 degrees with this normal plane, the value of the *angle for greatest sensibility* depending on the material of the reflecting surfaces. Results by Trotter[1] showing the performance of different materials which could be used for a Ritchie wedge are indicated in Fig. 29. These curves show the relative brightness of the surfaces when viewed from various angles or when making various angles with a normal to the rays of light. It will be seen that white blotting paper shows highest diffusibility as well as greatest brightness, the conclusion being that this material will make an instrument of high sensibility and the highest sensibility will be obtained by making the angle between the surface and the normal about 30 degrees.

It is obviously imperative in the use of this photometer that the light from the two sources strike the respective sides of the screen at the same angle. Any unequal obliquity of this nature will introduce an error and since the two sides are fixed relative to each other any increase in the obliquity of one side will decrease that of the other and, assuming the cosine law, will double the effect. Referring to the figure it will be seen that the chance of error may be reduced by making the angle of divergence of such value as to give maximum brightness since this point occurs on the flat part of the curve and maximum sensibility will be obtained at the same time.

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[1] *Illum. Eng.*, Lond., Vol. 2, p. 79, 1909.

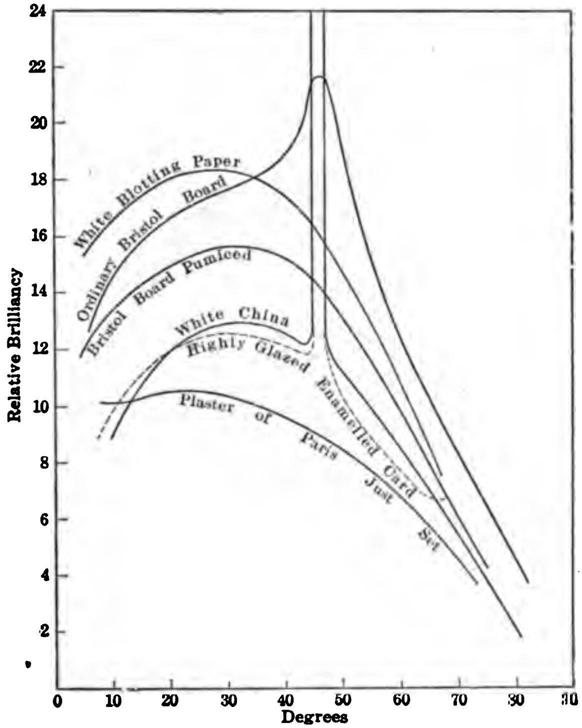

FIG. 29.—Effect of changing the inclination of the sides of the wedge (angles measured from normal to the rays).

The Elster-Joly diffusion photometer involves the use of two blocks of translucent substance as paraffin or milk glass so placed that each receives light from a different lamp. These blocks may be placed side by side with an opaque diaphragm between them (Fig. 30). They are viewed from the side and the photometrical setting obtained when both appear equally illuminated by the internally diffused light.

The Bunsen photometer is one of the oldest and simplest forms of photometric devices. Moreover, it is still widely used and constitutes a very efficient means of comparing the intensity of luminous sources. The screen differs from those used in the photometers previously described. In its simplest form this screen consists of a sheet of white paper a part of which is made translucent by paraffin or some other similar substance. The transparent portion is usually star shaped with its edges sharply defined. The light falling on either side of the screen is partly reflected and partly absorbed, while that falling on the treated part is partly transmitted. If the same per cent. is absorbed by both parts of the screen, a greater amount will be reflected from the untreated part. When the illumination on both sides of the screen is the same an equal amount of light is transmitted through the treated portion in each direction and if the light from each source is of the same hue both sides of the translucent portion of the screen, should appear identical. If the same per cent. of light be absorbed by both the treated and untreated portions of the screen, the entire surface of both sides of the screen will then appear identical when a photometrical balance is obtained, provided the color of the light from the two lamps is the same. The coefficient of absorption of the translucent portion is, however, usually greater than that of the untreated part and a contrast is observed when a setting is secured.

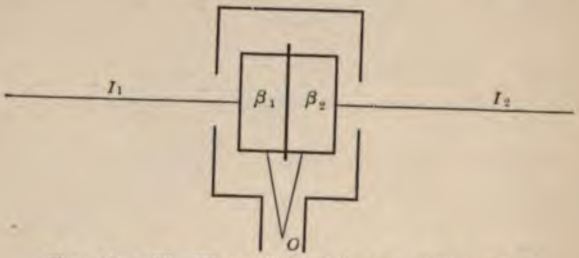

FIG. 30.—The Elster-Joly diffusion photometer.

Both sides of the screen are viewed simultaneously by means of two mirrors, m m', Fig. 31, properly arranged. It is essential that the mirrors have the same reflecting power and make the same angle with the screen. Both sides of the screen are used in obtaining a balance although either or both sides may be used if the absorption of both the treated and untreated portions be the same. The screen and mirrors being reversible a second balance can be obtained and the mean taken as the result thus eliminating any error which might arise from dissimilarity in either the sides of the screen or in the mirrors. Referring to the figure both sides of the screen *s s′* are viewed through the eye-piece *o* by means of the mirrors *m m′*.

The Leeson disk is an improvement over the Bunsen disk. It consists of a piece of thin translucent paper with opaque pieces with central star-shaped openings pasted on both sides of it.

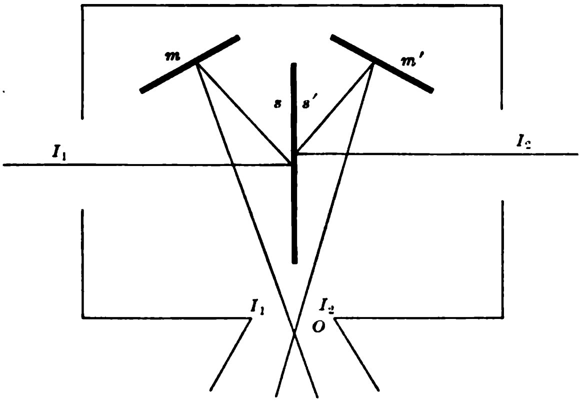

FIG. 31.—The Bunsen photometer.

The edges of the openings exactly coincide on the two sides of the screen. This screen like the Bunsen may be either of the contrast or disappearance type and, since the edges of the translucent portion are more sharply defined than those of the Bunsen, less difficulty is experienced in detecting slight inequalities in the illumination of the two sides of the screen.

The Lummer-Brodhun photometric device consists of a purely optical combination for viewing the two sides of the screen. A diffusing screen, *s s′*, Fig. 32, of high reflecting power is placed in, and with its plane normal to, the photometric axis. This screen is viewed on both sides by means of the optical device which presents both sides of the screen to the eye as adjacent or concentric fields.

The Lummer-Brodhun photometer is built both as an "equality of brightness" and as a "contrast" photometer. The arrangement of each is essentially the same. The light from the two sources, reflected from the sides of the screen *s* and *s'*, falls on the mirrors m_1 and m_2 and is reflected along a normal to the surfaces of the triangular prisms *A* and *B*. The observer, looking through the telescopic tube *O* directed normally to *B*, clearly views a divided field illuminated partly by one source and partly by the other.

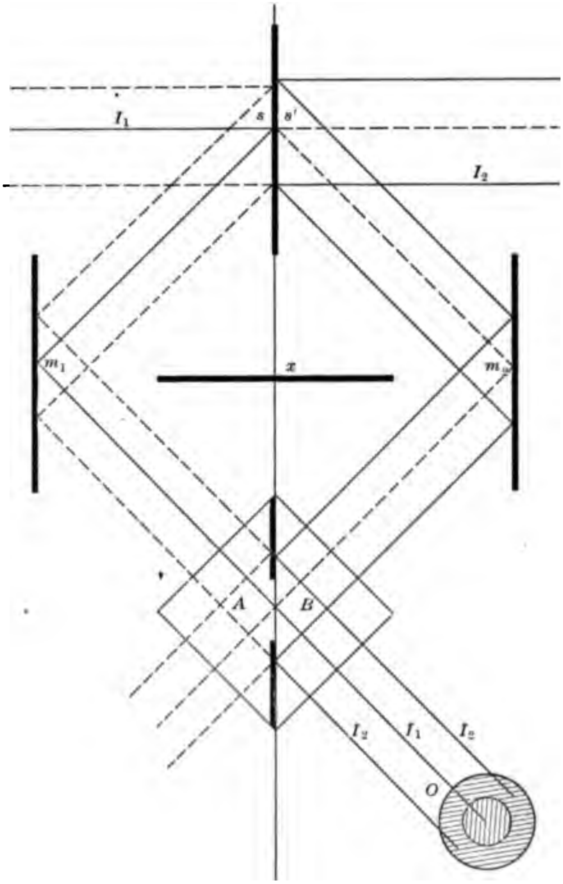

Fig. 32.—The Lummer-Brodhun "equality of brightness" photometer.

In the "*equality of brightness*" *photometer* the rays from I_1 pass directly through the central part of the prisms illuminating the central portion of the field as indicated in Fig. 32. The rays from I_2 pass in the same way into the prism *B*, the central rays pass directly through the prisms while the outer rays are reflected by the prism *B* and constitute the outer portion of the field of view. The paths of the light rays viewed through the eye-piece

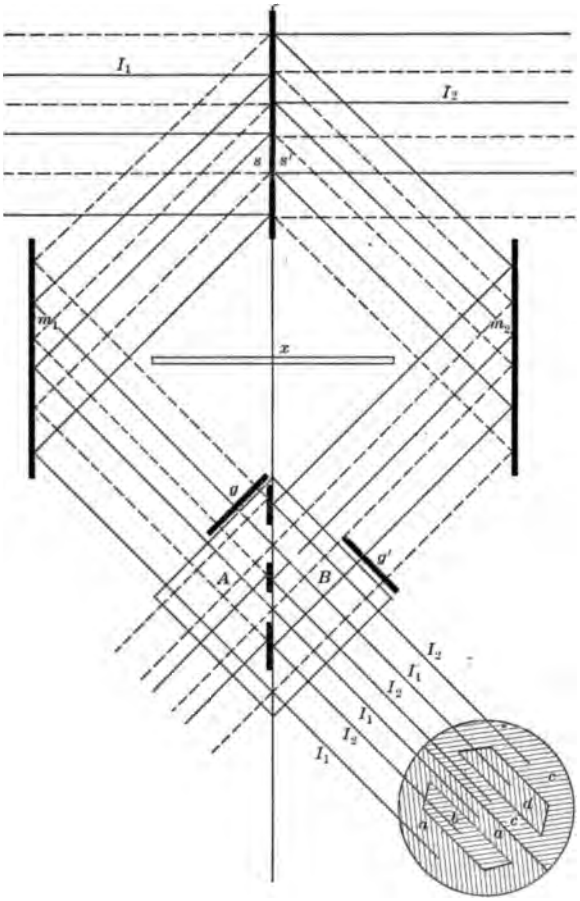

FIG. 33.—The Lummer-Brodhun "contrast" photometer.

are indicated by the full lines. Those shown dotted from I_1 are reflected by the prism *B* out of the line of vision while those represented by the dotted line from I_2 pass directly through the prisms out of view. With this arrangement one observes a two-part illuminated field as shown shaded at *O*.

In the *"contrast" photometer* the principle is the same but the field is divided up as illustrated by the shaded part of Fig. 33. The principle can be readily understood from the previous description. It will be seen that the parts of the field *a* and *d* are illuminated by the source I_1 while *b* and *c* receive light from the source I_2. Two thin pieces of glass *g* and *g′* intercept some of the light illuminating the fields *b* and *d* thus giving the contrast effect whereby this photometer receives its name. A photometrical balance is obtained when the intensity of *a* and *c* and of *b* and *d* are the same, *a* and *c* however will be brighter than *b* and *d* due to the small amount of light absorbed by the two pieces of glass *g* and *g′*.

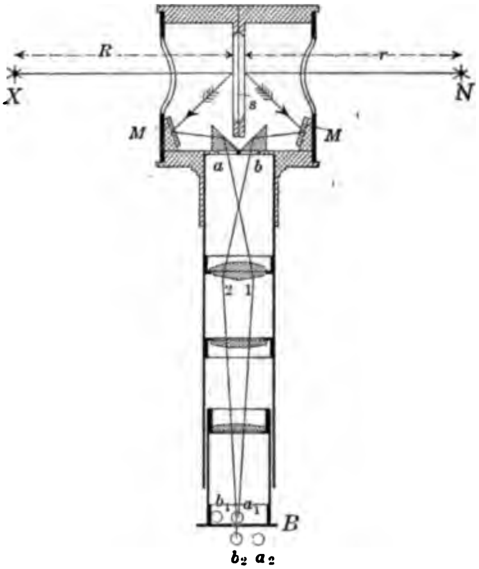

FIG. 34.—The Marten photometer.

The Marten Photometer Head.—A photometrical arrangement similar in principle to the Lummer-Brodhun is shown in Fig. 34. The two sides of the screen *S* are viewed through the telescopic tube, the prisms *ab* and the mirrors *MM*.

A recent type of the **Bechstein photometer**[1] is shown in Figs. 35 and 36. This photometer can be used either as a contrast or equality of brightness instrument. It has a Ritchie wedge of plaster-of-Paris G which receives the light from the two sources and which is viewed through the telescopic combination as shown. Two biprisms located at Z_1 and Z_2 at right angles to each other present the illuminated fields in the shape of four quadrants, alternate ones of which are illuminated by the same lamp. The angles of the prisms are so adjusted that the prism with its dividing edge vertical, Z_1, Fig. 36, divides the field into two equal portions, the left portion being illuminated by the light from the right-hand side of the wedge and the right-hand portion receiving light from the left side of the wedge. The influence of the prism Z_2 is the same as Z_1 except that it is 90 degrees displaced. The result is that the field of view is divided into four quadrants, with opposite quadrants illuminated by the same source. In the equality of brightness photometer the photometrical balance is obtained when the four quadrants are of equal brightness. In the contrast photometer the contrasting element may consist of two thin squares of glass, K_1 and K_2, Fig. 37, inserted in the path of the rays illuminating opposite quadrants, as shown in Fig. 38*a*. This glass absorbs a small

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[1] *Illum. Eng.*, Lond., Vol. 1, p. 498, 1908.

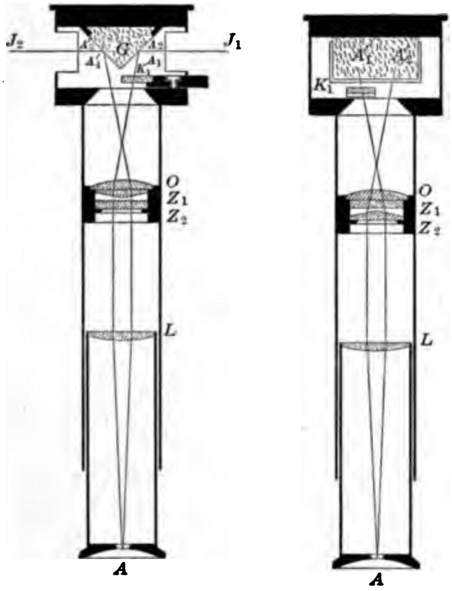

FIG. 35. FIG. 36.

FIGS. 35 and 36.—The Bechstein photometer.

amount of light, the right amount to produce the desired contrast. The unbalanced condition of the field is shown in Fig. 38*b*.

The **selenium photometer** takes advantage of the fact that the electrical condition of selenium changes under the influence of light. This property of selenium makes it especially fit for the construction of photometers, although nearly all attempts to construct a practical instrument have failed to yield satisfactory results. The disturbing influences are mainly the moisture and heat of the air and a gradual alteration in the condition of the selenium itself. One serious objection to this photometer is the long time it takes the resistance to assume a value corresponding to a given illumination. This is particularly true of the decrease of resistance.

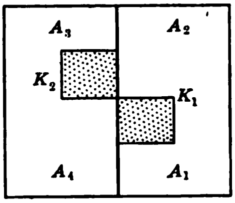

FIG. 37.

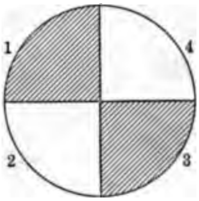

FIG. 38 a.

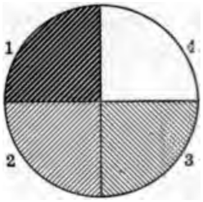

FIG. 38 b.

FIG. 37.—Contrasting elements.
FIGS. 38 a and 38 b.—Balanced and unbalanced condition.

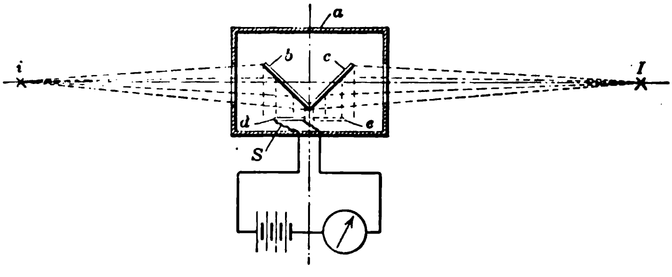

FIG. 39.—Diagram of the Selenium photometer.

The most successful selenium photometer is of German make. It consists of a device for exposing the cell alternately to the light from the standard and the source to be measured. The arrangement is shown in Fig. 39, where *S* is the selenium cell which is given an oscillating movement so as to pass directly from the illumination from the lamp *i* to the illumination from the lamp *I*. Both *i* and *I* are fixed on the photometer bench, while *b* and *c* are two mirrors which reflect the rays from the two lamps in the same direction, so that the oscillating selenium cell is successively lighted by the two lamps. The selenium cell *S* is in series with a battery and a galvanometer. If the two illluminations are equal, the current in the circuit is constant and the needle of the galvanometer remains at rest. If the two illuminations are unequal, the needle vibrates. By moving the photometer box or one of the lamps a balance is obtained in the ordinary way.

Since selenium does not agree with the luminous sensitiveness of the eye with respect to lights of different colors, it should only be used for comparing sources of the same spectral composition.

The **bolometer** consists essentially of a very thin wire, usually of iron coated with carbon. When exposed to the light it becomes heated, and the change in its electrical resistance affords a means for measuring the energy intensity of the light rays. The bolometer is apt to indicate the heat rays rather than the luminous rays.

Numerous attempts have been made to develop a *chemical photometer* but since the chemical action of the light is largely due to the ultra-violet or actinic rays, no success of any particular value has so far been achieved in this line.

Heterochromatic Photometry.—It has already been shown that the luminous value of a light depends upon its spectral composition and to some extent upon its intensity. Hence it is obvious that the comparison of the luminous qualities of two lamps differing in color at once become complicated. Also it has been found, as might be inferred from previous study of the eye, that a photometrical setting depends upon the size of the retinal area employed or, what amounts to the same thing, the size of the photometrical field.

Sources of light showing but slight color differences can be compared by any of the ordinary methods with an accuracy sufficient for commercial purposes. But when comparing lights differing greatly in color it is the general experience that different kinds of photometers give different relative values for colored lights, and even the same photometer used in two different ways, or comparing the lights at two different intensities on the screen, or viewing the screen from different distances may fail to give concordant results. Comparisons repeated on succeeding days by the same observer are likely not to agree with those at first obtained, and, what is still worse, the general conclusions reached by one observer, are liable not to concur with those derived by another.

The Flicker Photometer.—The flicker photometer has been designed for comparing lamps emitting light dissimilar in color, by presenting to the eye surfaces illuminated by each light source in rapidly alternating succession. Its principle is based upon the investigations of Rood[1] who discovered that when two surfaces differing in color are presented to view alternating and in rapid succession the sensation of color disappears or the color sensations are combined and the color difference lost, although the sensation of flicker may persist. Rood also found that when these two surfaces were equally illuminated no sensation of flicker was experienced.

Moreover, the rapid alternations of presenting first one and then the other illuminated surface tends to exercise the eye at its maximum sensitiveness and for this reason should reduce the personal variable. This seems verified by the results of experiments which show that this type of apparatus can be used for comparing light sources presenting the widest contrast in color with a consistency approaching that of the ordinary types of photometers when balancing lights of the same color. Such results are obtainable only with the normal eye, the fatigued eye showing a differential sensibility toward one of the lights.

Concerning the active part of the retina we have what is sometimes referred to in photometry as the "*yellow-spot effect*" the "yellow-spot" or fovea, as described in Chapter III, being the small central region of the retina. It will be remembered that the cones are the predominating light-perceiving organs in this region, and that it is at this point that greatest sensibility is obtained. With this point in mind together with the other peculiarities of the eye previously discussed, it will not be surprising to find that the readings obtained by photometers when comparing different colored lights may depend, within certain limits, upon the size of the photometrical fields, or, what amounts to the same thing, the distance of those fields from the eye of the observer. Investigations of this nature, obviously involving low intensities, by Mr. Dow,[2] show quite appreciable differences in readings due to these causes. Curves showing the influence of distance from the blocks of a Joly photometer to the eye when heterochromatic sources are compared are reproduced in Fig. 40. Curve 1 gives the apparent ratios of the candle-power of a Methven gas standard to an incandescent gas mantle; curve 2, a carbon-filament lamp to a Nernst lamp; and curve 3, a carbon-filament lamp to a mercury-vapor lamp. The values with the eye at 10 centimeters were reduced to unity in each case and the values at other distances given in per cent. of those at 10 centimeters.

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[1] *Amer. Jour. of Science*, Vol. 46, p. 173, Sept., 1893.

[2] *Illum. Eng.* Lond., Vol. 2, p. 610, 1909.

Another disturbing factor in heterochromatic photometry is known as the *"Purkinje effect."* It appears that the normal eye can distinguish details better in and is more sensitive to light of a greenish or bluish hue when the illumination is of low intensity while the greenish-yellow hue becomes more efficacious when cone vision comes into prominence and at higher illumination intensities. An investigation of the ratio of ruby-red to signal-green light, two colors distinct enough in color to show the Purkinje phenomenon effectively, by Mr. Dow,[1] shows that this factor becomes negligible at ordinary and high intensities.

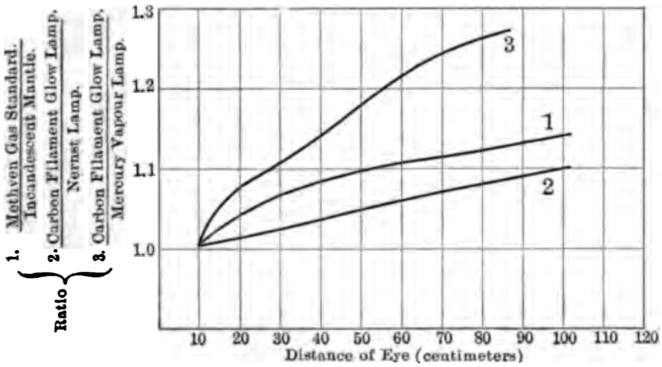

FIG. 40.—Influence of distance of eye from the blocks on the readings of a Joly photometer when heterochromatic sources are compared.

The flicker photometer appears to be influenced less by the yellow spot and Purkinje effects than the equality of brightness photometer, although it is less sensitive at low intensities. Comparative results showing this as obtained by Mr. Dow (Ref. Cit.) are shown diagrammatically in Figs. 41 and 42 where the ratios of the apparent candle-power of ruby-red light to signal-green light are given for the two classes of photometers. The wedge arrangement was used. Precautions were taken, in the case of yellow spot effect, to insure the use of the same portion of the retina for each instrument at successive distances from the screen. The retinal area of course decreased, as the distance increased. For obtaining the results shown in Fig. 41 the intensity on the screen remained constant.

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[1] *Elect. World,* Vol. 55, p. 465, Feb. 24, 1910.

In the case of Fig. 42, a comparatively large retinal area was used to secure greater sensibility and was of the same size for both photometers. It will be seen that with the equality of brightness method the values of the red light in terms of the green light become much less as the intensity of illumination is reduced which is in agreement with the Purkinje effect, but with the flicker arrangement these ratios show no marked variation in value.

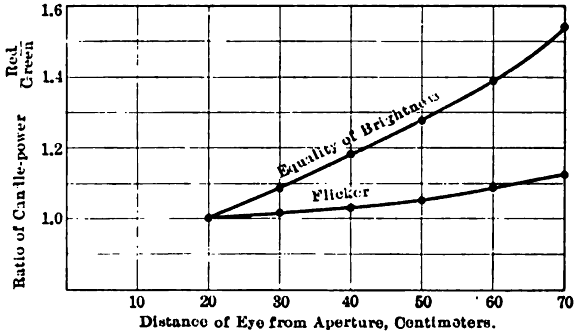

FIG. 41.—Comparison of "yellow spot effect" in flicker and equality of brightness photometers.

These physiological peculiarities arising from the use of the two types of photometers mentioned above have been verified, qualitatively at least, by several investigators. They indicate that where low intensities of illumination differing in color *must* be compared the flicker photometer will yield results less influenced by the yellow spot and Purkinje effects than will the comparison of brightness type of photometric apparatus.

The discrepancies in results obtained by different observers when different colored lights have been compared at high intensities have often been attributed to the Purkinje and yellow-spot effects, but it has been proven quite conclusively that these disturbing influences occur only at low intensities.

Dr. Bell[1] has shown that errors in heterochromatic work, with the equality of brightness photometer and at intensities above the range of Purkinje effect are the direct effect of *simultaneous contrast* which modifiies the apparent luminosities of two colored lights under comparison. When red and green, for example, are in juxtaposition the red appears redder and the green greener to a very noticeable degree if the circumstances are favorable. According to Titchener,[2] the increase in this contrast effect is always in the direction of the greatest opposition in colors.

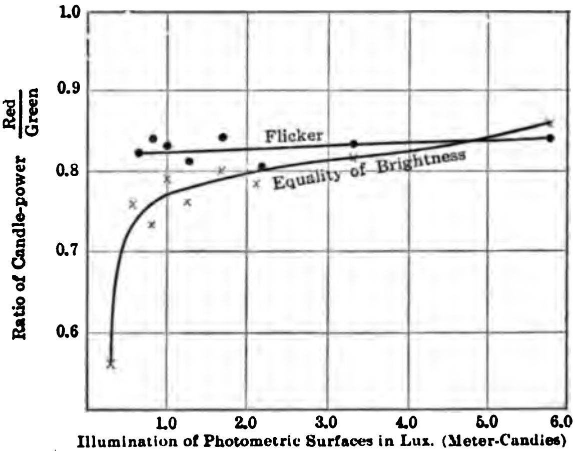

FIG. 42.—Comparison of Purkinje effect for flicker and equality-of-brightness brightness.

It increases with the saturation of the more prominent color and is always greatest when there is no simultaneous brightness contrast. The contrast effect is also sharper if the two colored surfaces are very close and not separated by visible boundary lines.

The *effect of simultaneous contrast* may be seen by placing a greenish light source, as a mercury arc for instance, on one end of the photometer bars on which is mounted a Lummer-Brodhun screen. When observed through the eye-piece the field appears a faint bluish-white similar to the color of the tube. If the mercury lamp is extinguished and the other side of the screen illuminated by a carbon lamp it will appear the usual yellowish-white. But if both lamps are lighted the sides of the screen become at once a conspicuous green and a brilliant orange, neither in the least resembling the color when either was used by itself. Hence it will be seen that due to contrast two colors in juxtaposition may be changed in hue to a surprising degree and abnormally thrown apart in the spectrum. Dr. Bell (Ref. Cit.) suggests Fig. 43 as conveying an approximate idea of what happens in such a case. Suppose two lights of the spectral regions *a* and *b* are being compared. When thrown on the screen in juxtaposition the hues are changed by contrast so that the red becomes redder and the green greener. The contrasts therefore correspond to colors of lessened luminosity, but lessened in unequal degrees, as for instance *a* shifting to *a′* and *b* to *b′*. The red is relatively more decreased and a photometrical balance based on the contrast values would show an apparent Purkinje effect at illumination intensities much higher than those within which the real Purkinje effect comes into play.

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[1] *Elect. World*, Vol. 59, p. 201, Jan. 27, 1912.

[2] "A Text-book of Psychology," page 76.

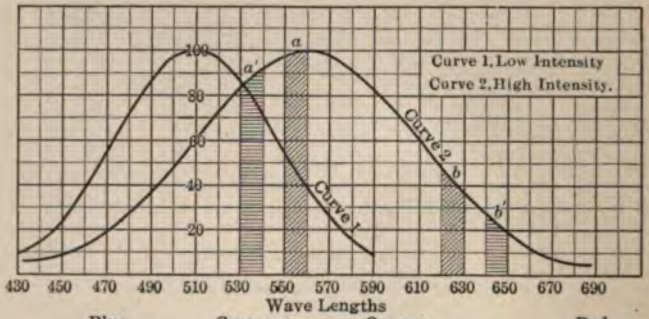

FIG. 43.—Luminosity at high and low intensities.

As illumination intensities are greatly increased even spectral colors become relatively less saturated and would consequently affect each other less by simultaneous contrast. As the lower limit of color vision is approached, on the other hand, the effects of contrast are less conspicuous so there would be a gradual transition into the actual Purkinje effect when the intensity is reached at which rod vision is dominant.

In the case of ordinary illuminants the red side of the luminosity curve is somewhat the steeper, hence an equal shifting of two contrasting colors would be of the nature explained above. In view of these conditions it is very probable that a large proportion of the aberrances found between different observers and by the same observer at different times when using the equality of brightness photometers, are mainly due to the phenomena here described.

It is in the flicker photometer that simultaneous contrast is eliminated. The color flicker disappears before the brightness flicker and when the balance is actually determined by the disappearance of the latter the colors have blended into a combination of the two from which simultaneous contrast has disappeared.

Color difficulties can also be reduced in the case of the Bunsen or Leeson screens by making the central portion of the field of such thin translucent material that there will be a marked color blending there, which will greatly reduce the color contrast.

The concluding paragraph of Dr. Bell's article (Ref. Cit.) is given below and sums up the question of the type of photometer to use for color photometry as far as color contrast is concerned.

"In making a judgment of brightness such as has to be made in an equality-of-brightness photometer the eye can depend only on what it sees and therefore makes its judgment on the apparent colors which it sees as influenced by contrast. Hence, if the contrast is of such a character as to shift the effective luminosities of the two color fields by unequal amounts, a color error will be introduced in the setting, and since the red side of the luminosity curve is usually steeper than the green side, the net result in comparing colors is often to produce a spurious Purkinje phenomenon perceptible even at fairly high illuminations. Even when this effect does not exist, the shifting of colors by contrast along the luminosity curve still takes place, and since different observers and the same observer at different times vary in perception of simultaneous contrast small and shifting color errors are constantly introduced in photometric observations which bear no relation to the genuine Purkinje phenomenon or to the absolute color sense of the individual observer. Perhaps the strongest claim of the flicker photometer rests on its freedom from errors of this kind, due to the fact that color blending is secured at a lower frequency than corresponds to the disappearance of brightness flicker, and in fact the flicker photometer is the only form in which simultaneous contrast is fully eliminated, which is another good reason for employing it in heterochromatic comparisons."

A series of researches by Dr. Ives[1] are of great value in solving

[1] Trans. Illum. Eng. Soc., Vol. 5, p. 711, 1910.

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the problems in heterochromatic photometry. In these investigations Dr. Ives used the flicker and equality of brightness arrangements for obtaining the luminosity curves showing the comparison of the spectral colors with white light. The apparatus was so designed that it could be used for both the equality of brightness and the flicker methods without disturbing any of the critical conditions.

The two illuminations used were 10 and 250 units, where a unit is the illumination of 1 meter-candle on a surface of magnesium oxide as viewed through an artificial pupil of 1 square millimeter area. Because of the small area of this pupil opening the effective illumination was probably about 0.1 these values. A circular photometrical field of 16 mm. in diameter at 24 cm. from the eye was chosen. This field subtended an angle of 4.5 degrees, approximately the size of the yellow spot of the retina.

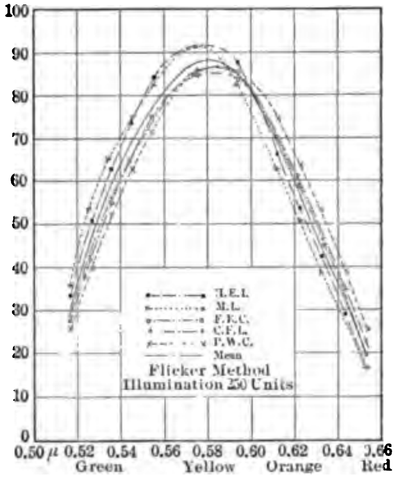


FIG. 44.—Relative luminosity of monochromatic light. (flicker method).

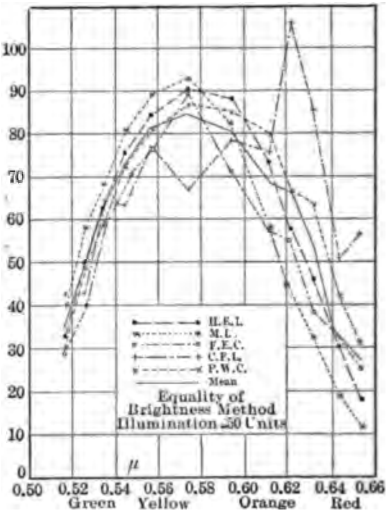


FIG. 45.—Relative luminosity of monochromatic light (equality of brightness method)

Readings were taken with each type of photometer for twelve points in the spectrum alternating on the red and green sides. The average readings of five observers and the average of the readings of the five observers at the high intensity of 250 units are shown by the curves of Figs. 44 and 45. It will be seen that the average of the readings of the five observers, shown by the full lines, are practically coincident for the two types of photometers. It will be seen, however, that there is considerable variation among the different observers when using the equality of brightness type. It should be remembered here that the different colors were being compared with white thus giving a less degree of contrast than the cases previously cited and also the two fields were not in exact juxtaposition. At low intensities (10 units) the luminosity curves obtained by the equality of brightness method shifted toward the blue end of the spectrum according to the Purkinje effect, whereas the luminosity curves obtained by means of the flicker device shifted toward the red end of the spectrum, this latter result being hitherto unobserved. The conclusion drawn from this investigation is that the flicker photometer should not be used for comparing low intensities, as the results so obtained must necessarily be incorrect. For comparing high intensities it may be expected that the results obtained by the two methods may possess approximately the same degree of accuracy. At high intensity it was found that the flicker arrangement showed far greater sensibility and there were less variations in the readings of different observers. These points of merit together with the greater ease of obtaining a photometrical setting and the less liability of change of the observer's criterion point to a decided superiority of the flicker photometer for heterochromatic photometry.

However, the question of heterochromatic photometry is still a debatable subject, and these investigations may be considered as preliminary steps in the solution of the problem as a whole. Quoting Dr. Ives from the paper referred to above where, in referring to the general coincidence of the luminosity curves obtained by the two methods at high intensity, he states: "This then would be an argument for choosing such an illumination as the standard one for making the heterochromatic comparisons necessary for the preparation of standards of different colors. For it must be clearly borne in mind that such comparisons can, from the nature of vision, hold exactly for only one illumination and size od field. The best solution to be hoped for must contain specifications of these two conditions. When the stage has been reached where one can give a definite candle-power rating to a colored illuminant for a certain illumination the first step will have been made. The second step will be when one can state also what the illuminant's candle-power will be at any other illumination, knowing it at the standard."

The Whitman Flicker Photometer.—One of the earliest flicker photometers was made by Whitman. Its construction is illustrated in Fig. 46. The disk *BD*, constructed as shown at the left, revolves around its axis *A*. When in the position shown at the right it presents to the eye of the observer at *O* a surface illuminated by I_1. When *B* has revolved from in front of the eye-piece the observer sees the surface *C* which is illuminated by light from I_2. In this way, by rotating the disk *BD* at its proper speed by means of a small motor or similar device, the two illuminated surfaces are presented to the eye in rapidly alternating succession, and a balance is obtained by varying the intensity on one surface or the other as in the case of the ordinary type of photometer head.

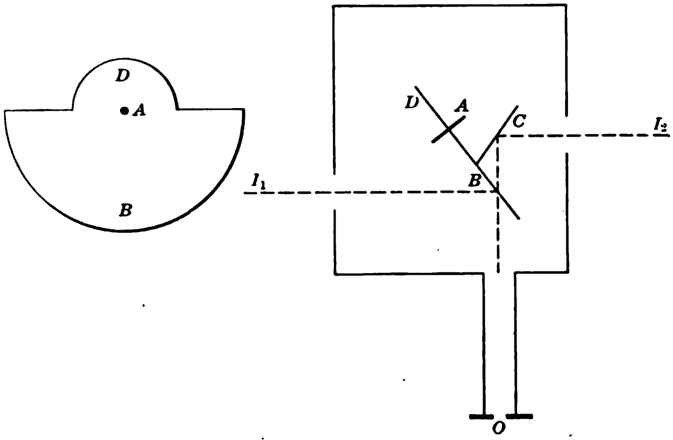
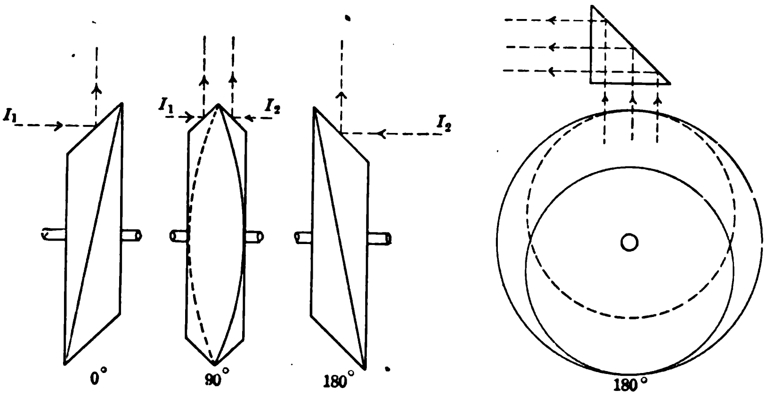


FIG. 46.—The Whitman flicker photometer.

The Simmance-Abady flicker photometer consists essentially of a thick disk of plaster of Paris driven by a clockwork mechanism. The edges of the disk are beveled as shown in Fig. 47 and these beveled surfaces are viewed from above as they revolve by means of a 45 degree mirror or prism and the sight tube as shown in Fig. 48. It will be seen from the construction of the disks that the observer sees first a surface illuminated by one lamp and then a surface illuminated by the other presented alternately and in rapid succession when viewing the disk through the sight tube and the prism. The speed of the disk can be adjusted by a regulator until highest sensibility is obtained. This adjustment is simple and offers no difficulty. One form of this instrument is so constructed that it can be tilted and lamps photometered at various vertical angles.

The Wild flicker photometer[1] consists of a rotating disk of white paper, one-half of which is made translucent by treatment similar to the Bunsen screen. The observer looking through the telescope (*H*, Fig. 49) observes in the mirror first the treated and then the plain portion of the screen. The screen may be rotated by a spring constructed on the principle of a clock mechanism. This instrument is exceedingly simple and of great sensitiveness when equipped with a suitable screen. Another advantage claimed for this instrument is that the rays of light from the two sources strike the surfaces of the disk normally as is the case with most of the high sensibility photometers, and that, therefore, the possibility of "angle-errors" is greatly reduced.



Section in the photometric axis.

Section normal to the photometric axis.

FIGS. 47 and 48.—The Simmance-Abady flicker photometer.

The Bechstein flicker photometer[2] is of the type in which a lens oscillates in front of the dividing line between the illuminated surfaces in the photometer. The optical arrangements are shown diagrammatically in Fig. 50. The two sides of the wedge *G* are illuminated by the two sources of light. The path of the rays from the photometric surfaces to the eye is through the lens L_1, through the prismatic arrangement *Kk*, to the eye-piece at *H*. The prismatic arrangement, *Kk*, consists of a prismatic annulus, *K*, the angle of which is so chosen that, in the position shown, only the rays from the right side of the wedge are brought to the eye-piece. Within the annulus *K* is fixed the disk *k*, which is likewise prismatic, having an angle of inclination equal, but in the reverse direction, to that of the annulus. Consequently, only the rays from the left side passing through this disk are brought to the eye-piece. The result is that the eye sees a field of view consisting of a circular area inside of an annular area. The inner area is illuminated by rays from the left-hand source, and the outer ring by means of rays from the right-hand source. If the prismatic arrangement *Kk* is kept stationary, we can employ the photometer on the "equality of brightness" principle, and obtain a balance by judging when ring and center appear equally illuminated. In order to obtain the flicker, the prismatic disk, *Kk*, is rotated about a horizontal axis by means of a band from a small motor passing over the pulley *R*. It is obvious that when the disk has rotated through 180 degrees, the rays of light

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[1] *Illum. Eng.*, Lond., Vol. 1, p. 500, 1908.

[2] *Illum. Eng.*, Lond., Vol. 1, p. 499, 1908.

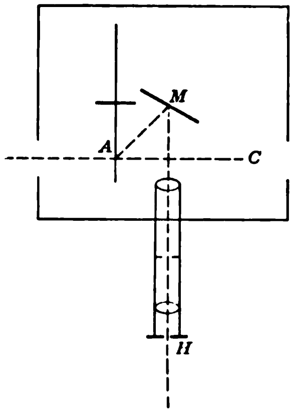

FIG. 49.—The Wild flicker photometer.

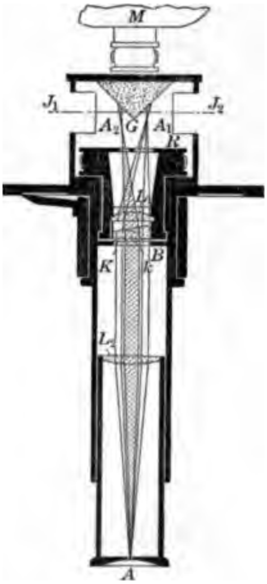

FIG. 50.—The Bechstein flicker photometer.

which formerly illuminated the center now illuminate the ring, and *vice versa.*

Hence, when the disk is rotated at a suitable speed, both ring and center will appear alternately dark and bright, and a flicker will be seen which disappears only when the center and ring are equally bright. One model of this instrument is so constructed that the observer can reverse sides of the screen without being required to change his position at the bench.

The Speed of a Flicker Photometer.—In practice little difficulty is experienced in obtaining the proper speed of the flicker photometer. This speed should be the minimum at which flicker can be made to disappear when a balance is obtained. In other words, the proper speed is that at which a flicker is most perceptible in the case of a slight unbalancing but imperceptible when a balance is secured.

Sensitiveness of Photometers.—In choosing a photometer one of the most essential considerations is its sensitiveness. Investigations along this line by Wild[1] are given in the following table. In obtaining these results the comparison device was moved to the right and brought back slowly until a balance was just noted. It was then moved a considerable distance to the left and again brought back until a balance was just obtained. The difference between these two settings was considered a measure of the sensitiveness of the photometer.

TABLE 24

Sensitiveness of Photometers

Photometer	Carbon lamps both same efficiency per cent.	Carbon and tungsten lamps 4.3 and 1.4 wpc. per cent.	Pentane and incandescent gas mantle per cent.
Ritchie wedge	2.4	5.0	8.2
Joly prism (paraffin)	2.5	4.8	8.3
Lummer-Brodhun	0.7	3.5	8.0
Bunsen ordinary single	1.2	3.6	8.7
Bunsen ordinary double	1.5	5.0	9.8
Bunsen special single	0.4	3.3	7.8
Bunsen special double	1.0	4.5	9.0
Trotter, badly made	3.5	6.2	11.5
Trotter carefully made	0.8	4.8	9.6
Simmance-Abady flicker	1.7	1.8	2.1
Wild flicker	0.8	0.8	1.0

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[1] *The Electrician.*, Vol. 60, p. 122, Nov. 8, 1907.

The Ritchie wedge was the Simmance-Abady flicker photometer with the wheel at rest. Since the edge between the two illuminated surfaces was slightly blunt the two fields were not in absolute juxtaposition.

The Joly photometer consisted of two blocks of paraffin separated by a thin sheet of paper. Again the surfaces were not in absolute contact.

The Lummer-Brodhun was of the "equality of brightness" type and for lights of the same color was superior to the others except the special Bunsen.

In the ordinary Bunsen photometer the edges of the grease spot were not sharply defined. By the single method only one side of the screen was used for obtaining the balance.

The special Bunsen employed a screen of white blotting paper partially treated with wax, the special features being that the wax was very evenly laid on, was of the same thickness on both sides, and the edges were sharply defined so that the transition from the reflecting to the translucent portion was extremely sudden.

The sensitiveness of the Trotter form of screen depends on the care exercised in cutting the card. If the edges are the least bit rough the sensitiveness falls off.

In the Simmance-Abady flicker photometer it was found that there was considerable residual flicker after the best balance was obtained due to the bluntness of the edges of the wheel and to the fact that different parts of the wheel are different distances from the lights.

In the Wild flicker device these two undesirable features were eliminated but one of less magnitude was brought in—namely, there was noticeable a slight residual flicker due to the variation of the angle of view when the eye moved across the field.

In conclusion it will be seen that two photometers of the stationary type give better results than any of the flicker instruments when the lights are of the same color, but when color differences are involved, as in comparing different types of lamps, the flicker arrangement becomes superior from the standpoint of sensitiveness.

The Precision Photometer.—Where candle-power measurements of *highest possible accuracy* are required, as in obtaining the value of the intensity of lamps to be used for secondary standards, in research work respecting the performance of illuminants, in making measurements likely to become the basis of litigation or, in fact, any photometric work where a high degree of accuracy is required, recourse should be made to the dark room and the standard bar photometer. The arrangement of the apparatus and auxiliaries for such measurements is shown in Fig. 51.[1] which represents part of the apparatus used for precision measurements at the Bureau of Standards.

For this class of work a room with its walls, floors and ceiling painted a dull black, and from which all stray light is excluded is desirable, although a room with light colored walls may be used if the necessary screening of reflected light is provided and other precautions observed. Having the dark room, the next item is the photometer. This may consist, as shown in the figure, of two bars of tubular or angular cross-section supported on a frame, and the whole device mounted on a bench or table. The standard bars are graduated to read in millimeters and are usually from 2.5 to 4.5 meters in length. These bars support three carriages, two for the lamps and one for the photometric device. These carriages are provided with clamps, by means of which they may be kept in a fixed position, and with holders for the lamps and photometer head. These holders may be raised or lowered to place the centers of the light sources and the screen in the photometric axis and at the same distance above the bars. A carriage carrying a lamp may be placed at either end of the bars and the photometric device moved along the bars to obtain a setting, or one lamp may be placed at the zero end of the bars and the photometer head and the other lamp, with their carriages rigidly connected, moved together. This latter method is more convenient since the equation for calculating the results is thereby simplified. In the figure are shown the rheostats for adjusting the voltage impressed upon the lamps, and at the left, the potentiometer for determining the correct value of this voltage. The galvanometer is shown against the wall and the translucent scale, on which a spot of light is thrown by the mirror of the galvanometer, may be seen just back of the potentiometer. A rotating sectored disk may be seen in the photometric axis to the left of the photometer screen.

A series of *screens* are shown, the function of which is to intercept the stray light reflected from the walls and ceiling. Those

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[1] *Bull. Bur. Stand.*, Vol. 2, p. 1, 1906.

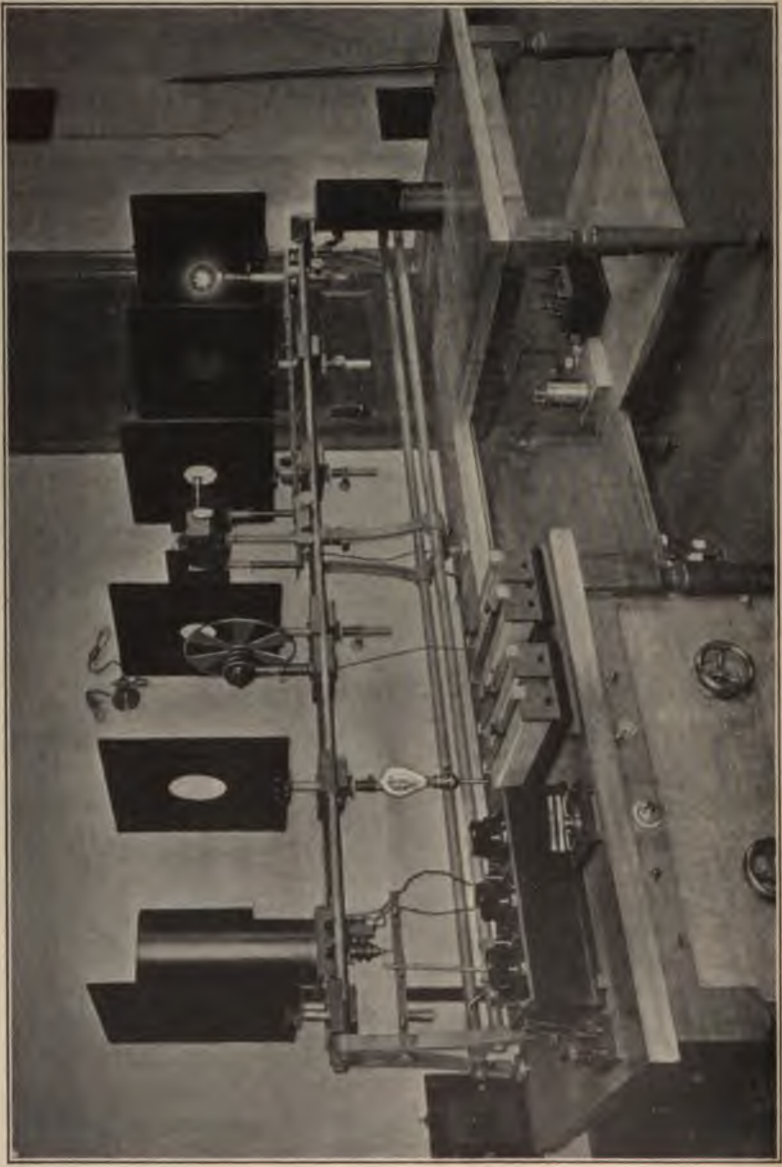

FIG. 51.—A complete photometer bench.

between the two lamps have openings at their centers and they are so placed that the direct rays from the lamps reach the photometer screen unobstructed. These screens may be made of black velvet or of any material and painted a dull black. Unless the material of which the screens are made is very thin the edges around the holes should be chamfered so as not to reflect light onto the photometer screen. Screens should also be placed so that no light will shine into the eyes of the observer.

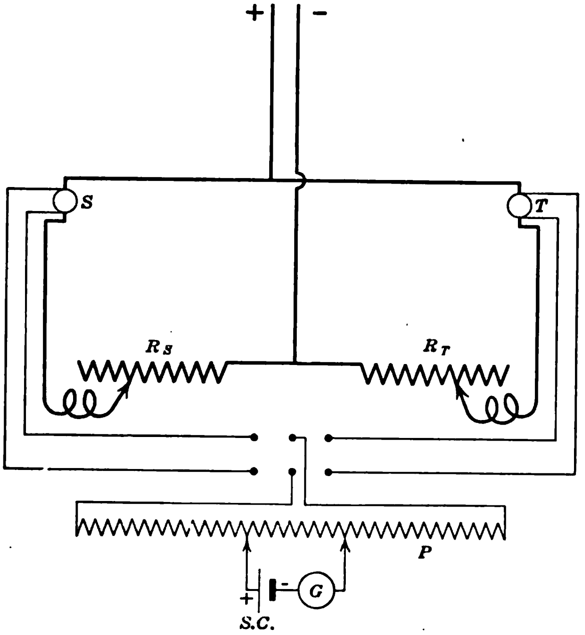

FIG. 52.—Wiring diagram for precision photometer.

Housings are also shown for the two lamps. These, however, are not necessary in a room with dark walls and ceiling but they should be lined with black velvet or dull black paint if used.

There are two methods in vogue for using this apparatus in the photometry of luminous sources, namely, the *direct* and the *substitution methods.* In the direct method the standard is placed on one side of the photometer screen and the test lamp on the other, as may be inferred from the above discussion. When a number of lamps are to be tested it becomes desirable to check the working standards from time to time and the substitution method may be employed. This method may be conveniently carried out by placing the standard lamp at the zero of the scale and the comparison lamp in the carriage attached to the photometer head. After checking the candle-power and adjusting the voltage of the comparison lamp, the standard lamp may be removed and the test lamp put in its place. This method will be found convenient when the two sides of the photometer screen are not exactly alike since it does away with the necessity of reversing the screen and averaging the readings.

The conventional diagram for connecting the electrical circuit for this apparatus with the potentiometer for determination of voltage adjustment is shown in Fig. 52. The potential across the lamps S and T may be controlled by the rheostats R_s and R_t, P represents the potentiometer and $S.C.$ and G the standard cell and galvanometer, respectively.

A Recording Candle-power Scale.—A form of candle-power scale conceived and designed by Middlekauff[1] eliminates the tedious computations usually accompanying photometric work and the difficulty experienced in using the ordinary commercial photometer scale for precision work. The scale is so constructed that it will adapt itself to the adjustment of the comparison lamp within reasonable limits and is simple in construction and operation as will be understood from the following explanation with reference to Fig. 53. The scale proper is the rectangular area $ABCD$ and is calculated on the basis that a 16-candle-power lamp is to be measured with the photometer screen at a distance of approximately 120 cm. from the lamp, a range of 6 cm. either side of 120 cm. being allowed in the construction of the scale. The sides AB and CD of the rectangle are two ordinary linear candle-power scales calculated to suit the arrangement of the photometer for adjustments of the comparison lamp. The points of graduation were determined by means of the equation

$$x^2/a^2 = X/16 \text{ or } X = \frac{a}{4}\sqrt{x} \qquad (36)$$

where a is the distance of the 16-candle-power point of graduation from the lamp and x is the corresponding distance of any other point (X-candle-power) on the scale that is being calculated. The rectangle was made 30 cm. in length so as to include a greater range of candle-power values. The balance for lamps having values above or below those indicated on the top and bottom of the scale, as for instance, 8-candle-power and 32-candle-power lamps, may be thrown within this range by means of rotating sectored disks placed on the proper side of the photometer screen.

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[1] *Bull. Bur. Stand.*, Vol. 7, p. 12, 1910.

By drawing horizontal lines through the figure it is obvious that the rectangle can be made to include an indefinite number of linear candle-power scales and corresponding to any fixed adjustment of the comparison lamp that will throw the 16-candle-power balance at any point within the 12 cm. range between the limits of the 16-candle-power diagonal line (*i.e.*, between 114 cm. and 126 cm. from the lamp), and there will be, in the rectangle, a linear scale that will exactly fit the comparison lamp adjustment.

Two important reference lines *XX* and *SS* are drawn across the figure perpendicular to each other and to the sides of the rectangle, intersecting the 16-candle-power diagonal at its middle point, which is also the middle point of the rectangle and is at exactly 120 cm. from the lamp located at zero of the photometer scale, when properly mounted for practical use. This rectangular scale so constructed on a sheet of paper is mounted on a cylinder and the line *XX* placed under the printing point attached to the photometer carriage, the photometer carriage *p* being placed so that the test side of the screen is at 120 cm. on the photometer bars. Then with a standard 16-candle-power lamp, burning at its correct voltage, at zero on the test side of the screen, and the comparison lamp burning at the proper voltage to match the standard in color, the rods connecting the photometer and comparison lamp carriages are adjusted so as to throw the intensity balance anywhere within 6 cm. either side of the line *XX*, and the apparatus is ready for making measurements. The only adjustment requiring any degree of care is that of placing the line *XX* at its proper distance of 120 cm. from the lamp to be photometered.

Settings are now made on the standard lamp and are recorded by a corresponding group of dots printed near the top of the sheet. The standard is then removed, the cylinder is turned by hand a distance sufficient to separate the group of dots just made from those to follow, and other standards or lamps to be standardized are placed in turn in the socket and for each lamp the settings are recorded by a corresponding group of dots, as before. The record is thus continued down the paper until all of the lamps of a set are photometered.

A reproduction of a complete record sheet of a series of measurements actually made in this manner on a set of six standards and seventeen lamps compared with them is shown in Fig. 53. The number of the lamps, their voltages and their candle-power, as read from the scale are written along the margin, each lamp being recorded opposite its corresponding group of record dots.

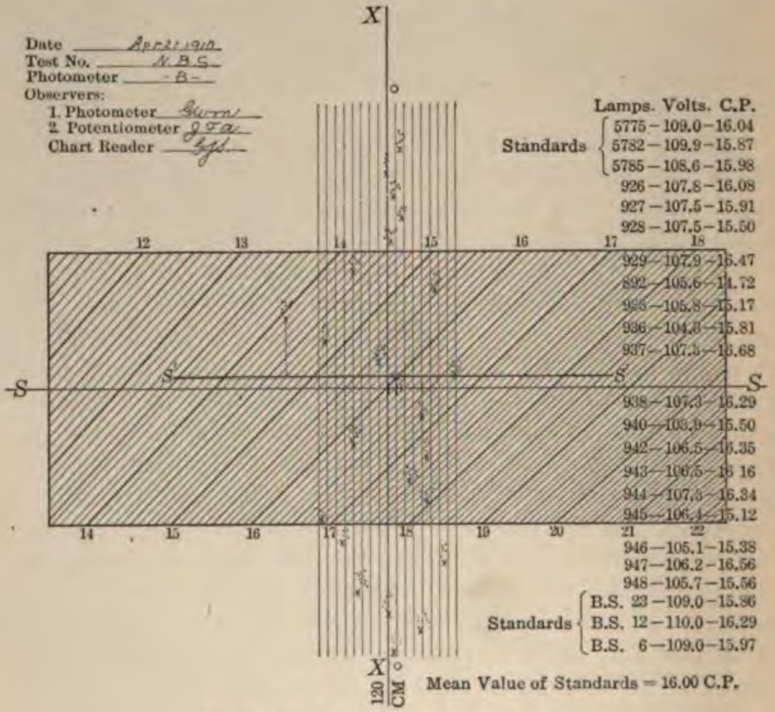

FIG. 53.—A complete record sheet.

The proper position of the candle-power scale $S'S'$ from which the candle-power values were read was determined in the following manner: The preliminary relative candle-power readings of the six standards, whose mean value was known to be 16.00 candle-power, were made from the scale SS, and the readings recorded by a mark on the scale as shown. Through the mean of these six readings, which was found to be 16.10, the line oo was drawn perpendicular to SS, and through the point of intersection of the line oo and the 16-candle-power diagonal the desired scale $S'S'$ was drawn parallel to the standard scale SS. All values, including those of the standards, were read from this $S'S'$ scale and recorded. The values of the standards in the order recorded were 16.00, 15.90, 15.98, 15.82, 16.33 and 15.96 respectively. A comparison of these values with those read from the scale and recorded on the chart gives a fair indication of the accuracy of the measurements.

The advantages of such a device for recording settings are obvious. There are no tedious adjustments required and the photometer settings are recorded rapidly and accurately. The eye is not unnecessarily fatigued by alternate reading of the photometer screen and scale, and the observer is almost entirely free from prejudice in making the settings. The record is simple, permanent, and complete, and is easily read, checked and filed for future reference.

Summary of Chapter VI

Assuming the inverse square law.

$$I_x = I_s d^2_x / d^2_s \qquad (29)$$

If the distance between the screen and the standard lamp is fixed,

$$I_x = K d^2_x \qquad (30)$$

and with the scale graduated to the square of the distance,

$$I_x = Kr \qquad (31)$$

The extent to which the inverse square law may be applied to linear and surface sources and be within experimental error are:

$d_x = 5$ times the length on the tubular source.

$d_x = 15$ times the diameter of a circular source.

There are two general types of photometers:

1. Stationary apparatus for precision work.
2. Portable photometers for illumination measurements.

Four types of photometric devices:

1. "Visual acuity."
2. "Equality of brightness."
3. "Contrast."
4. "Flicker."

Four methods of obtaining a photometrical balance:

1. Varying the distance between the lamps and the screen.

2. Use of absorbing media.
3. Rotating sectored disk.
4. Combinations of two or more of the preceding.

The Purkinje and contrast effects are troublesome in photometry when using the "acuity," "equality of brightness," or "contrast" screens for comparing different colored lights. The "flicker" photometer shows an effect, at low intensities, opposite to the Purkinje effect.

At high intensities the "equality of brightness" and "flicker" photometers give practically the same results, although the readings are more consistent and the photometrical setting more easily determined by means of the latter.

In regard to the photometry of lamps giving the same colored light preference is given to the "equality of brightness" or "contrast" types. The choice depends on their relative sensibility and this in turn depends upon the sensitiveness of the screen and the clearness of the optical system.

CHAPTER VII

PORTABLE PHOTOMETERS AND APPARATUS FOR OBTAINING THE DISTRIBUTION OF LIGHT FROM A SOURCE

Much attention is being given to the illuminating value of lamps and to the distribution of light from lamps equipped with various types of reflectors. In fact it is so obvious that in order to predetermine the details of an installation or to compare lamps on the proper basis, data of such a nature become of vital importance.

Various types of portable photometers have been designed and constructed for making illumination measurements, and several devices are in use for determining the distribution of light from an equipment. These are discussed on the following pages and a careful study of the use and manipulation of the different types of apparatus should equip the reader with a practical knowledge of this part of the subject.

The principles of portable photometry are essentially the same as those of ordinary photometry discussed in the preceding chapter. They make use of some one of the photometric devices there described, and can in general be used for candle-power measurements with possibly a less degree of accuracy. Several methods are employed for obtaining a photometrical balance. Some of these are:

1. Varying the distance between the comparison lamp and the screen.
2. The use of absorbing media.
3. Dispersion lenses.
4. Variable diaphragms.
5. Polarization media.
6. Inclination of the illuminated surface.
7. Variation of the intensity of the comparison lamp.
8. Combinations of two or more of the foregoing methods.

The most accurate and generally satisfactory system of obtaining a balance is the combination of the first and second of this list. The *absorbing medium* is used to increase the range of the instrument or to approach a balance by definite known steps, while the *variation of the distance* between the screen and the comparison lamp is used for a gradual approach and for obtaining the final setting.

In making illumination measurements a *"test plate"* consisting of a plain, white surface is used to receive the illumination to be measured. To give high sensibility to the apparatus, this plate should be made of material having a high co-efficient of reflection, and should possess a high diffusing quality so that the intensity when received from an angle will obey the cosine law. It should be of material of such a character that it will not introduce errors due to selective absorption and it should be so placed that neither the instrument nor the observer will intercept any of the light rays which would and should fall upon it. Furthermore, the instrument should be so designed that the test plate can be placed at any angle in a vertical plane passing through its center. The test plate may be of translucent material and viewed from the rear. The same specifications as to freedom from selective absorption and to diffusibility apply also to this type of test plate.

The *ideal photometer* should possess a test plate of plain white diffusing substance, the most reliable comparison source of light, the best means of varying the intensity of light admitted to the comparison device, and a photometrical device of highest sensibility. For making light surveys, and for measuring illumination in general, portability becomes of importance and a direct-reading scale whereby the values of illumination or candle-power, as the case may be, are indicated by the photometrical setting without calculation will be found a great convenience.

The manipulation of a portable photometer for illumination, candle-power or brightness measurements are given in the following discussion. There are also given descriptions of types of photometers employing the different methods of obtaining a photometrical balance.

The portable photometer most extensively used in this country is known as the **Sharp-Millar universal photometer**[1]. This instrument represents the endeavor of the inventors to embody in one piece of apparatus the features which go to characterize the ideal photometer—namely, a sensitive photometric device, the best means of obtaining a photometric balance, adaptability to the use of a reliable source of light for a comparison standard, portability, and simplicity of operation. A further requirement, in the case of the measurement of illumination, is a proper test plate to receive the illumination which is to be measured. Such a plate should have a surface of sufficient diffusibility so that its illumination varies according to the cosine law and be so located that neither the instrument nor the operator will intercept any light which would otherwise fall upon it. With due regard to these features the photometer was constructed as shown in Fig. 54. For a sensitive photometric device a modified form of the Lummer-Brodhun arrangement of the comparison of brightness type was adopted, and located at *B* where it is viewed through a telescopic eye-piece in the side of the box.

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[1] *Elect. World,* Vol. 51, p. 181, Jan. 25, 1908.

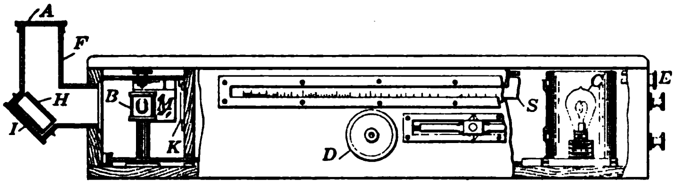

FIG. 54.—Side elevation of Sharp-Millar photometer.

The photometrical balance may be obtained by varying the distance between the comparison lamp and the screen. An electric lamp was chosen for the comparison source as would naturally be expected and either a low voltage lamp of the battery type or one of higher voltage to receive power from the service mains may be employed. When a battery lamp with a tungsten filament is employed the writer has found it very convenient to use primary cells for the source of power and maintain the intensity of the standard constant by means of a low-reading ammeter. Then by the use of lamp cord or similar conductors the photometer may be used to measure the illumination over a considerable area without having to move the meter and battery. Portability is insured by constructing the body of the instrument of wood and minimizing the number of detachable parts.

A side elevation of the photometer is shown in Fig. 54. The comparison lamp is located at *C* and can be caused to slide along a track, running lengthwise of the box, by means of a cord which is attached to the supports of the lamp and passes through pulleys and around a drum which is operated by the knob *D*. The light from this lamp falls upon a milk glass plate at *K* which is viewed through the optical device. The intensity of light upon this plate is made to vary inversely as the square of the distance from the comparison source by making the inner surface of the instrument a dull black and by interposing a system of moving screens as shown in Fig. 55. These screens are of fiber, supported on brass rods and have apertures of sufficient size to permit the direct rays from the lamp to pass unobstructed. At the same time they are of sufficient size to intercept any light reflected from the sides of the box.

The scale upon which the indications of the photometer are read, is made of translucent celluloid, graduated with an inverse-square scale, and placed in a longitudinal opening in the side of the box. This scale is equipped with a shutter which may be lowered or raised by means of an external knob, and which may be used to exclude external light from the interior of the box. To facilitate taking readings in dark places, a slit is cut in the side of the housing containing the comparison lamp, so that when the shutter is raised the shadow of the pointer, which is inside the box, is cast upon a brightly illuminated scale. On the other side of the housing is a small tube and cross-hair which, when the comparison lamp is properly placed, throws a line in the middle of a spot of light on the side of the box. In this way the proper position of the lamp is determined.

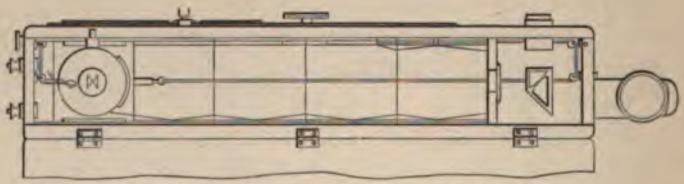

FIG. 55.—Plan of Sharp-Millar photometer.

Beneath the scale and toward the right of Fig. 54 is shown a resistance for varying the current of the lamp. At the rear end of the box are four terminals, two for the supply line and two for the voltmeter if one is used.

The elbow tube at the end of the box has a number of important functions to perform. It fits friction-tight on a collar fastened in the end of the box, and hence may be turned about a horizontal axis and set at any angle, which angle is indicated on a semi-circular scale on the end of the box. The use of a tube of this description furnishes the simplest means of measuring illumination or light coming from any direction. In the elbow of the tube is fixed a reversible plate, one side being a diffusely reflecting surface used in the measurements of candle-power, and the other a mirror used in connection with a test plate on the end of the tube for measuring illumination. In measuring candle-power, the diffusely reflecting surface is turned toward the inside of the tube, and the end of the elbow tube is open, the tube serving to screen off stray light from the plate. The distance between the plate and the source of light must be known.

The photometer being portable and the elbow tube adjustable to any angle, this arrangement lends itself to determinations of the vertical distribution of luminous intensity of light sources, and is also applicable to the measurement of the candle-power of street lamps, either close at hand or at a considerable distance from the lamps.

To measure general illumination, the translucent illumination test plate is slipped onto the end of the tube and the plate at the elbow is reversed, making its mirrored side effective. The illumination on any required plane can then be measured, the light having unobstructed access to the test plate. It has been found that the test plate of translucent glass, ground on its upper surface so that its power of regular reflection is destroyed, is well adapted to the purpose.

It has been found to be most convenient to make the scale direct-reading in foot-candles. The same scale is equally applicable to the measurement of illumination and of candle-power. The foot-candle scale in question has a range from 0.4 to 20 foot-candles. This range can of course be increased considerably by changing the candle-power of the comparison lamp, either by altering its voltage or by using a lamp of different size; but the range given with any one lamp cannot be considered sufficient to meet all the conditions which are present in practical work. It is not, in general, advisable to change the voltage of the comparison lamp.

The change from one comparison lamp to another is an inconvenient operation. Recourse has been had, therefore, to the use of absorbing screens. Two of such screens are used, one of which transmits substantially 10 per cent. of the light falling upon it, while the other transmits only 1 per cent. of the light. These two screens are attached to a movable holder, which is shown in detail in Fig. 56. By turning the knurled head, which is exposed when the cover of the photometer box is lifted, either screen may be interposed between the prism and the milk glass window, or between the prism and the elbow tube, or both screens may be turned to such a position that they intercept no light. Evidently when the screens are used, the total range of measurement, with but one comparison lamp, is from 0.004 foot-candle to 2000 foot-candles, within which range all measurable lights and illuminations can be said to fall. It is to be understood, of course, that the coefficient of absorption of these screens must be determined exactly by measurement.

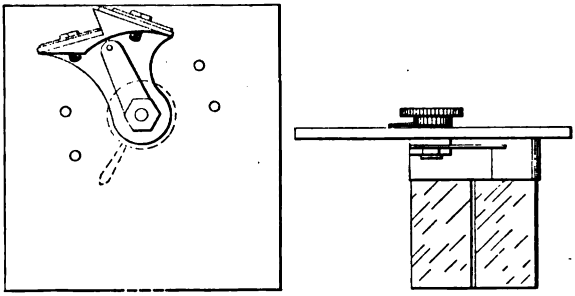

FIG. 56.—Plan of screen system.

In the design of this instrument no endeavor has been made to meet the condition which obtains in most photometers, namely, that the light from the comparison lamp and from the lamp or illumination to be measured shall transverse similar and symmetrical paths, and that any absorption which is suffered by one light must be suffered by the other also. It is considered that this condition is of no importance whatever as long as the photometer is calibrated and used after the substitution method, which is undoubtedly the correct method for all photometric comparisons. The proper method of calibrating the instrument for the measurement of candle-power or illumination is by employing a known candle-power or illumination produced by a standard lamp. The voltage or current of the comparison lamp is adjusted by means of the slider rheostat on the box to such a value that the pointer on the scale indicates the known candle-power or illumination. The instrument then becomes direct reading for candle-power or illumination as the case may be, all effect of lack of symmetry, etc., being eliminated.

As has been said, the performance of the photometer in practice has been found to be very satisfactory. It has been used in widely different classes of work, as for instance, measurement of candle-power of incandescent lamps, laboratory measurements of arc and mercury-vapor lamps, street measurements of the candle-power of arc lamps and of incandescent gas lamps, measurements of illumination in rooms illuminated by incandescent lamps, arc lamps, Moore tube, etc. The choice of photometric device has been found to be an unusually good one, since its sensibility is so high as to enable very quick settings to be made, and it also enables extremely feeble illuminations or candle-power to be measured, such as are met with, for instance, in photometering an arc lamp at a distance of 250 ft. Moreover, the method of varying the distance of the comparison lamp is very convenient and quick, and the scale being illuminated from the inside can be read very quickly, so that a rapidly fluctuating source of light, such as the arc lamp, can be followed and the amount of fluctuation can be determined.

When measuring illumination in a room lighted by incandescent lamps, it is advisable to operate the comparison lamp from the same circuit that supplies the illuminating sources, since, by so doing, the effect of variations in voltage are minimized.

The Sharp-Millar photometer just described is made in two sizes. The standard apparatus is about 23 in. in length. A larger one is also made for precision work which is approximately 39 in. long.

A third model of smaller size has been designed for fieldwork where portability is of importance.[1] In some respects this instrument is slightly less accurate than the standard instrument. The length is 14.5 in. and the cross-section 2.5 in. by 2.5 in. The plan of the small model is illustrated in Fig. 57, showing the essential features. *P* is the reversible tube, bearing test plate and diaphragm; *T* is the elbow tube; *G* and *G* are absorbing screens; *S* is the photometric device, *D* the diffusing glass, *I* the interrupter for the telephone circuit, *L* the comparison lamp carriage, *R* an adjustable rheostat, *W* a screw-driven cursor on slide wire in bridge, *K* the knob for the comparison lamp drive. The troublesome feature in all small portable photometers is the comparison lamp. In the new design the large resistance-temperature coefficient of the tungsten filament is taken advantage of. The lamp is connected so as to constitute one arm of a Wheatstone bridge. The other arms are composed of small coils of manganin wire. Between one lamp terminal and one coil a portion of this wire is extended to form a slide wire, and a scale is arranged to indicate the position of the cursor on this wire. The coils have such a resistance that when the lamp is receiving its proper current its resistance will be such that the balance point of the bridge will come within the range of the cursor on the slide wire. Any change in the current through the lamp is accompanied by a corresponding change in its resistance and the bridge is thrown out of balance. As a convenient means of detecting the point of balance of the bridge, a low-resistance telephone receiver is used. In series with the receiver is an interrupter consisting of a wheel into the periphery of which pieces of insulating material have been set at frequent intervals. The two bridge arms which are in parallel with the lamp and its corresponding arm have approximately five times the resistance of the latter, so that the amount of additional current which they require produces no undue drain on the battery. The telephone receiver is provided with a head band so that both hands are left free for the operation of rotating the contact wheel and adjusting the external rheostat until a condition of silence in the telephone is attained. The adjustment of current in this way can be made to within about 1 milliampere (one-third of 1 per cent.), which is substantially the limit of accuracy of a high-grade ammeter. Therefore the

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[1] *Illum. Eng.*, N. Y., Vol. 6, p. 534, 1911; *Illum. Eng.*, Lond., Vol. 5, p. 7, 1912.

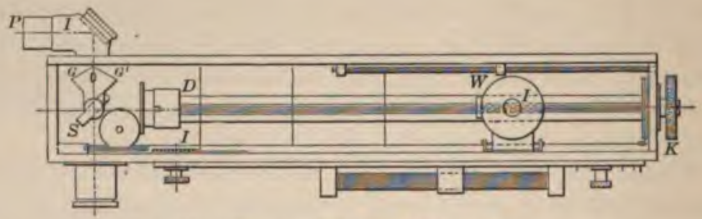

FIG. 57.—The small Sharp-Millar photometer.

measurement involves in the shape of apparatus external to the photometer only the telephone receiver, which can be readily carried in the pocket if desired. The optical system has been somewhat changed. In place of the modified Lummer-Brodhun prism as a photometer screen, a circular mirror of thin glass, silvered on the back, with a small round hole cut in the silvering, is employed. The mirror is placed at an angle of 45 degrees to the axis of the box and is viewed through the eye-piece in the ordinary way. A single small storage cell or two dry cells give the current for the lamp. The calibration and the use of the instrument have been described.

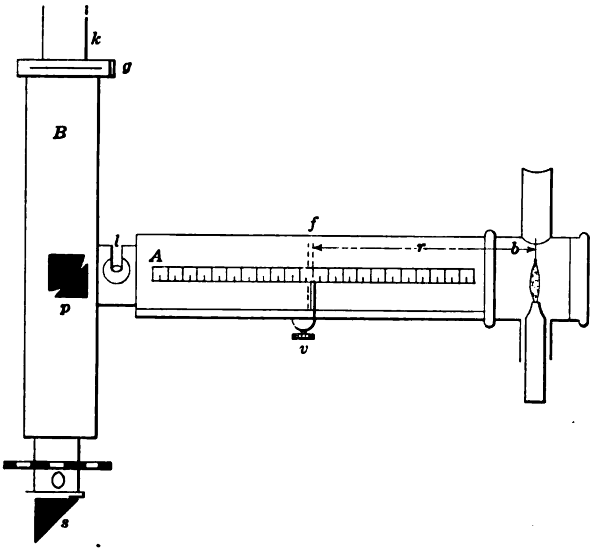

Fig. 58.—The Weber photometer.

The **Leonard Weber photometer** is especially adapted for the measurement of the intensity of illumination. It consists of a tube about 30 cm. long (Fig. 58) supported on an upright rod so that it can be raised or lowered or turned in any desired direction. The standard source of light is contained in a lantern fastened to one end of the tube *A*. Within this tube is a circular piece of glass which can be moved lengthwise of the tube and its position read on a scale. At right angles to this tube, and supported at the end of the tube opposite the lantern, is a second tube *B* which can be rotated in a vertical plane, and its position relative to the vertical can be read on a graduated circle. A photometric train of the Lummer-Brodhun type which is contained in the tube B in its axis of rotation, receives light from the opal-glass plate in tube A and reflects this light toward the eye-piece o in the tube B, so that the outer portion of the field of vision is illuminated by light from the standard source. The central zone of the field receives light from the source to be measured, through the prism p and the tube k. The intensity of this light can be decreased by means of one or more opal plates placed at g in order to make it comparable with that from the standard lamp. The tube B in making measurements is turned toward the light to be measured and it differs not whether the light be direct or reflected and diffused. A white screen of rough finish is usually used in connection with this apparatus and it can be set up at any desired inclination to the source of light. The photometer may be placed at any convenient position and the tube B directed toward the center of the screen, the only restriction being that the field of vision receive no rays other than those emanating from the screen. The opal-glass plate in tube A is now moved until both the central and the outer portions of the field of vision appear equally luminous. The distance r of this plate from the standard source of light, at the time of equal illumination on the comparison screen, is read on the scale in millimeters and the intensity of light on the screen calculated from the formula

$$I = KR^2/r^2 \tag{37}$$

where R is the distance of the light from the screen and K is the constant previously determined as follows: A standard candle is placed exactly one meter distant from the white screen and the tube B pointed toward its center and so placed that the eye perceives nothing but the light coming from the screen. The light intensity on the screen will then be 1 meter-candle and having obtained a balance the constant can be easily calculated. If other distances or standards are taken the difference must be taken into consideration. The photometer can be taken apart and packed in a small box 24×12×6 in. which recommends it as a portable piece of apparatus and as such a miniature incandescent lamp is found very satisfactory as a standard; it should, however, be calibrated before and after each test.

The Weber photometer is now made in a more portable form and with some modifications as shown in Fig. 59. The screen is fixed relative to the photometric device, but may be placed at any angle in a vertical plane, the photometric device moving with it around the horizontal axis of the comparison lamp. A photometric balance is obtained by moving the standard lamp along the photometric axis. This photometer is designed to employ the Hefner lamp as a standard. The result indicated by the position of the standard lamp when a balance is obtained, is in terms of a unit of illumination intensity. The range is increased by the use of absorbing screens.

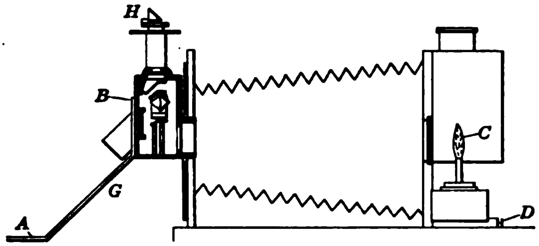

FIG. 59.—Portable form of Weber photometer.

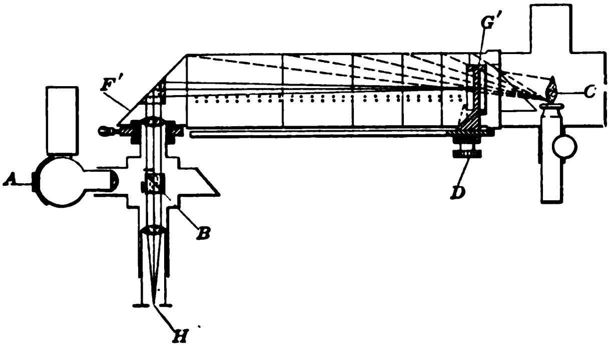

FIG. 60.—The Bechstein form of Weber photometer.

Another modification of the Weber photometer known as the **Bechstein form** is shown in Fig. 60. A translucent plate *A* receives the light to be measured and is viewed through the optical device *H* and the prism *B*. The comparison lamp *C* is a benzine lamp in this case. The photometric balance is obtained moving by the translucent plate *G* along the photometric axis;

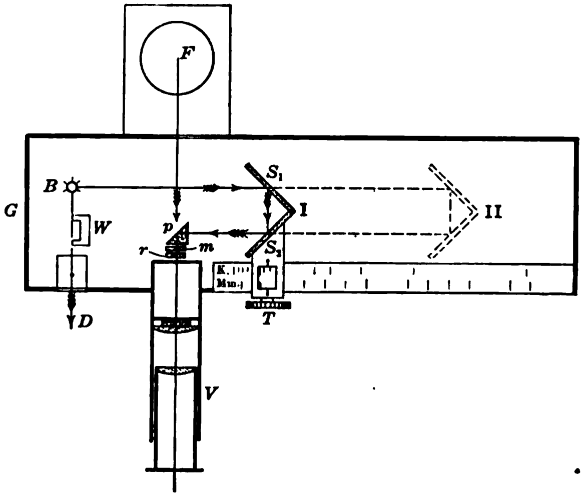

FIG. 61.—Marten's photometer (horizontal section)

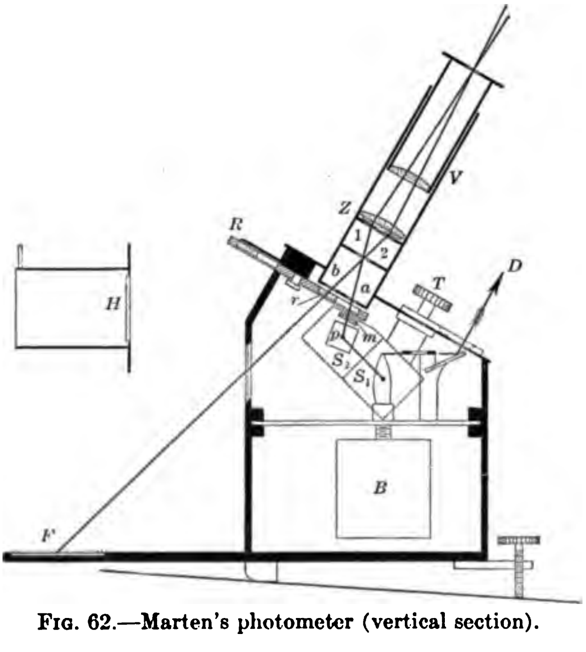

FIG. 62.—Marten's photometer (vertical section).

the light transmitted by this plate is reflected by the prism *F* and is received at the eye-piece through the prisms *B*. This photometer may be calibrated similar to the Weber and its range can be extended by increasing the density of the plate at *A*.

The Marten Portable Photometer.—A horizontal sectional view of the Marten portable photometer is shown in Fig. 61 and a vertical section in Fig. 62. The light to be measured falls on the test plate *F* which is observed through the sight tube *V* through which may also be seen the surface of the plate *p*, illuminated by the comparison lamp *B*, through a pair of mirrors S_1 and S_2. These mirrors may be moved nearer or away from the lamp by means of a rack and pinion device *T* thus varying the intensity on the plane *p*. The photometrical setting is indicated by a pointer attached to the mirrors and is read directly from a scale graduated proportionally to the square of the distance through which the light from *B* passes in reaching *p*.

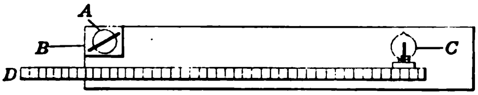

FIG. 63.—Ryan's photometer.

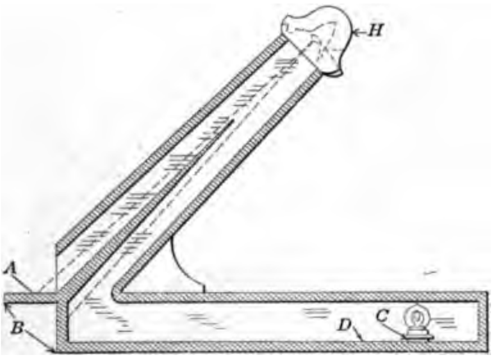

FIG. 64.—The Burnett photometer.

In **Ryan's photometer,** Fig. 63, the light to be compared falls at *A* upon one surface of the photometric device, *B*, which consists of a translucent block, diagonally divided by a thin, opaque film. The comparison lamp, *C*, illuminates the other half of the block. The distance between this lamp and the block is varied by means of the rod, *D*, upon which is calibrated a scale which indicates the results of the photometric settings. The photometric device, *B*, is viewed through the side, equal portions of the two halves of the block being present in the field of vision.

A vertical section of the **Burnett photometer**[1] is shown in Fig. 64. The light to be studied falls upon a test-plate, *A*, which is one surface of a photometric wedge. The other surface of the wedge is illuminated by the comparison lamp, *C*, which can be moved along the horizontal axis of the box. The two surfaces of the wedge which constitute the photometric device, *B*, are viewed from the position, *H*, through a box, divided by a partition which constitutes a well-defined line of separation between the two illuminated surfaces of the wedge. The results are indicated on a scale on the horizontal box which is calibrated directly in foot-candles.

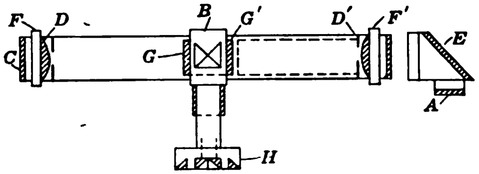

FIG. 65.—The Blondell and Broca photometer.

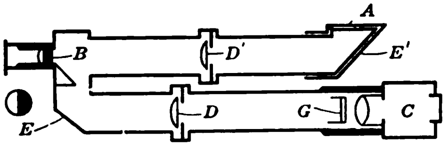

FIG. 66.—The Mascart photometer.

The **Blondel** and **Broca photometer**[1] is shown in Fig. 65. That portion of the light which is transmitted through the test-plate, *A*, is reflected from the 45-degree mirror, *E'*, through the lens *F'*, upon the ground-glass plate, *G*, which forms part of the photometric device. Before the lens, *F'*, is placed a "cat's-eye" diaphragm, *D'*. A comparison light source, *C*, which is not described, is placed to the left of the apparatus. The light passes through the lens, *F*, and falls upon the ground-glass plate, *G*. A second adjustable "cat's-eye" diaphragm, *D*, is placed before the lens, *F*. The photometric device, *B*, consists of crossed prisms through which the ground-glass plates, *G* and *G'*, are viewed, from the binocular arrangement, *H*. Equality of illumination is produced by the adjustment of the diaphragms, *D* and *D'*, which are equipped with means for indicating the size of the aperture.

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[1] Trans. Illum. Eng. Soc., Vol. 2, p. 546, 1907.

In the **Mascart photometer**,[1] shown in Fig. 66, the light to be studied falls on the translucent screen, *A*, which constitutes the test-plate. That which is transmitted passes through the 45-degree mirror, *E'*, a convex lens, and falls upon the upper half of the translucent screen, *B*. A comparison lamp, *C*, illuminates translucent screen, *G*. Light which is transmitted through this screen passes through a convex lens, the 45-degree mirror, *E*, the totally reflecting prism immediately above *E*, and falls upon the lower part of the screen *B*, where it is viewed from position *H*. Both of the lenses just mentioned have adjustable

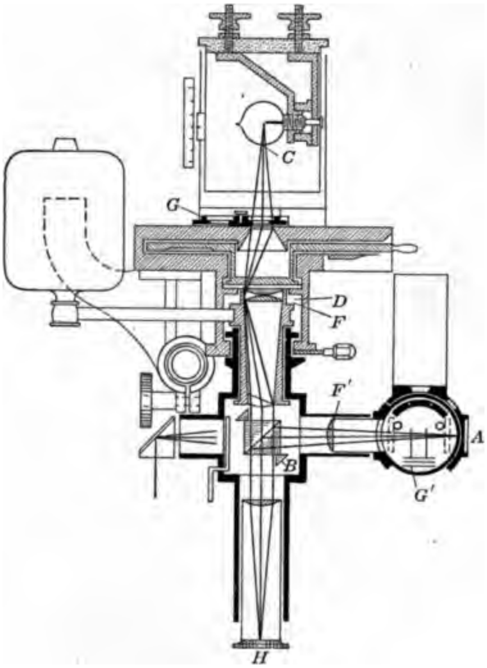

Fig. 67.—The sector type photometer.

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[1] Trans. Illum. Eng. Soc., Vol. 2, p. 546, 1907.

diaphragms, D and D', which are used to vary the intensity and effect equality of illumination upon screen B. The adjustment of these diaphragms indicates the result of the photometric settings. The comparison lamp, C, is a flat-flame gasoline lamp. The instrument is calibrated by means of a Carcel lamp, whose light falls upon the test-plate, A.

The **sector type photometer,**[1] designed by Walter Bechstein, is shown in Fig. 67. The test-plate, A, is placed at the point where it is desired to study the illumination. This plate can be rotated throughout a vertical plane about the photometric axis. B is a Lummer-Brodhun cube through which the lower surface of the test-plate, A, is viewed. Through the prisms, B, may be viewed also the translucent plate G, which is illuminated by the comparison lamp, C. Equality of illumination is obtained by varying the size of the opening in the variable sector disk, D, and is determined by viewing the plate, G, through a portion of the lenses, F, which can be rotated. The result of the photo-

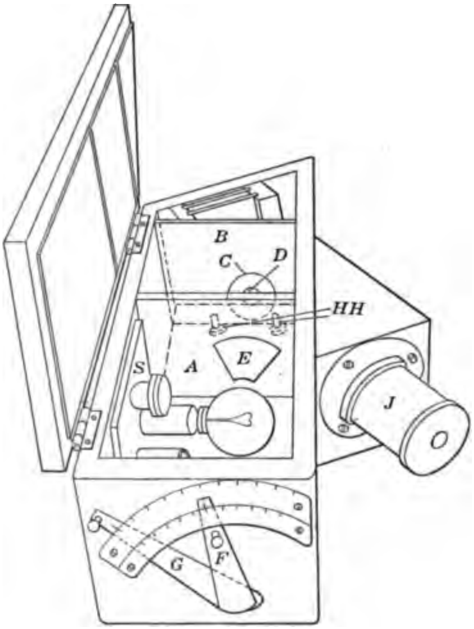

Fig. 68.—The "Lumeter."

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[1] Trans. Illum. Eng., Soc., Vol. 2, p. 546, 1907.

metric setting is indicated by the size of the opening in the disk, *D*. This device is in effect a rotating sector disk except that the beam of light instead of the disk is rotated, thus facilitating precise adjustment of the sector disk.

The **"Lumeter"** is the name applied to an ingenious little device of English make for determining values of brightness. It consists of a small box 8.5×2.75×2.5 in. divided into two chambers, *A* and *B*, Fig. 68. *A* contains a small incandescent lamp and is lined with a matt-white surface; *B* is a dark chamber with a matt-white screen *C* and a small circular aperture *D*. By means of a tubular focusing eye-piece which enters the back chamber obliquely, the matt surface white screen can be observed, the circular aperture *D* appearing in the center of this field. A hole at the far end of the box allows an uninterrupted view of outside objects through the opening *D* in the center of the matt-white screen *C*. The light from the incandescent lamp passes through the translucent screen *E* and illuminates the screen *C*. This translucent glass is segmental in shape and of sufficient density to be of uniform intensity on the far side. The amount of light allowed to pass through this glass opening is proportional to its area and this area is varied by means of two sectors which are operated by two indicating levers *F* and *G* on the end of the case. One of these segments, *G*, Fig. 69, closes the whole aperture with the exception of a segmental band in the center thus reducing the area from 1 to 0.1. The other sector, *F*, can be used to reduce this area from 0.1 to 0.01, thus it will be seen that with the proper candle-power lamp in the chamber *A* the indicator attached to the segment *G* may read tenths and the indicator attached to *F* hundredths of a foot-candle which will be the values of the intensity of illumination of the screen *C*.

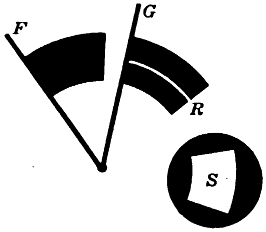

FIG. 69.—The sectors for the lumeter.

To operate this photometer it may be held in the hand and pointed at the object to be examined so that the object or portion of it may be seen through the opening *D* when looking through the eye-piece. Then by means of the indicators which regulate the illumination of the screen *C* the intensity of the screen *C* is varied until it matches the intensity of the object which appears in its center. The intensity of the light reflected from the object under examination is then given on the scales by the positions of the indicators.

For measuring the brightness of objects of greater intensity than 1 foot-candle on the screen *C*, slides of smoked glass are provided which may be placed between the object being measured and the opening *D*. These slides are operated by the knobs *HH*, one slide *g* giving a maximum range of 10 foot-candles and both slides together increasing it to 100 foot-candles. The position of the standard lamp can be varied so that if a lamp of the right candle-power be chosen it can be so placed that the instrument will be direct reading. This position can be determined or the instrument may be calibrated by illuminating an auxiliary screen, which comes with the apparatus and is presumably of the same material as *C*, with a standard lamp placed at a known distance, and matching this screen with *C*. It is obvious that when once calibrated in this way the Lumeter may be used with the auxiliary screen to measure illumination intensities. In this case the screen should be kept at the same angle relative to the instrument as when it was calibrated.

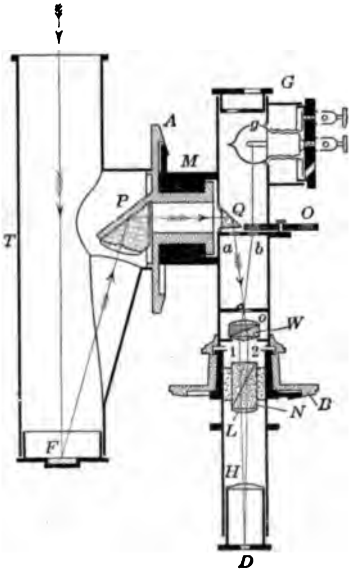

FIG. 70.—Marten's polarization photometer.

Marten's polarization photometer made by Schmidt and Haensch is shown in Fig. 70. As the name implies, the photometrical setting is secured by employing the polarization principle. The light from the unknown source falls upon *F* and passes through the prisms *P* and *Q* to the prisms *W* and *N*, these being a Wallaston and Nichol prism respectively. The direct rays from the standard comparison lamp pass direct to the prisms *W* and *N*. At *W* the rays from the two sources are divided into beams polarized in planes at right angles to each other. If the plane of the prisms at *N* is at 45 degrees to the planes of the polarized rays the light from both sources will be reduced in the same proportion. By turning the prisms at *N* from this position the intensity of one field will be increased in brightness and the other decreased. It is obvious that a scale attached to the circle *B* can be so calibrated that the position of a pointer attached to the prisms at *N* will indicate the measurement when a photometrical setting is obtained. With an opaque diffusing screen at *F* the apparatus is used to measure candle-power. By substituting a plate of milk-glass for *F* and closing the other end of the tube or by placing a mirror at *F* and a plate of milk-glass at the other end of the tube the instrument may be used to measure illumination intensities. By leaving an opening at *F* it may be used to measure surface brightness. The range of the instrument is increased by means of a set of smoked-glass

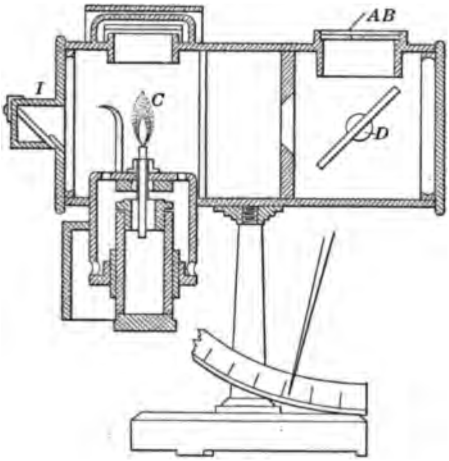

FIG. 71.—The Preece and Trotter photometer.

screens in the disk *O* any one of which may be placed before the opening *b*.

The **Preece** and **Trotter photometer**,[1] of which a vertical section is shown in Fig. 71, has a test-plate, *A*, upon which the light to be studied falls and which is also a portion of the photometric device, *B*. The comparison lamp, *C*, illuminates a piece of bristol board, *D*, which may be inclined at any desired angle. Through slits in the surface, *A*, portions of the screen, *D*, may be viewed. Photometric balance is obtained by equalizing the brightness of the surfaces, *A* and *D*. *I* consists of a mirror and gage for adjusting the height of the flame of the lamp, *C*.

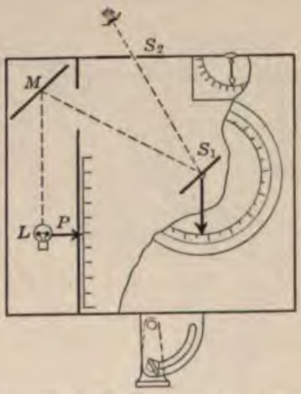

Fig. 72.—Trotter's universal photometer.

The **Trotter universal photometer**[2] is shown in Fig. 72. The comparison source is an incandescent lamp the distance from which to the mirror *M* can be varied as indicated in the figure. The rays from this lamp are reflected by the mirror onto a white diffusing screen *S* which can be rotated so that the light strikes it at various inclinations. It will be seen that the range of the instrument is made greater by having it made so as to vary the distance between the comparison lamp and the mirror as well as rotating the illuminated screen. The illumination to be measured falls upon the surface *A* and is compared with the illumination of the screen *S* viewed through an opening in the test-plate *A*.

In Fig. 73 is shown the arrangement of the **Marshall "Illu-**

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[1] Trans. Illum. Eng. Soc., Vol. 2, p. 546, 1907.

[2] *Illum. Eng.*, Lond., Vol. 1, p. 502, 1908.

minometer."[1] The test-plate, A, is also the upper surface of the photometric device, B. This consists of a screen, the translucency of which is varied by using different thicknesses of paper. It is illuminated from beneath by the comparison electric lamp, C, the intensity of whose light is varied by means of the rheostat, D. The lamp, C, is operated from two dry cells within the instrument, a spring switch being placed in the circuit so that the cells are in use for a minimum time only. To effect a photometric setting, the amount of current through the lamp, C, is varied until disappearance of contrast is obtained at the screen, B, when viewed from above at an angle of about 45 degrees to its surface. The resistance of the lamp circuit is then measured by means of a simple Wheatstone bridge with a portable galvanometer, both inclosed within the instrument. One arm of this bridge consists of a tightly drawn resistance wire, supported over a millimeter scale. The contact is moved along this exposed wire until the point of no deflection is obtained and the foot-candle value then calculated from the scale reading by means of an interpretation curve.

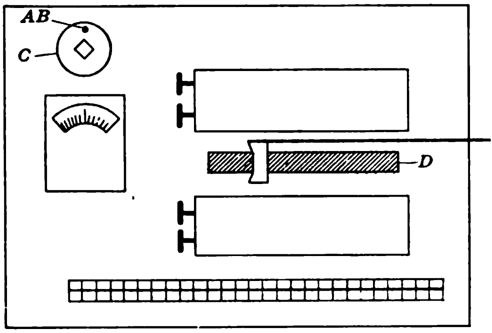

FIG. 73.—The Marshal "illuminometer."

A device known as the **luxometer** is illustrated by Fig. 74. Its operation is obvious. The intensity of the sources in the direction of the hole in the box are compared by observing the brightness of the illuminated spots on the piece of white paper placed in the bottom of the box. Such a device as this must be used with caution and used only for comparing lamps equipped with the same type of reflector since it gives no clue as to the general distribution of light but only the comparative values in one and only one direction. It is obvious that this method should not be used for the comparison of reflectors.

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[1] Trans. Illum. Eng. Soc., Vol. 2, p. 546, 1907.

A simple device known as the **illumination tester** for obtaining an idea of the illumination of interiors is indicated diagrammatically by Fig. 75.[1] The illumination is received on the test-plate *c* which is viewed through an opening *a* in *bg*. Directly above *a* is located a convex lens *d* which focuses the light passing through it at *a*. Above the lens *d* is a mirror *e* which can be turned about a horizontal axis. The observer looking along the direction *bac* observes the surface *c* through the opening at *a*. He then turns the mirror *e* in such a way that an image of a portion of the sky is formed on *bg* near the aperture *a*. The results are merely comparative, indicating whether the illumination is greater or less than the amount thus received through the mirror and lens from the sky.

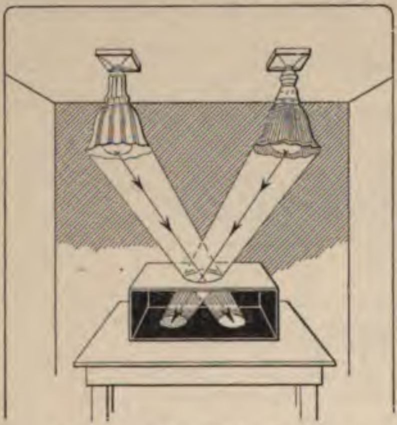

Fig. 74.—The "luxometer."

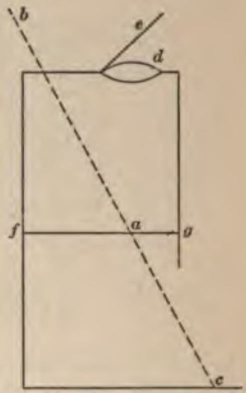

Fig. 75.—The illumination tester.

In Fig. 76 may be seen the general arrangement of the **Harrison portable flicker photometer**[2] which was designed for measuring street illumination. The comparison source is an incandescent lamp *C* which receives energy from a battery, and which illuminates the revolvable screen *D*. *A* is a sector-disk which may be driven at any desired speed by means of a small air-blast motor. On this screen is received the illumination to be measured and through the openings between the sectors may be seen the screen *D* by means of the mirror *M*.

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[1] *Illum. Eng.* Lond., Vol. 1, p. 505, 1908.

[2] *Illum. Eng.*, Lond., Vol. 1, pp. 504 and 1051, 1908.

Looking through the eye-piece *H* the observer sees first the illuminated white surface of a sector of *A*, and then in the mirror *M* the illuminated surface of the screen *D*. A photometrical balance is obtained by altering the inclination of the screen *D* until no flicker is perceived by the eye. The illumination of the screen *D* is indicated by the position of the pointer which moves over a scale graduated in foot-candles. In the position shown in the diagram the photometer is arranged to measure the intensity of illumination normal to a direction of 45 degrees to the vertical, but by tilting the photometer the intensity in any desired direction can be measured.

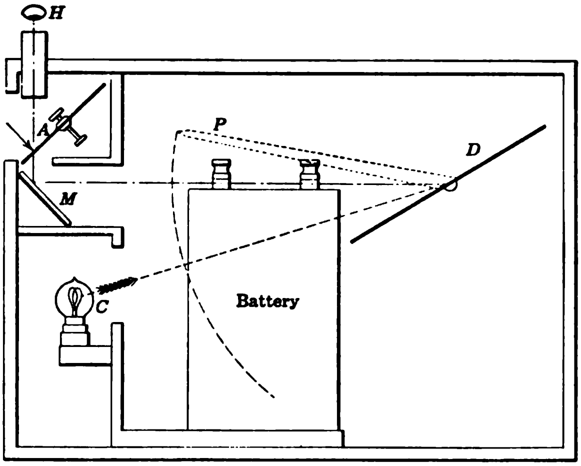

Fig. 76.—The Harrison portable flicker photometer.

By setting the disk so that part of the mirror may be seen, one is enabled to use the photometer as a direct comparison instrument. In this case the disk is set so as to receive light from the source to be measured and placed beside this may be seen the screen *D* in the mirror *M*. The range of this instrument is increased by having lamps of different candle-power so arranged that any one can be put in circuit by inserting a plug.

Determination of the Mean Horizontal Intensity of a Source of Light.—There are the present time several methods for the determination of the so-called mean horizontal intensity of incandescent lamps, *i.e.*, the mean intensity in the plane perpendicular to the axis of the lamp and passing through its center. The oldest of these methods, and one which is equally applicable to other light sources, consists in measuring the intensity at equal angular intervals in the horizontal plane and either taking the mean of these observed values or plotting the observations and determining the mean radius vector of the curve drawn through the plotted points.

A second method is an abbreviation of the first. According to this second method photometric measurements are made in a single fixed direction and the mean horizontal intensity is computed from mean horizontal reduction factors previously determined for each different type of lamp. This method is not recommended where accurate results are desired because of the difference which exists in lamps having a single type of filament. This difference occurs partly in the shape of the filament and partly in the non-uniformity of the glass globe.

A third method, and one which is used almost universally in this country in all practical determinations of the mean horizontal intensity of incandescent lamps, consists in spinning the lamp about its axis at a uniform speed of 180 revolutions per minute. For this purpose a device has been designed having a revolvable socket to which electrical connections are made by means of brushes and collector rings. The socket is supported by an arrangement whereby readings can be taken not only in the horizontal plane but in other directions in a vertical plane through the axis of the lamp with the lamp revolving about its axis. In this way, the mean intensity in different directions in a vertical plane may be determined.

This method possesses two possible sources of error, one caused by the distortions of the filament due to centrifugal force and the other due to flicker which is perceptible in nearly all types of lamps when rotating at 180 r. p. m. Investigations have shown, however, that at 180 r. p. m. changes in intensity due to centrifugal action lie within the range of experimental error. But with most types of lamps the speed cannot be raised until flicker disappears without introducing error due to distortion of the filament. The principal source of error[1] appears to be due to flicker and this error differs with different individuals. The conclusion drawn from the investigations cited above is that where the horizontal distribution curve deviates considerably from a circle so that a bad flicker results when the lamp rotates at 180 r. p. m., some other method for determining the mean horizontal intensity should be employed.

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[1] *Bull. Bur. Stand.*, Vol. 2, p. 426, 1906.

A fourth method of obtaining the mean horizontal intensity of a lamp consists in rotating a pair of mirrors about the lamp which is held in a stationary position. The lamp is mounted with its axis horizontal and coincident with the photometric axis, the tip of the lamp being turned toward the photometer screen. The two mirrors, inclined at approximately 90 degrees to each other, reflect to the photometer screen, light emitted from the lamp in a direction perpendicular to its axis. The direct rays from the lamp are prevented from reaching the photometer by a small screen. The mirrors rotate about the the axis of the lamp and reflect to the screen a quantity of light proportional to the mean horizontal intensity of the source. This method necessitates, of course, the determination of the reflection coefficients of the mirrors, which is accomplished by

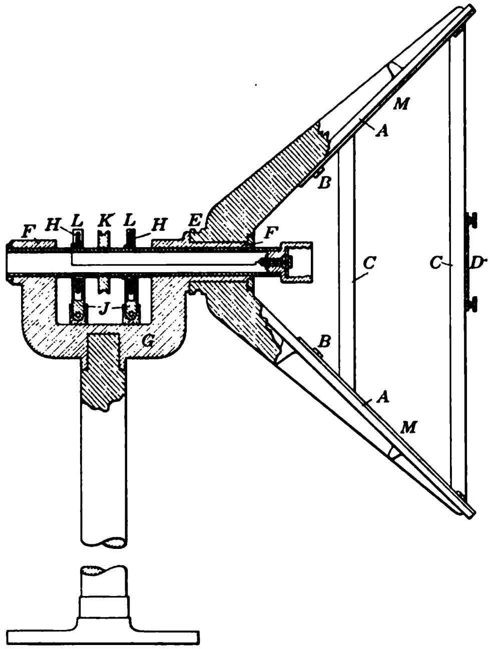

FIG. 77.—Rotating mirrors.

means of a standard lamp, the mean horizontal intensity of which has been previously determined by the first method mentioned above.

The arrangement of the rotating-mirror apparatus is shown in Fig. 77.[1] For experimental purposes it was so constructed that the lamp could be rotated independently of the mirrors. The mirrors may be rotated by a motor belted to pulley *E*. The mirrors are held to the frame by the bolts *B* and the frame prevented from spreading by the stay rods *C*. The screen *D* intercepts the direct rays from the lamp.

Apparatus for Obtaining Values for the Polar Curve.—In determining the distribution of light around a lamp which must be burned in a certain position, as for instance an arc lamp which cannot be inclined without introducing variations in its candle-power values, it becomes necessary to resort to other means than those heretofore described.

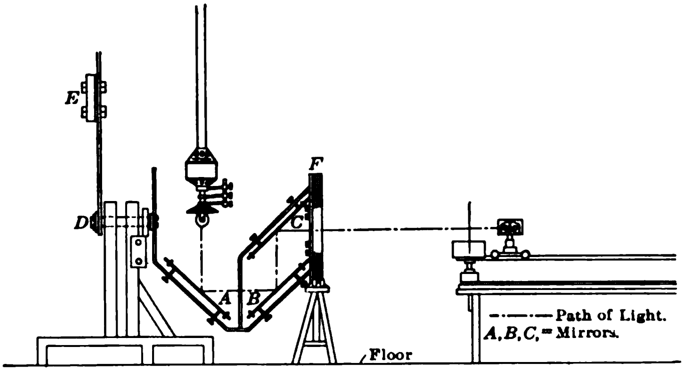

FIG. 78.—Mirror reflector photometer.

A *device for this purpose consisting of three mirrors A*, *B*, and *C* revolvable about an axis *D* is shown in Fig. 78. It will be seen that the beam of light follows a path indicated by the broken line and is compared on the photometric device shown in the right-hand side of the figure. It will be seen that the mirrors can be rotated so as to measure the candle-power in a vertical plane. It is obvious that this device can be used for photometering any ordinary source of light and can be calibrated by substituting a source of known candle-power in the same way as with the portable photometers.

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[1] *Bull. Bur. Stand.*, Vol. 2, p. 419, 1906.

A *simpler arrangement* is shown in Fig. 79 where only two mirrors are used. The manipulation of this arrangement is the same as the preceding.

Numerous other contrivances are used for a similar purpose. *A single mirror* may be used with an arrangement of apparatus as indicated in Fig. 80.[1] In this case the lamp is moved around the mirror and the mirror turned with the lamp so that the rays from different directions around the lamp will strike it at the same angle.

Another arrangement in which the *lamp remains stationary and the mirrors revolve* is shown in Fig. 81.[1] This method embodies many desirable features and deserves more than passing

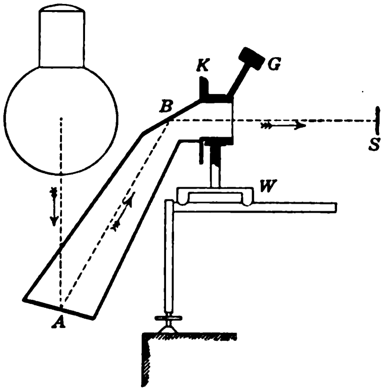

Fig. 79.—Mirror for determining the distribution of light.

notice. It will be seen that the direct rays from the lamp are intercepted by a black screen placed in the photometric axis. The light which is measured is reflected by the mirrors and strikes the photometer screen at an acute angle. This necessitates *calibrating the apparatus* which may be accomplished either by placing a source of known candle-power in the same place as the test lamp, or by removing the intercepting screen and the mirrors and obtaining the constant of the apparatus by comparing the horizontal intensity obtained in this way with the readings obtained when the mirrors were in the horizontal plane and the screen in position. It is obvious that the position of the mirrors as shown is merely to illustrate the construction and when in actual operation each is similarly placed on opposite sides of a vertical line passing through the center of the source of light. Thus if it be desired to determine the candle-power 15 degrees below the horizontal a mirror should be placed on opposite sides of the lamp 15 degrees below the horizontal.

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[1] Trans. Illum. Eng. Soc., Vol. 6, p. 641, 1911.

Obviously one mirror can be used instead of two, but two mirrors offer an advantage in arc lamp photometry by reducing the fluctuations in intensity due to unsteadiness of the arc and the traveling of the arc around the electrodes.

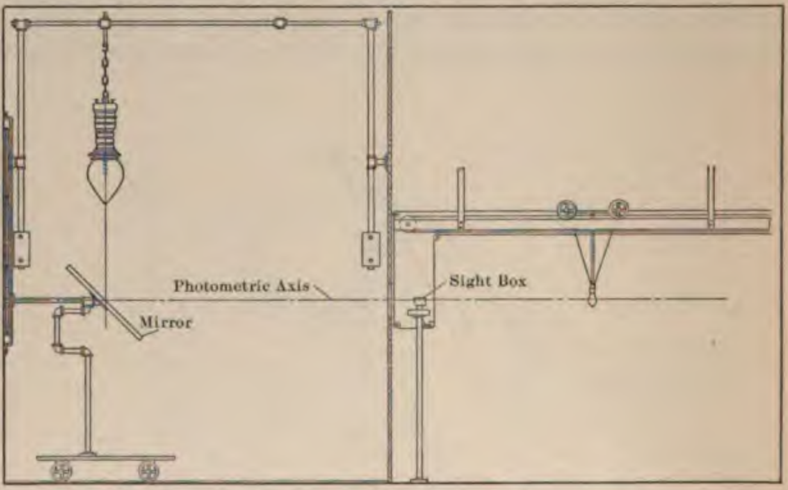

Fig. 80.—Single-mirror Crane-type photometer.

It will be seen from the figure that a *rotating sector disk* driven by a small motor is placed in the photometric axis. This is used to increase the range of the apparatus by decreasing the intensity on that side of the photometric sight box. The sight box and standard lamp are suspended from a track located above the photometric axis. Reflected light due to the standard lamp is intercepted by opaque screens placed between the standard lamp and the sight box and having openings along the photometric axis large enough to let the direct rays pass through. A photometrical balance may be obtained by changing the size of the openings in the rotating sector disk and varying the distance between the standard lamp and the sight box. The sight box should remain in the same position as when the apparatus is calibrated so that the incident angle of the light rays will be the same. For very low intensities or with a high candle-power standard the sector disk can be placed on the other side of the sight box. In this way the range of the apparatus can be varied to meet almost any value of intensity.

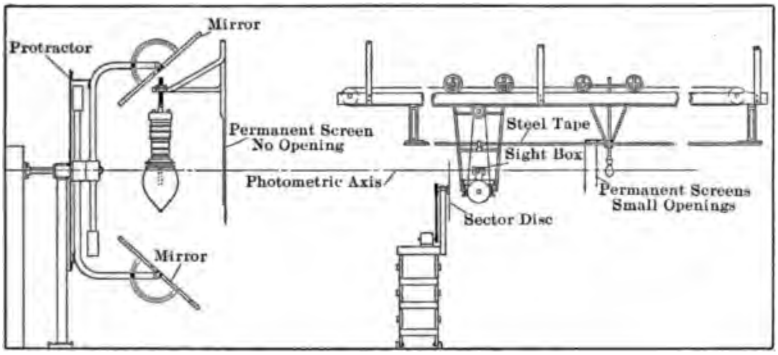

Fig. 81.—Arrangement of twin-mirror photometer.

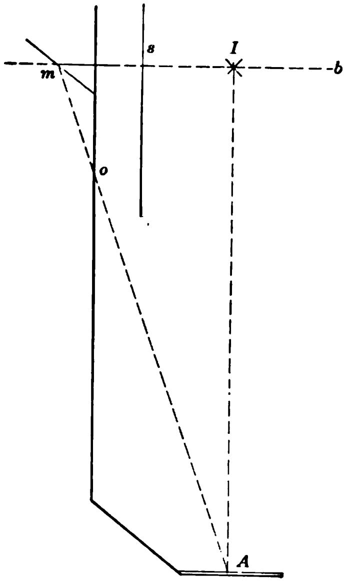

Fig. 82.—Revolvable test-plate for determining the distribution of light.

Still another arrangement for the same purpose but simpler in construction is illustrated graphically by Fig. 82. Assume the source to be located at *I* and a *test-plate A revolvable about the photometrical axis ab* to receive the illumination to be measured.

The illumination on the test-plate A may be viewed through a photometric device placed on the photometric axis and the mirror m located so as to rotate with the test-plate. Since the illumination varies inversely as the square of the distance, the candle-power can be calculated from the illumination measurements or the device may be calibrated to read the candle-power values directly, in the same way as in the case just described. It is evident that a mirror must be substituted for the screen in the photometric device on the side receiving the light from the test-plate. The test-plate must obviously be far enough from the source that the source may be considered a point source, since the nature of the reflection from the test-plate will differ from that from a mirror.

CHAPTER VIII

LIGHT-FLUX CALCULATIONS AND SPHERICAL PHOTOMETRY.

In order to compare luminous sources on the basis of the total amount of light emitted and to be able to determine the amount of light available for illuminating purposes it becomes necessary to understand some of the methods used and the underlying principles involved in obtaining the *total amount of light flux emitted*, or any part thereof, by any lighting equipment.

The various methods used in practice for determining the value of the candle-power in different directions from the source have been described in the preceding chapter. These values are usually represented graphically by curves plotted to polar coördinates. These curves merely show the distribution of light in one plane (usually the vertical plane) around the source of light as a center. They simply indicate the value of the candle-power in definite directions from the source and have no significance as a representation of the qauntity of light.

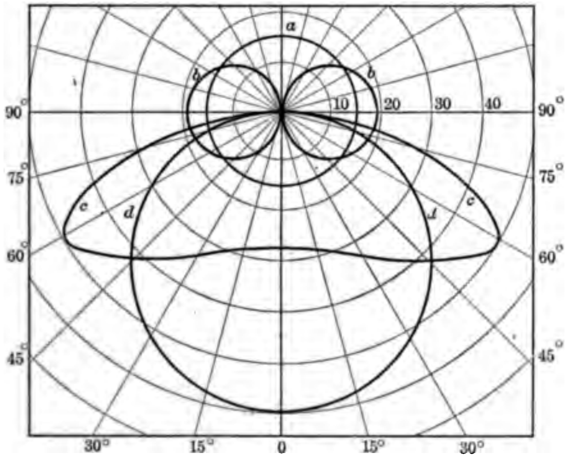

FIG. 83.—Polar curves giving the same amount of light.

Polar Curves and Spherical Surfaces.—An interesting comparison indicating the misleading conceptions likely to arise in the study of polar diagrams is illustrated by the polar curves of Fig. 83.[1] The four curves *a*, *b*, *c*, and *d* represent theoretical distributions of light in a vertical plane. The maximum values in these four cases are approximately in the ratios of 15–19–50–60. However, if these curves represented the distribution of light from luminous sources in a vertical plane the mean spherical candle-power or the *total flux of light would be the same for each.*

The fundamental theory of this section of the subject is based on the study of spherical surfaces, in junction with which it becomes necessary to determine either the mean spherical intensity in candle-power or the zonal or total flux of light in lumens. In these spherical calculations it is assumed that the luminous intensity is equal in azimuth and varies only in one plane through the source which, in the following discussion, will be a vertical plane.

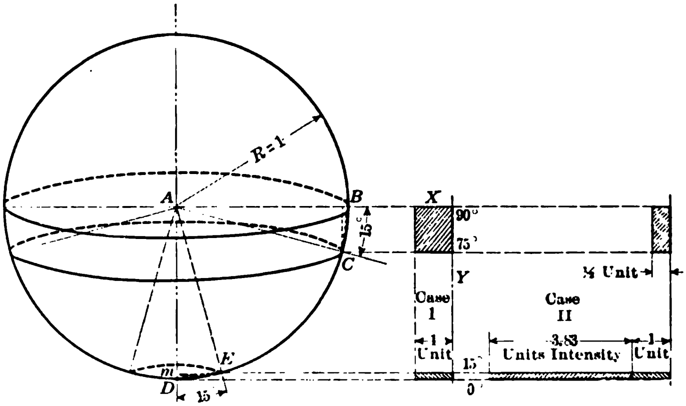

FIG. 84.—Relative areas of zones shown graphically.

If we assume the source of light to be surrounded by a sphere of radius *r* with the source as a center, and further consider this sphere divided into a number of zones in such a manner that the illumination of similar parts of each zone is uniform, the total flux of light embraced by a zone will be equal to the product of the average intensity and the area of the zone. From a summation of these products for each zone the total value of light flux emitted by the source may be obtained and this divided by the area of the sphere ($4\pi r^2$) will give the mean spherical candle-power.

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[1] *The Armour Engineer*, Vol. 3, p, 1, Jan., 1912.

The light flux in the lower hemisphere will be the sum of the products of the zones and their intensities and this sum divided bp the hemispherical area ($2\pi r^2$) will give the mean lower hemispherical candle-power.

The area of a zone subtended by an angle embracing the first 15 degrees below the horizontal is 7.66 times as great as the area of the zone extending 15 degrees from the vertical. With the same intensity in each zone, the total flux of light embraced by the former zone will be 7.66 times that passing through the latter. If now the source of light is of uniform intensity in all directions and by means of a reflector half of the light from the zone subtended by the first 15 degrees below the horizontal be redirected downward through the zone extending 15 degrees from the vertical, the intensity in the latter zone will be increased to 4.83 times its former intensity.

These results are shown graphically in Fig. 84,[1] where case I represents the normal condition and the shaded parts the relative amounts of light in the two zones. Case II shows graphically the relative amounts and intensities of light in the same zones obtained by the use of the reflector. Thus the quantity of light depends not only upon the intensities in the different directions, but upon the areas of the zones which the various intensities illuminate.

The *area of a zone* defined by the angles a and ada is

$$2\pi \cos ada, \tag{38}$$

and the quantity which it receives will be

$$F_1 = \int_{a_1}^{a_2} 2\pi I \cos ada \tag{39}$$

a_1 and a_2 being the angles with the vertical which locate the meridians determining the zone. The mean intensity for the zone is equal to the total quantity of light flux divided by the area.

If the spherical surface is divided into n zones subtended by equal angles, then the total amount of light from the lamp will be

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[1] Trans. Illum. Eng. Soc., Vol. 1, p. 245, Mar. 1906.

$$F = F_1 + F_2 + \ldots\ldots + F_n = \int_0^{0+\frac{\pi}{n}} 2\pi I_1 \cos a da + \int_{0+\frac{\pi}{n}}^{0+\frac{2\pi}{n}} 2\pi I_2 \cos a da$$

$$+ \ldots\ldots + \int_{\pi-\frac{2\pi}{n}}^{\pi-\frac{\pi}{n}} 2\pi I_{n-1} \cos a da + \int_{\pi-\frac{\pi}{n}}^{\pi} 2\pi I_n \cos a da. \tag{40}$$

If the intensity is uniform in all directions we have

$$F = \int_0^{\pi} 2\pi I \cos a da, \tag{41}$$

and the mean spherical intensity

$$I_{ms} = \frac{\int_0^{\pi} 2\pi I \cos a da}{4\pi} = \tfrac{1}{2} I \int_0^{\pi} \cos a da \tag{42}$$

Unfortunately, the law according to which the intensity varies is too complex to allow the integration to be effected directly. Hence it becomes necessary to resort to methods involving approximations.

It can be shown by spherical trigonometry that the areas of the zones of a sphere are to each other as their altitudes. Thus the luminous flux in any zone of an imaginary sphere surrounding a source of light is proportional to

$$2\pi I(\cos a_1 - \cos a_2), \tag{43}$$

where $a_1 - a_2$ is the angle subtending the zone of reference, a_1 and a_2 being angles measured from the vertical, and I is the average intensity of illumination in that zone.

This equation forms the basis of the graphial methods of Rousseau, Kennelly, Macbeth, Wohlauer and others for obtaining the mean spherical candle-power and luminous flux in lumens from a source having its distribution of light equal in azimuth.

The Rousseau Diagram.—The Rousseau diagram is the oldest of the various methods by means of which the mean spherical or hemispherical candle-power and the luminous flux as a whole or in part may be obtained. In its construction advantage is taken of the proportionality of the areas of the zones of a sphere to their respective altitudes. The values of the altitudes of zones subtended by equal angles are laid off to scale along the vertical axis of the diagram. These may be determined graphically, as shown in Fig. 85, where the sphere is divided into 15 degree zones and the zonal boundaries projected on the vertical or by laying off horizontal lines at distances, to scale, from the center line equal to the cosines of the various angles. For every 10 degrees these values are as follows:

Table 25

Cosine Values for Constructing the Rousseau Diagram

Degrees...	0	10	20	30	40	50	60	70	80	90
Cosine.....	1.0	.985	.939	.866	.766	.643	.500	.342	.173	0

If, on the horizontal lines drawn from the terminals of the successive altitudes, we lay off to scale the values of the candle-power in the direction of the corresponding angles, then the area enclosed by the curve, determined by these values of the candle-power at various angles, and the vertical, will represent the total luminous value of the source of light and the proportional part of the total light in any zone will be clearly shown. The value of this area in terms of the product of the scales to which the curve was plotted divided by the sum of the ordinates or altitudes will give the mean spherical candle-power and this value multiplied by 4π will give the flux in lumens. In the same way, areas corresponding to any zone or zones divided by the altitude or altitudes corresponding to the area will give the mean candle-power throughout the respective portion of the spherical area. To obtain the value of the mean spherical or mean hemispherical candle-power without a planimeter, the area enclosed by the curve may be divided horizontally by 20 lines bisecting areas of equal heights. Without appreciable error we may assume that

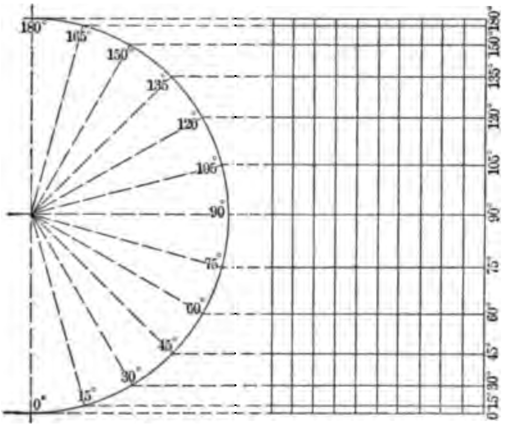

Fig. 85.—Construction of Rosseau diagram.

the average width of each section is equal to the distance across the middle of that section. Hence by reading the lengths of these horizontal lines and by dividing the sum of their values by 20 we obtain the approximate value of the mean spherical candle-power. If the hemispherical candle-power is disired, consider only the areas corresponding to that hemisphere and divide the sum of the values of the 10 lines by 10.

The use of the Rousseau diagram for showing the relative light values of different sources is shown in Fig. 86. The areas enclosed between the curves *a*, *b*, *c* and *d*, and the vertical on the left represent the total flux of light from four sources having distribution curves as indicated by *a*, *b*, *c* and *d*, respectively, of Fig. 83. It will be seen that the four areas determined by the curves *a*, *b*, *c* and *d* are equal. Since these areas represent graphically to scale the value of the light flux we have the same value of the lumens or mean spherical candle-power for each.

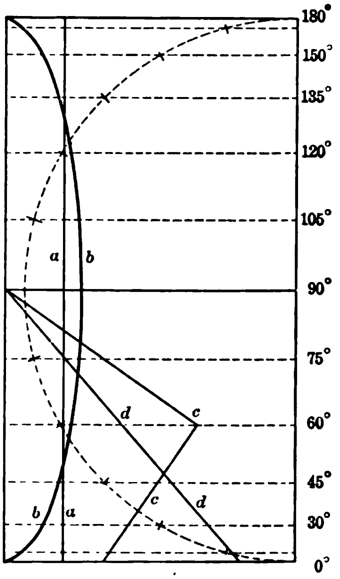

FIG. 86.—The Rousseau diagram applied to curves of Fig. 83.

Macbeth's "Polar Flux" Paper.—In the method of determining mean spherical intensities just discussed it will be seen that the spherical area surrounding the source of light was divided into zones of equal areas and the candle-power in the direction of the zonal centers of these areas assumed as the average for that zone. This method forms the basis of Macbeth's "polar flux" paper, which is the ordinary polar coördinate paper on which are placed the radial lines representing the zonal centers of 20 zones of equal areas. The construction of this paper is shown in Fig. 87. The vertical distribution curves may be plotted as usual. The values of the candle-power read along these additional lines shown dotted in the figure, and the sum of these readings divided by the number of readings will give approximately the average candle-power throughout the zones which they represent. In order to use this method on polar curves constructed on ordinary polar coördinate paper, these radial lines may be drawn as in Fig. 88,[1] on transparent celluloid of convenient size, to place over the polar curves. The values of candle-power may be read and the results determined as on the "polar flux" diagram.

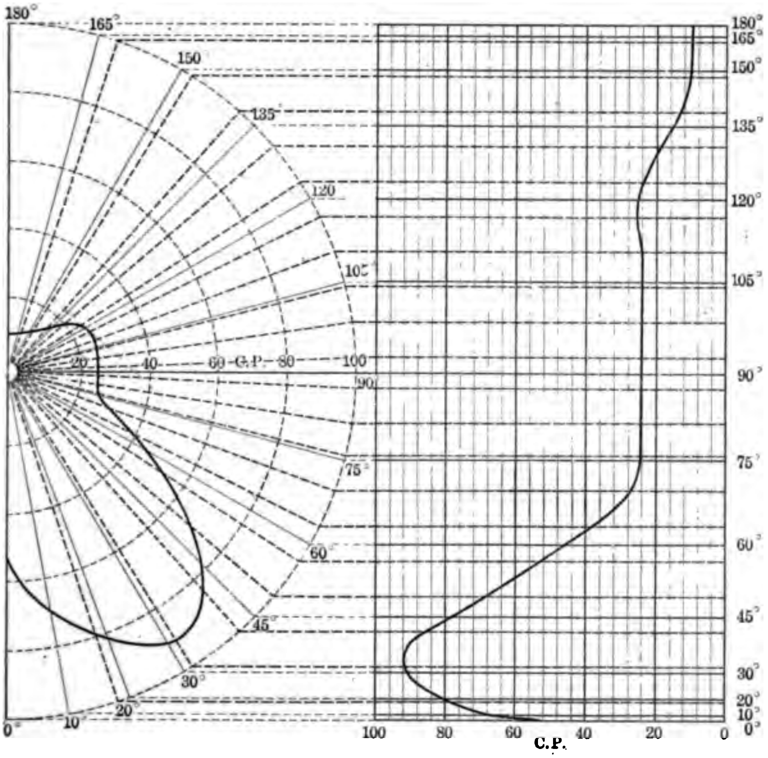

FIG. 87.—Construction of Macbeth's "polar flux" paper.

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[1] *Illum. Eng.*, N. Y., Vol. 3, p. 27, Mar., 1908.

The Kennelly Diagram.—The Kennelly diagram[1] possesses the advantage of yielding the value of the mean spherical intensity in terms of a linear quantity. Moreover, only an angle protractor and a pair of compasses are necessary for its construction. This method consists in determining graphically an evolute from the polar curve of the luminous source, together with its involute and then projecting this involute upon a vertical line.

The method may be best understood by means of an example. In Fig. 89 is shown one of the common distribution curves for a tungsten equipment. The construction of the diagram is adapted to zones of 15 degrees. Find the radii of the midzones, represented by the dotted lines, *oz*, *ou*, *ov*, *ow*, *ox*, *oy*, respectively. Then with *o* as a center and with a radius *oz* describe the arc *tz* through an angle of 15 degrees. Draw the radius *ol* from the end of this arc and measure from *l* along *ol* a distance *la* equal to *ou*. With *a* as a center and radius *ou* describe a second 15 degree arc so that *ak* makes an angle of 15 degrees with *al*. Along *ak* lay off *bk* equal to *ov* and with *b* as a center continue the involute 15 degrees. Draw *bi*, lay off *ci* equal to *ow*, draw arc *ih* and line *hc*, lay off *hd*, draw arc *hg* and line *gd*, lay off *eg*, draw arc *fg*, and line *ef*. The line *ef* should be vertical.

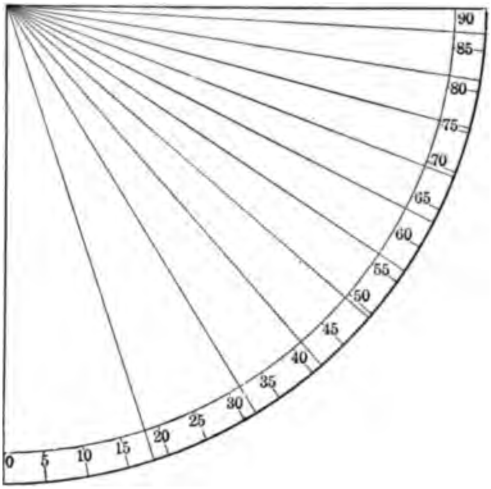

FIG. 88.—Device for obtaining mean hemi-spherical candle-power.

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[1] *Elect. World*, Vol. 51, p. 645, Mar. 18, 1908; Trans. Illum. Eng. Soc., Vol. 3, p. 243, Apr., 1908.

The method of procedure in the upper hemisphere is the same as in the lower hemisphere as indicated in the figure. Project the points *f* and *f'* upon the vertical. Then half the distance *mm'* between the extreme projections, to scale, is the value of the mean spherical candle-power, the length *mo* the mean lower hemispherical candle-power, and the length *m'o* the mean upper hemispherical candle-power. These values are approximately mean spherical candle-power, 36.5; mean lower hemispherical candle-power, 52; mean upper hemispherical candle-power, 21. The total flux in lumens is 4π times the mean spherical candle-power, as in the other methods.

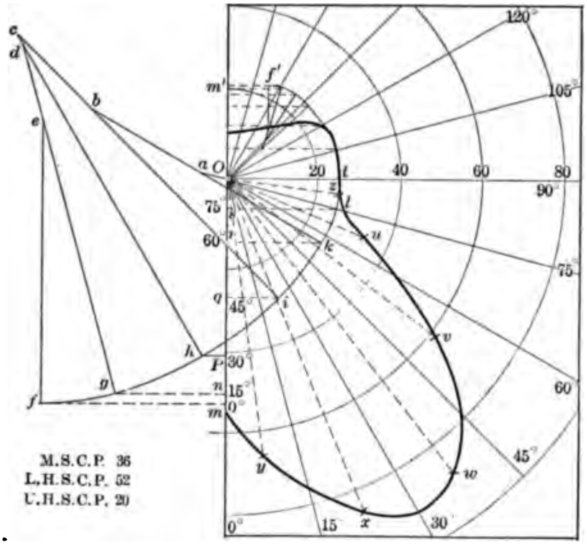

FIG. 89.—The Kennelly diagram.

Theory of the Method.—The mean spherical intensity of a luminous source having equal intensities in azimuth and occupying a virtual point is

$$I_m = 0.5 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_\theta \cos\theta \, d\theta \text{ candle-power or hefners} \qquad (44)$$

where I_θ is the luminous intensity at the elevation θ. Let there be n equal zones each of $\frac{\pi}{n}$ radians. Then the above equation becomes:

$$2I_m = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{2}+\frac{\pi}{n}} I_\theta \cos\theta \, d\theta + \int_{-\frac{\pi}{2}+\frac{\pi}{n}}^{-\frac{\pi}{2}+\frac{2\pi}{n}} I_\theta \cos\theta \, d\theta + \ldots .$$

$$+ \int_{\frac{\pi}{2}-\frac{2\pi}{n}}^{\frac{\pi}{2}-\frac{\pi}{n}} I_\theta \cos\theta \, d\theta + \int_{\frac{\pi}{2}-\frac{\pi}{n}}^{\frac{\pi}{2}} I_\theta \cos\theta \, d\theta, \text{ in hefners.} \qquad (45)$$

In each of the above n terms, I_θ can differ but little from its value at the midzone. We can, therefore, write without much error,

$$2I_m \simeq I_{\frac{\pi}{2}+\frac{\pi}{2n}} \int_{-\frac{\pi}{2}}^{-\frac{\pi}{2}+\frac{\pi}{2}} \cos\theta \, d\theta + I_{\frac{\pi}{2}+\frac{3\pi}{2n}} \int_{\frac{\pi}{2}+\frac{\pi}{n}}^{-\frac{\pi}{2}+\frac{2\pi}{n}} \cos\theta \, d\theta + \ldots .$$

$$+ I_{\frac{\pi}{2}-\frac{3\pi}{2n}} \int_{\frac{\pi}{2}-\frac{2\pi}{n}}^{\frac{\pi}{2}-\frac{\pi}{n}} \cos\theta \, d\theta + I_{\frac{\pi}{2}-\frac{\pi}{2n}} \int_{\frac{\pi}{2}-\frac{\pi}{n}}^{\frac{\pi}{2}} \cos\theta \, d\theta, \text{ in hefners.} \qquad (46)$$

The construction of the involute curve may be seen to express the relation

$$s \simeq \int_0^{\frac{\pi}{2}} I_\theta \, d\theta, \text{ in units of length} \qquad (47)$$

and

$$\int_{\frac{\pi}{2}}^{0} I_\theta \, d\theta, \text{ in units of length} \qquad (48)$$

in the upper and lower hemispheres respectively.

When the zones are taken of 20°, it is advantageous to select as their midangular elevations +80°, +60°, +40°, +20°, +0°, −20°, −40°, −60°, −80°. This leaves no uncompleted arc at either terminus, and it avoids any discontinuity in the involute curve at the horizontal line. The first arc is drawn from +10° to −10° with the radius equal to the mean horizontal intensity.

The "Fluxolite" Paper.—The "fluxolite" diagram[1] devised by Mr. Wohlauer offers a convenient means of determining the luminous flux and spherical candle-power. The value of the flux is obtained by simply adding a number of linear dimensions drawn to scale and multiplying the same by some constant. This constant depends for its value upon the number of angular subdivisions of the spherical area.

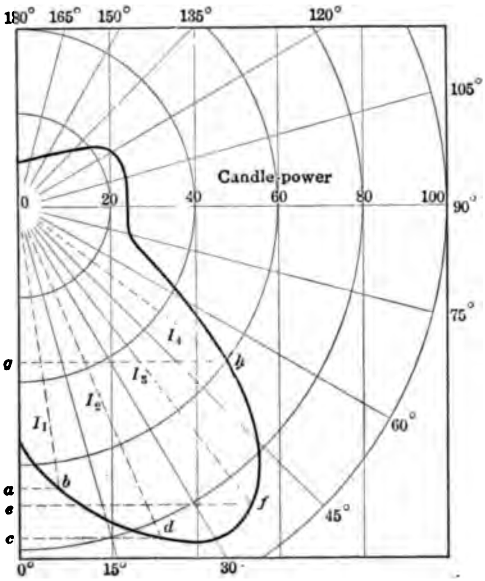

FIG. 90.—Wohlauer's "fluxolite" paper.

It can be shown geometrically that the altitude and hence the area of a zone is proportional to the sine of the angle, measured from the vertical axis, which bisects the zone. Hence if the imaginary spherical area be divided into n numbers of equi-angular zones and assuming the midzone intensity to be the average for the zone, then the flux in any zone will be

$$F = KI \sin a, \qquad (49)$$

where I is the average intensity of the zone, a the bisecting angle measured from the vertical axis and K a constant the value of which depends upon the number of zonal subdivisions.

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[1] *Illum. Eng.*, N. Y., Vol. 3, p. 655, Feb., 1909; Vol. 4, p. 491, Nov. 1909; Vol. 4, p. 148, April, 1909; Vol. 5, p. 132, May, 1910.

Referring to Fig. 90 and representing the flux in successive zones from the nadir by F_1, F_2, etc., and the average intensities by I_1, I_2 etc., and the midzone angles by a_1, a_2, etc., we have

$$F_1 = KI_1 \sin a_1 = KL_{ab}, \tag{50}$$

$$F_2 = KI_2 \sin a_2 = KL_{cd}, \tag{51}$$

$$F_3 = KI_3 \sin a_3 = KL_{ef}, \tag{52}$$

or

$$F = F_1 + F_2 + \ldots \; F_n = K(L_{ab} + L_{cd} + L_{ef} + \ldots, \text{etc.}). \tag{53}$$

Thus the flux in any zone is equal to the horizontal projection of its midzone intensity multiplied by the constant and the total flux in lumens is equal to the sum of the several projections multiplied by the constant.

The mean hemispherical candle-power may be obtained by dividing the value of the flux in that hemisphere by 2π, and the mean spherical candle-power may be determined by dividing the value of the total flux by 4π. The values of K for various angular subdivisions are given in the following table:

TABLE 26

Constants for Light-flux Calculations

Angle embrace of zone........	5	10	15	20	25	30
Value of K..................	0.548	1.098	1.64	2.18	2.72	3.25

In the example just cited K is equal to 1.64, a being equal to 15 degrees.

The polar diagram is constructed with vertical lines spaced equal to the polar scale to facilitate the evaluation of the projections of the various midzone intensities.

By referring to the values of the constants given above we will see that for zones subtended by 10 degree angles the value of K is 1.098. If now the polar curve, Fig. 90, was plotted on polar coördinate paper so dimensioned that 1.098 in. would equal some multiple of the candle-power, then the lumens could be determined directly by measuring the distances *ab*, *cd*, *ef*, etc., in inches and multiplying by the value of the multiple referred to above.

Macbeth's Polar Flux Scale.—Since these constants refer to the relation between the candle-power scale and the scale of the distances from the vertical to the intersection of the midzone radial lines and the polar curve, it follows that for a certain design of polar coördinate paper one may construct a scale of convenient size and shape and graduated according to the above relation and with the common scale values indicated

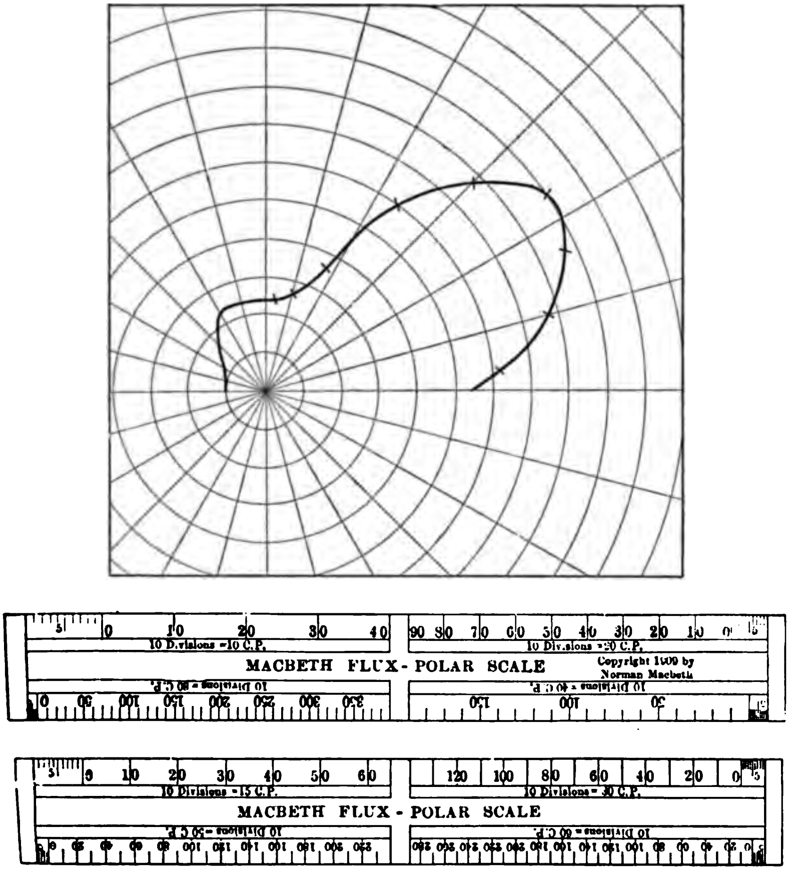

FIG. 91.—Macbeth's flux scale.

whereby the flux in lumens may be determined for any polar curve on that design of paper. Such a construction is indicated in Fig. 91, which shows the polar flux paper and polar flux scale as designed by Mr. Macbeth. It will be seen that this scale (3/4 in. by 5 1/4 in.) has eight sections to correspond to eight different values of candle-power per division on the polar paper for which the scale is designed. By choosing the proper section of the scale the distances from the vertical to the points *a*, *b*, *c*, etc., on the polar curve, measured by means of the scale, will equal approximately the number of lumens embraced by the corresponding zones. By continuing this process for the entire polar curve and adding the results the total flux will be obtained.

Where the curve is plotted on other paper or when the flux scale is not available a most convenient method and, in the opinion of the writer, the simplest existing method of obtaining the flux entire or in part, is to place the edge of a sheet of paper on the vertical or 0° line and on the adjacent edge mark off the distance from the vertical to the point *a*. Then move this mark to the vertical and again mark off on the same edge the distance from the vertical to *b*. By continuing these operations through 90 degrees it is obvious that the sum of the lengths laid off on the paper will be equal to the sum of the horizontal projections of the different 10 degree midzone intensities. The candle-power scale to which the polar curve is plotted can be used for measuring the sum of the lengths laid off on the paper, and this value multiplied by 1.1 will give the light flux in lumens. It should be remembered that this method can be used for finding the flux in any zone or zones desired.

Modification of Wohlauer's Method.[1]—Still another application of the constant 1.098 is indicated by Fig. 92, where the radial line OK is drawn making an angle, XOK (24° 3′) the secant of which is 1.098. The spherical surface is divided into ten-degree zones and the average zonal values of the candle-power represented by the radial lines, Oa, Ob, etc. If these midzone values be projected vertically onto the line OK and the extremity of each projection be continued around O along the arc of a circle into the horizontal the value of the flux in lumens in any zone will be represented by the corresponding distance, to the same scale as the polar curve is plotted, along the horizontal OX measured from O. As an example of the manipulation of this method consider the zone between the angles of 40 and 50 degrees from the vertical. Assume Oe, equal to 78, to be the average intensity in this zone. Its projection on OK is equal to Oe' and this value transposed to OX will be Oe'' or 60.5 lumens. In the same way the flux in the 50–60 degree zone will be 77 lumens. By adding the lumens in the different zones together the total lumens in the lower hemisphere may be obtained. The flux in lumens in the upper hemisphere may be found by projecting the midzone values upon the line *OK* or upon another line 24° 3′ above the horizontal, and transposing to the horizontal as above.

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[1] *Elect. Rev. and West Elect.*, Vol. 58, p. 440, Mar. 4, 1911.

The simplicity of this method is manifest. The only apparatus necessary is a pencil and piece of paper and the only calculation is simple addition of the nine or eighteen values obtained. A right-angled triangle will be found useful in securing the projections upon the line *OK* but a corner of the sheet of paper on which the curve is plotted may be conveniently substituted.

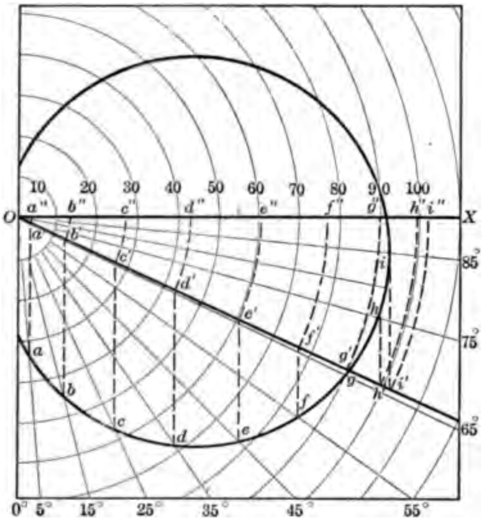

Fig. 92.—Modification of Wohlauer's method.

It is obvious that this method may be applied to any distribution curve plotted upon polar coördinate paper. In this case a series of vertical lines together with the radial line, Fig. 93, laid on a quadrant of transparent celluloid, tracing cloth, or similar transparent material will greatly facilitate operations.

Since the secant of 24° 3′ is approximately 1.1 (1.098) the midzone values may be projected directly onto the horizontal line and the sum of the projections multiplied by 1.1, and the same results obtained as above.

The mean hemispherical candle-power may be found by dividing the lumens in the hemisphere by 2π (6.28) and the mean spherical candle-power by dividing the total light flux of the lamp by 4π (12.56).

"A Set of Constants" Method.—A method of obtaining approximate values of the mean spherical or hemispherical candle-power[1] or the luminous flux in lumens of the various zones, by means of an ordinary slide-rule and set of constants will be found very convenient, especially where the results of photometrical tests are given in numerical values. These constants are calculated to correspond to a certain number of midzone directions in a vertical plane at which the candle-power is usually measured.

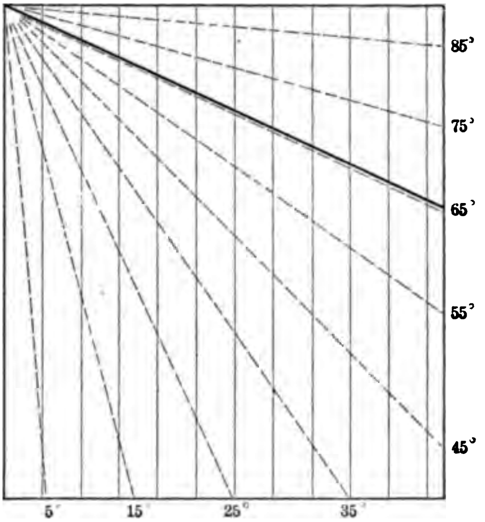

FIG. 93.—Auxiliary Diagram applicable to any polar curve.

These values corresponding to every 15 degrees from the vertical are as follows:

TABLE 27

Constants for Calculating Light-flux, 15° zones

	Candle-power constant		Flux constant	
Cos 0° − cos 7° 30′	= .00856	× 2π	= .0538	for 0° from vertical.
Cos 7° 30′ − cos 22° 30′	= .06756	× 2π	= .424	for 15° from vertical.
Cos 22° 30′ − cos 37° 30′	= .1305	× 2π	= .820	for 30° from vertical.
Cos 37° 30′ − cos 52° 30′	= .1846	× 2π	= 1.16	for 45° from vertical.
Cos 52° 30′ − cos 67° 30′	= .226	× 2π	= 1.42	for 60° from vertical.
Cos 67° 30′ − cos 82° 30′	= .252	× 2π	= 1.58	for 75° from vertical.
Cos 82° 30′ − cos 90°	= .1305	× 2π	= .820	for 90° from vertical.

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[1] Trans. Illum. Eng. Soc., Vol. 4, p. 436, May, 1909.

By similar calculations those for every 10 degrees from the vertical are

TABLE 28

Constants for Calculating Light-flux, 10° zones

Midzone angles	Candle-power constant	Flux constant
0	.00381	.0239
10	.03926	.190
20	.0596	.374
30	.0872	.547
40	.1120	.705
50	.1335	.841
60	.1509	.949
70	.1638	1.025
80	.1716	1.075
90	.0872	.548

Thus to determine the mean hemispherical intensity and the flux in the lower hemisphere proceed as indicated in the following table:

TABLE 29

Example of Use of Constants

Degrees from vertical	Candle-power	Candle-power constant	Component of M. H. S. C. P.	Flux constant	Lumens
0	8.0	.00856	.0685	.0538	.43
15	8.8	.0675	.594	.4244	3.73
30	10.6	.1305	1.383	.8202	8.69
45	13.1	.1846	2.418	1.1598	15.20
60	14.7	.2261	3.324	1.4205	20.85
75	16.2	.2521	4.080	1.5843	25.65
90	16.7	.1305	2.180	.8202	13.70
			M. H. C. P. 14.04		Lumens 88.25

It should be noted that these values of the lumens are for zones extending 7° 30′ above 0°, below 90°, and either way from the 15°, 30°, 45°, 60° and 75° directions.

To facilitate the above calculations these constants may be applied to the construction of a slide rule as shown in Fig. 94. The slide of this rule is divided into sections proportional to the constants for the 15-degree subdivisions, as given above, the sum of which is equal to unity, and each section subdivided into 10, or a multiple of 10, equal parts.

The upper scale or candle-power scale is subdivided uniformly and has a length equal to the slide. The lower scale or lumen scale is graduated into sections 2π times those in the upper scale.

To determine the mean hemispherical candle-power or lumens is the question of one or two minutes. The zero of the slide is set at the zero of the other scales, the cross-hair on the rider is placed over the value of the candle-power in the vertical direction deduced to scale, in the 0-degree section. The zero of the 15-degree section is then placed under the cross-hair, the rider remaining fixed. The cross-hair is then moved, the slide remaining fixed, to the value of the candle-power in the 15-degree direction on the 15-degree section and the zero of the 30-degree section placed under the cross-hair. These manipulations are repeated for the 30-, 60-, 75- and 90-degree sections. The mean hemispherical candle-power can be read off the upper scale and the hemispherical lumens read off the lower scale under the final position of the cross-hair. Another advantage of the rule is that the lumens may be read off the lower scale at any time during the procedure, but one should remember that the value covers the spherical area 7° 30′ beyond the designating angle.

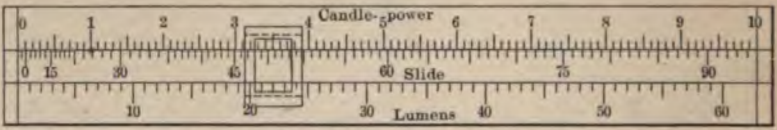

FIG. 94.—Slide-rule for calculating the mean spherical candle-power or lumens.

The mean spherical candle-power of a source may also be found by multiplying the horizontal candle-power by the spherical reduction factor provided the mean spherical reduction factor for that type of lamp is known, and the lumens found by multiplying this product by 4π.

Thus

$$F=4\pi f I_h, \tag{54}$$

where

F = number of lumens,
f = spherical reduction factor,
I_h = horizontal candle-power.

Spherical Photometers.—It is obvious that where the spherical light values of a large number of lamps are required and the spherical reduction factor is not accurately known, and especially in the photometry of lamps where the light fluctuates considerably, it is desirable to employ means whereby the result can be obtained by one photometrical settng. This is accomplished very satisfactorily by means of an integrating photometer of either the mirror or the spherical type.

The Mathews' Photometer.—The mirror photometer[1] was first designed by Professor Mathews for use in the laboratories of Purdue. By means of it the mean spherical candle-power of any source can be determined by one reading, but it is especially intended for arc light photometry.

The theory of the method as given by Professor Mathews is as follows: A spherical surface concentric with the lamp is considered and its area divided into zones. Then if I_a be the intensity at an angle a to the vertical, the product of the intensity I_a and the area of an elementary zone of radius r is

$$I_a dS = I_a \times 2\pi\ r\ R\ da \qquad (55)$$

$$= 2\pi\ R^2 I_a \sin a\ da \qquad (56)$$

The mean spherical intensity is

$$I_{ms} = \int \frac{I_a\ dS}{4\pi\ R_2} \qquad (57)$$

$$= \tfrac{1}{2} \int_0^{\pi} I_a \sin a\ da. \qquad (58)$$

The mean hemispherical intensity is

$$I_{mhs} = \int_0^{\frac{1}{2}\pi} I_a \sin a\ da. \qquad (59)$$

If the intensity I_a be taken at n equal intervals through 180 degrees or n' equal intervals through 90 degrees in a vertical plane, then we may write for the mean spherical intensity

$$I_{ms} = \frac{\pi}{2n} \sum_0^{\pi} I_a \sin a. \qquad (60)$$

and for the mean hemispherical intensity.

$$I_{mhs} = \frac{\pi}{2n'} \sum_0^{\frac{1}{2}\pi} I_a \sin a. \qquad (61)$$

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[1] Trans. Amer. Inst. Elect. Eng., Vol. 18, p. 677, 1901; Trans. Amer Inst. Elect. Eng., Vol. 20, p. 1465, 1902; *Bull. Bur. of Stand.*, Vol. 1, p. 255, 1905.

These expressions are quite correct if n and n' are sufficiently large.

In order to produce on the photometer screen an illumination proportional to the mean spherical intensity of the source it is necessary to direct toward the photometer beams of light from various angles in the vertical plane. It is furthermore essential that these rays, representing the intensity of light at the respective angles, be reduced in intensity in the ratio of the sine of the angle which the rays make with the vertical, in order that the light from each mirror shall be proportional to the area of the zone which it represents. In developing this method a ring of 24 large trapezoidal mirrors, placed one every 15 degrees, surrounds the arc. The inclination of the mirrors to the arc is such that 24 images of the lamp are presented to the eye placed at the photometric device. Direct rays from the lamp are intercepted by a black screen. The reduction of the light in the ratio of the sine of the vertical angle is accomplished by means of a polygonal glass disk composed of as many sectors as there are mirrors and which are smoked sufficiently to give the desired absorption.

One essential condition to the success of this piece of apparatus is that the screen shall receive through each sector only the light from its corresponding mirror. The sector disk can be adjusted by replacing the screen by a cardboard pierced by a small opening and by sighting through this hole the disk can be placed with reference to the mirrors. Mirrors of good quality French plate glass were used and their reflection coefficients were carefully determined for the angle of incidence at which they were used, the maximum difference from the mean value of 0.815 was only ±2.9 per cent. which was corrected by smoking the corresponding sectors. The smoking of the disks was accomplished by burning small quantities of turpentine in a receptacle in which the sector, with a guardring to insure even density, was placed in a vertical position.

In the photometer described by Professor Mathews, a plan of which is shown in Fig. 95, the light from the standard lamp, because of the small size of the room, was reflected onto the screen by means of another mirror, M. The illumination on the side of the screen toward the arc lamp, being very intense, was reduced by means of a rotating disk provided with a large number of slots.

The complexity involved in determining the working equations for a photometer of this type can be best understood from the deduction given by Professor Mathews for the apparatus illustrated in Fig. 95. He lets

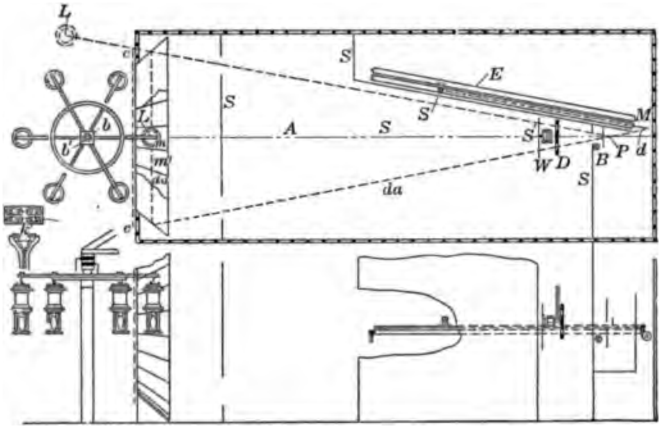

Fig. 95.—The Mathews photometer.

d_a = distance from arc lamp to the photometer screen.
d_s = distance from standard to photometer screen.
K_o = reflection constant of the horizontal mirror.
K_g = transmission constant of clean glass.
K_s = reflection constant of mirror of the standard.
K_w = transmission constant of the sector wheel.
n, n' = number of mirrors.
f = factor due to the lack of normal incidence.

The illumination due to the arc is

$$i = \frac{2K_oK_gK_w f \sum_0^\pi I_a \sin a}{d_a^2} \tag{62}$$

and that due to the standard lamp of intensity I_s is

$$i_s = \frac{f K_s I_s}{d_s^2}. \tag{63}$$

Equating these two expressions, we have for the mean spherical intensity

$$I_{ms} = \frac{\pi}{2n} \sum_0^\pi I_a \sin a \tag{64}$$

$$I_{ms} = \frac{\pi K_s}{4n\, K_0\, K_g\, K_w} \cdot \left(\frac{d_a}{d_s}\right)^2 I_s. \tag{65}$$

Similarly for the mean hemispherical intensity

$$I_{mhs} = \frac{\pi}{2n'}\sum_0^{\frac{\pi}{2}} I_a \sin a \tag{66}$$

$$= \frac{\pi K_s}{4n'\, K_o\, K_g\, K_w} \cdot \left(\frac{d_a}{d_s}\right)^2 I_s \tag{67}$$

With the following numerical values

$d_a = 865$ cm., $n = 12$, $n' = 7$,

$K_o = 0.812$, $K_g = 0.690$, $K_s = 0.901$, $K_w = 0.090$

These equations reduce to

$$I_{ms} = 1.17 \left(\frac{865}{d_s}\right)^2 I_s \tag{68}$$

and

$$I_{mhs} = 2.01 \left(\frac{865}{d_s}\right)^2 I_s, \tag{69}$$

which are the working equations for this apparatus.

If the arc is fed by hand, a smaller number of readings are necessary in order to obtain a fair average value of the candle-power than when the arc regulates normally. To obtain a distribution curve it suffices to uncover the mirrors in pairs and make sittings in the usual way. The smoked-glass sectors must of course be removed and the proper mirror constants used in working up the results. Tests checked by other methods indicate that this apparatus is very reliable considering the fluctuating values of the light from the automatically fed arc.

The Leonard Photometer.—The Leonard photometer is of more recent design than the Mathews photometer although similar in theory and construction, the chief difference beween the two being, that the Leonard photometer has its mirrors located at intervals corresponding to the middle points of successive zones of equal areas. Thus the average of the light thrown upon the screen represents directly the mean spherical, or the mean hemispherical candle-power, as the case may be, without intervention of smoked sectors.

The Globe Photometer.[1]—Prof. Ulbrich's spherical photometer, which also permits the measurement of the mean spherical candle-power of a lamp by a single setting, consists in part of a large sphere, with its inner surface painted white. The source of light to be measured is placed inside of this sphere and the illumination at an opening in the surface of this sphere (the direct rays being intercepted by a screen) is proportional to the mean spherical candle-power of the lamp. A globe of this kind integrates very successfully the illuminating effect of any source. It involves in addition a photometric device and a standard incandescent lamp.

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[1] Trans. Illum. Eng. Soc., Vol. 3, p. 502, Oct., 1908.

The general theory of the spherical photometer,[1] according to Dr. Bloch,[2] is as follows: When a source of light is placed inside a spherical shell having a matt surface, the light received by any part of the interior surface can be considered in two parts—(a) that coming directly from the lamp, and (b) the light received from the remainder of the interior surface of the sphere after one or more reflections. The quantity (a) is that which is measured in the ordinary photometer which determines the intensity of light emitted in any one direction, and is not considered at all in the globe photometer. According to the theory of the globe photometer the quantity (b) is constant all over the surface of the shell, and is proportional to the total amount of light emitted by the lamp, quite independent of its position in the shell.

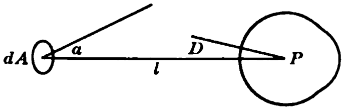

FIG. 96.—Theory of the globe photometer.

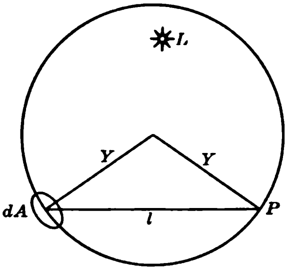

FIG. 97.—Theory of the globe photometer.

The theoretical argument for this photometer is as follows: Assume a surface P (Fig. 96) to be illuminated by radiation from a small luminous area dA, the bightness of which is B; then the light received on a unit area at P is

$$\frac{B\,dA\cos a\cos b}{l^2} \qquad (70)$$

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[1] See also "The Hollow Sphere," Chapter X.

[2] *Elek. Zeit.*, Vol. 26, pp. 1047, 1074, Nov., 1905.

a and b being the angles which l makes with the normals to the two surfaces. If now we let Fig. 97 represent a section through the photometer into which is inserted for measurement a lamp L, indicated in the figure, and consider the illumination of the surface at the point P by light reflected from a small area dA, and the circle shown be a section through the point P and the area dA, then the light received on dA from the lamp directly will be $E_A\,.dA$, and the $\int E_A\,.dA = F$, the total light emitted by the lamp. The intensity of dA is $k\,E_A$, since the surface is perfectly matt and throws the light in all directions equally, k being the reflection constant for the surface. The light received from this surface dA at P once reflected is

$$F_A = \frac{k\,I_A\,dA}{l^2}\,\frac{\cos^2 a}{} = \frac{k\,I_A\;dA}{4\,r^2}\,. \tag{71}$$

Hence the illumination at P due to once reflected light from the whole surface of the sphere will be

$$\frac{k}{4\,r^2}\int E_A\;dA = \frac{k}{4\,r^2}\,F. \tag{72}$$

It follows at once that the illumination at P by light twice reflected will be

$$\left(\frac{k}{4\,r^2}\right)^2 F. \tag{73}$$

The total illumination at P due to reflected light is

$$F(k/4\,r^2 + (k/4\,r^2)^2 + (k/4r^2)^3 + \ldots) = K\,F, \tag{74}$$

where K is the constant of the instrument and depends on the size of the sphere and the quality of the interior surface.

Thus we can see that the illumination of the interior of the sphere is theoretically uniform and proportional to the total light emitted by the lamp; therefore, if we screen off a small area from the direct rays of the lamp, the remaining illumination of that area is proportional to the mean spherical candle-power of the lamp.

If the lamp be removed to the surface of the sphere so that the center of the source is directly in the surface, then a single measurement will give a value proportional to the mean hemispherical candle-power, the ratio of proportionality being one-half that in the determination of the mean spherical candle-power.

There are two sources of error in this method, one is likely to occur by the lamp not being in the center of the surface, while the other is due to the non-transparent fittings of the lamp which intercept part of the light. Experiments show that errors due to the latter cause greatly outweigh those due to the former, but that a compromise can be made sufficient for all practical purposes if the lamp is not supported at the center but at a distance of 8 or 15 per cent. of the diameter from the top of the inner surface of the sphere, also by making the globe very large.

The construction of this photometer is shown in Fig. 98 where *L* and *N* are the two lamps, *P* the photometric device, *S* a screen whose aperture can be varied and *M* is the window of some translucent material protected from the direct rays of the lamp by the screen *B*.

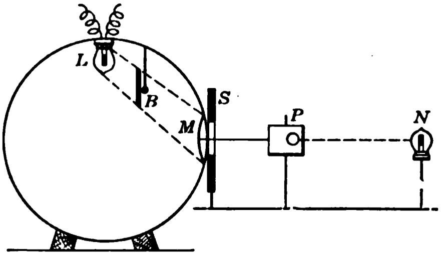

Fig. 98.—The globe photometer.

Experiments with globe photometers give results very consistent with those obtained from "point-to-point" readings, and tests of a great many different sources of light show that the mean spherical candle-power can be obtained by this arrangement with an error of not more than 3 per cent. The apparatus can be calibrated by means of a lamp of known mean spherical candle-power placed at the center of the sphere. The most satisfactory surface for the inner side of the sphere appears to be a coating of lithopone (barium sulphate), and more consistent results are obtained by using a translucent screen of milk-glass. This instrument highly recommends itself for the photometry of arc lamps where the distribution of light is not desired, since the fluctuating value of the candle-power due to the continual changing of the arc, makes arc-lamp photometry, by the ordinary distribution methods, slow and tedious.

The Box Photometer.—The box photometer, consisting of a large square box with a white interior, has been investigated but results with respect to accuracy and theoretical correctness seem to favor the globe photometer. Its only advantage appears to be in the mechanical simplicity of its construction.

The Kennelly Lumenmeter.—An ingenious arrangement has been designed by Dr. Kennelly for determining the mean spherical intensity or the total luminous flux from a lamp by a single reading. It can be used for the photometry of either incandescent or arc lamps but when used for the former it has an auxiliary device for rotating the lamp.

It consists essentially of two revolving mirrors and a disk containing a series of openings proportional to the areas of the zones from which the light passing through them comes when the mirrors revolve. The mirrors are placed 180 degrees apart and revolve in a vertical plane about the source of light as a center. The openings in the sectored disk are placed in the paths of the rays reflected by the mirrors onto the photometer screen. The theory of this instrument involves that of the rotating sectored disk and the mirror photometer. It is obviously immaterial in the case of the rotating sectored disk whether the disk rotates and the beam of light is at rest or *vice versa*. In the case of the mirror photometer the intensity of the light striking the screen from the various zones is made proportional to the light flux in the respective zones. The mirrors of the lumenmeter are driven by a motor and must revolve at sufficient speed that no flicker is noticeable on the screen.

CHAPTER IX

ILLUMINATION CALCULATIONS—POINT SOURCE

Having become familiar with the various methods of obtaining the amount of light available for illuminating purposes we may now proceed with the several methods of performing illumination calculations. It is the common practice in this country to consider the intensity of illumination on a horizontal plane as the basis of comparison or computation. This plane, known as the "working plane," is where the luminous effect of the source is utilized. In stores this working plane would be considered as even with the tops of the counters or 42 in. from the floor. In offices and similar interiors the plane would be that of the desk top or about 30 in. from the floor.

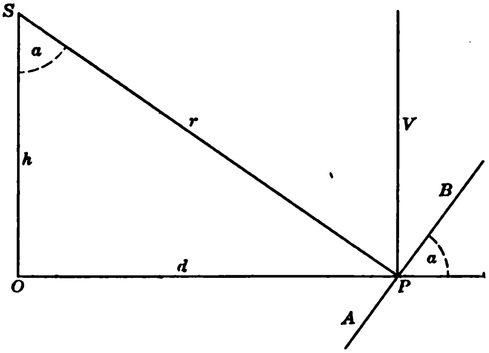

FIG. 99.—Illumination calculations by the point-by-point method.

The Point-by-point Method.—One of the first methods of determining the value and distribution of illumination is known as the "point-by-point" method. While reflection from walls and ceiling will in most cases make this method impractical for illumination calculations, it will, however, be found very useful in determining the distribution of illumination from a luminous source with respect to uniformity or for comparison with other sources or for approximately determining the location of lamps for a desired distribution of light.

These calculations involve trigonometric equations, but sets of constants may be derived and tabulated so that results may be calculated by simple arithmetic. If we assume the source of light to be located at a point S, Fig. 99, and let

I_a = candle-power a degrees from the vertical.
E_n = illumination on a normal surface.
E_h = illumination on a horizontal surface.
E_v = illumination on a vertical surface.
d = horizontal distance from vertical through lamp to point whose illumination it is desired to ascertain.
h = distance to lamp from horizontal plane.
l = distance direct from source to point considered.
a = the angle which the light rays make with the vertical.

Then the *illumination intensity at a point on a normal plane AB* will be

$$E_n=\frac{I_a}{l^2}=\frac{I_a}{h^2+d^2}=I_a\frac{\cos^2 a}{h^2}\text{ foot-candles} \tag{75}$$

$$\text{since } l=\frac{h}{\cos a}\text{ or } l^2=\frac{h^2}{\cos^2 a}. \tag{76}$$

The *intensity of light on a horizontal plane OP* making an angle a with this normal plane will be $E_h=\frac{I_a\cos a}{l^2}$, and by substituting as before

$$E_h=I_a\frac{\cos a}{l^2}=I_a\frac{\cos a}{h^2+d^2}=I_a\frac{\cos^3 a}{h^2}\text{ foot-candles} \tag{77}$$

The horizontal distance d from the vertical, may be found from the relation

$$d=h\tan a. \tag{78}$$

By the same reasoning the intensity of *illumination on a vertical plane EF* will be

$$E_v=I_a\frac{\sin a}{l^2}=I_a\frac{\sin a}{h^2+d^2}=I_a\frac{\sin^3 a}{d^2}\text{ foot-candles,} \tag{79}$$

$$\text{since } l=\frac{d}{\sin a}. \tag{80}$$

If it is desired to know the candle-power which will furnish a certain illumination the foregoing equations may be transposed, giving

$$I_a=E_n l^2=E_n(h^2+d^2)=E_n\frac{h^2}{\cos^2 a}, \tag{81}$$

the *candle-power required to illuminate a surface normal* to the rays,

$$I_a = E_h \frac{l^2}{\cos a} = E_h \frac{h^2+d^2}{\cos a} = E_h \frac{h^2}{\cos^3 a}, \qquad (82)$$

the *candle-power necessary to give a horizontal illumination* E_h, and

$$I_a = E_v \frac{l^2}{\sin a} = E_v \frac{(h^2+d^2)}{\sin a} = E_v \frac{d^2}{\sin^3 a}, \qquad (83)$$

the *candle-power sufficient to illuminate a vertical surface d* feet from the source with an illumination of E_v, foot-candles.

The values of sin a, cos, a, tan a, $\sin^2 a$, $\cos^2 a$, $\sin^3 a$, and $\cos^3 a$ are given in *Table* 1, *of the appendix* and will be found convenient in calculations of the nature of those above.

Illumination on horizontal areas and the candle-power to produce that illumination are the most common calculations in the distribution of light. In street lighting and in interior lighting where the lamps are equipped with opaque reflectors which throw the light downward on floors or surfaces dark in color, these equations are applicable and will give very approximate results. A set of constants per unit candle-power for various heights of suspension of the source above the plane of reference may be tabulated making preliminary determinations of such an installation comparatively simple. It will be seen that in the equation $E_h = I_a \frac{\cos^3 a}{h^2}$ the values of $\frac{\cos^3 a}{h^2}$ can be determined for various values of a and h, and designated by K, then $E_h = KI_a$ and $I_a = \frac{E_h}{K}$.

These values of K for various heights, distances and angles are given in *Tables* 2 *and* 3 *of the appendix* together with an illustration of their use.

Point-by-point Method—Graphical.—The calculation of illumination at a point on a plane may be performed graphically,[1] supplemented by a few simple calculations, without the use of the preceding equations. As already indicated the illumination directly under the source would be

$$E_n = \frac{I_v}{h^2} \qquad (84)$$

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[1] *Electrical World*, Vol. 55, p. 1215, May 12, 1910

or from the values indicated by the polar curve of Fig. 100 when the lamp is placed 8 ft. above the plane

$$E_n = \frac{52}{8^2} = 0.81 \text{ foot-candle,} \qquad (85)$$

When the point is not directly under the lamp as at M the graphical method is applied as indicated in Fig. 100, based on the equation

$$E_h = \frac{I_a \cos a}{h^2 + d^2} \qquad (86)$$

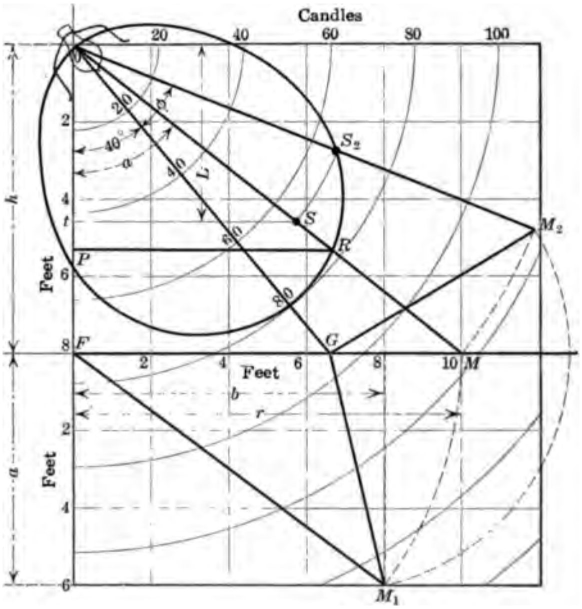

FIG. 100.—Graphical solution of the point-by-point method.

where E_h = the horizontal illumination in foot-candles.

I_a = the candle-power a degrees from the vertical.
h = height of lamp above the working plane.
d = distance from beneath the source.

It will be seen that the polar diagram is cross-sectioned and h and d laid off to scale. The point R on the polar curve is determined by drawing the line OM. The vertical component of the candle-power, $I_a \cos a$, will be represented by OP, in this case 48 candle-power and E_h at M will be,

$$\mathrm{E}_h = \frac{48}{8^2 + 10^2} = 0.3 \text{ foot-candle.} \qquad (87)$$

If the *source of light is inclined* and the point under consideration is not in a vertical plane through the axis of the source of light the case becomes somewhat more complicated, still the same equation holds and the solution is simple if certain facts are kept in mind. Consider in this case the point at M_1, Fig. 100, 6 ft. from the vertical plane through the axis of the source and 10 ft. from beneath the source. The plane of M_1 makes the angle M_1DG with the plane OFG. The candle-power in the direction of M_1 is determined by rotating a line between O and M_1 around the axis OG into the vertical plane. This line falling at OM_2 indicates the candle-power $OS_2 = 66$ in the direction of M_1 which when transferred to ES on OM has the vertical component of candle-power in the direction of M_1 equal to $Ot = 42$ and by applying the above equation we have

$$\mathrm{E}_h = \frac{42}{8_2 + 10_2} = 0.255 \text{ lumen per square foot.} \qquad (88)$$

Chart for Determining Point-by-point Values.—The equation $E_h = \frac{I \cos^3 a}{h^2}$ may be applied to the construction of charts and diagrams whereby the various calculations may be performed graphically in much less time. By transposing the above equation we have

$$E_h h^2 = I \cos^3 a \qquad (89)$$

Since h may be considered constant we have the equation of a hyperbola and by constructing a chart composed of hyperbolic curves the value of any one of the four quantities contained in the above equation may be determined graphically by moving along the proper curve from one set of coördinates representing $E_h h^2$ to the other set of coördinates representing $I \cos^3 a$. Such a chart by Mr. Austin[1] is shown in Fig. 101, the values of I and h^2 being the ordinates while E_h and $\cos^3 a$ are the abscissæ.

The manipulation of this chart will be best understood by an example. Assume the lamp to be placed 15 ft. above and 10 ft. to the side from the point where it is desired to determine the intensity of illumination. From the angle diagram in the upper right-hand corner the angle a is found to be 35 degrees. Assume also that a lamp is used which gives 300 candle-power, 35 degrees

[1] *Electrical World*, Vol. 56, p. 124, Nov. 24, 1910.

from the vertical. Find the intersection on the chart of the ordinate equal to 300 in the candle-power column and the abscissa $\cos^3 35$ degrees in the angle a column. This is at a, near

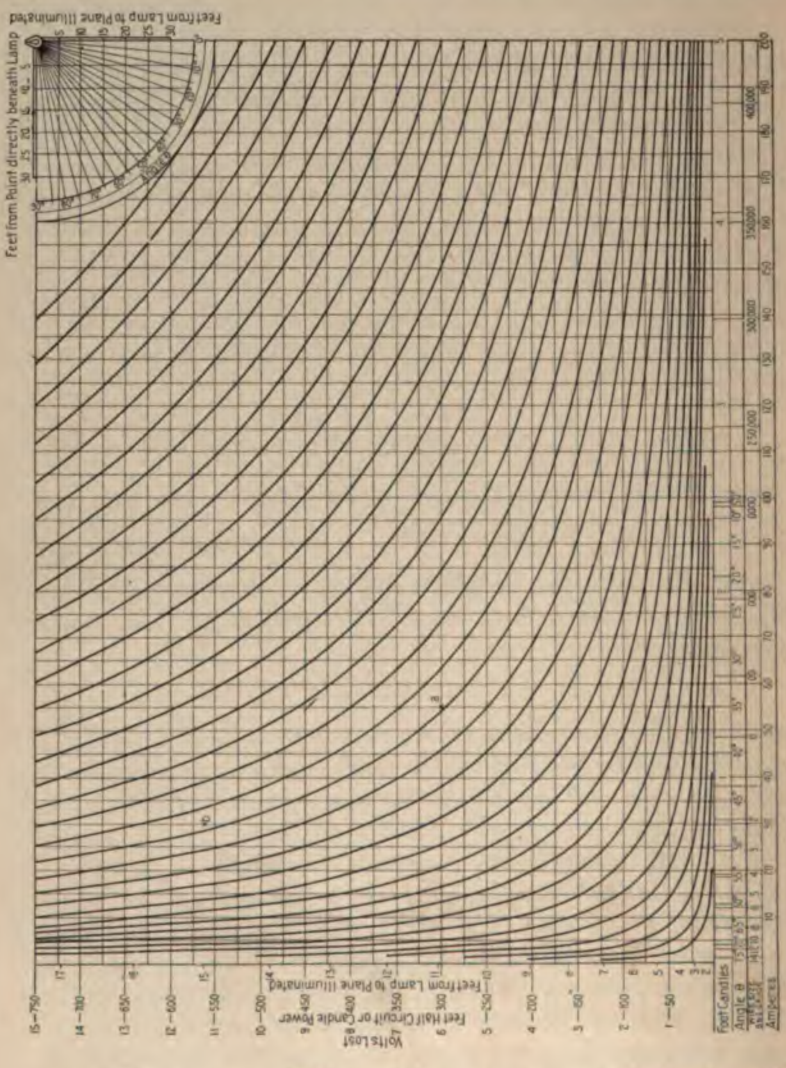

FIG. 101.—Chart for calculating intensity of illumination.

one of the curves. Follow this curve to a point b where its projection on the vertical falls at 15, the height of the lamp above the plane. Then by dropping a vertical line to the foot-candle scale at the bottom of the chart the value of $E_h = .75$ is obtained. This same diagram may be used for determining sizes of wire or loss in voltage in a manner similar to the above since $E = \frac{10.8\,IL}{d^2}$ or

$$d^2E = 10.8\,IL = 21.6\,ID \tag{90}$$

where d is the diameter of the wire in mils, E the loss in volts, 10.8 the mil-foot resistance of copper, I the current in amperes, L the length of wire in feet equal to $2D$.

The Slide-rule for Calculating Illumination.[1]—The ordinary slide-rule may be used for calculating illumination intensities and will be found convenient when tables or diagrams are not available. If the normal or horizontal illumination is required the formulæ

$$E_n = \frac{I_a \cos^2 a}{h^2} \tag{75}$$

and

$$E_h = \frac{I_a \cos^3 a}{h^2} \tag{77}$$

are used respectively. It is evident that after the values of $\cos^2 a$ and $\cos^3 a$ are obtained, the calculations can be performed by the ordinary processes of multiplication and division. The determination of a, $\cos^2 a$, and $\cos^3 a$ are quite simple if the height of the lamp h and the horizontal distance d are known. This deduction can perhaps be best understood from an example. Suppose

$$h = 9.5 \text{ and } d = 7, \text{ then } d/h = \tan a.$$

From this we can obtain a from the slide-rule by the use of the "T" tangent scale on the back of the slide. In the case above, place the right-hand index of the tangent scale over 9.5 and find over 7 the angle a equal to 36° 20′. To determine the value of $\cos a$ we resort to the "S" sine scale since $\cos a = \sin(90 - a)$. Then $90 - a = 53° 40'$. Set the slide so that the indices of slide and stock correspond and read over 53° 40′ on the "S" scale, the cosine of 36° 20′ equal to 0.805. Set the runner over this value of $\cos a$ transferred to the lower scale and find the value of $\cos^2 a$ under the runner on the upper scale. To find the value of $\cos^3 a$ turn over the slide and multiply the value of $\cos^2 a$ obtained above by the value of $\cos a$. If h is greater than d or if $h = 7$ and $d = 9.5$ we set the "T" scale as before with the index on the larger number but we find over 7 not the value of a, but the value of the complement of a or $(90° - a)$. Having found a the value of $\cos a$ can be found by means of the sine scale. The remainder of the process is the same as before.

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[1] *Illum. Eng.*, N. Y., Vol. 3, p. 152, May, 1908.

When the first digit of the greater number is less than the first digit of the smaller number, the left-hand index of the "T" scale should be set over the greater number. Thus if $H = 12$ and $d = 8$, place left-hand index of "T" scale over 12 and read over 8 the angle a, 53° 40′. The remaining calculations are performed as before.

The Macbeth "Calculator."[1]—The "calculator" is the name applied to an instrument designed by Norman Macbeth. It involves the principles of the slide-rule, by which the solution of many of the various mathematical problems which occur in illuminating engineering may be accomplished.

The calculator consists of two concentric revolvable disks having circular scales printed upon them, and a runner of transparent celluloid also turning about the center and having on its surface a fine radial line.

The computations which can be performed by this instrument are based on equations

$$E_n = \frac{I_a}{l^2} \text{ and } E_h = \frac{I_a \cos^3 a}{h^2}, \tag{91}$$

which have already been discussed. Any of the desired values in these equations may be obtained by use of the calculator if the others are known. The calculator may also be used for ordinary multiplication and division.

The arrangement of the several scales upon the two disks is shown in Fig. 102. The large disk carries two scales; the scale on the outer circle is used for reading both intensity of light in candle-power, and intensity of illumination in foot-candles. The inner scale is used to determine the distance d from the foot of the perpendicular drawn through the light source.

The smaller disk carries nine circular scales, beginning with the outer scale marked "height" gives the heights of the light sources above the given plane. The next two circles form a single scale which is marked "horizontal and vertical illumination angles" and is used in problems in which it is required to determine either the horizontal or vertical intensity of illumination from rays striking at a given angle, or for determining the angle of the incident rays when the height of the light source and the distance to the point from the vertical, are given.

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[1] *Illum. Eng.*, N. Y., Vol. 3, p. 21, Mar., 1908.

The next two circles toward the center form a single scale marked "normal illumination angles" which is used in problems requiring determination of the intensity of normal illumination when the candle-power and angle are given, or when the intensity of illumination and height are given to read candle-power at all angles.

The remaining four circles constitute a single scale for determining the angle when the height and distance are given, or the distance corresponding to any given height when the angle is given.

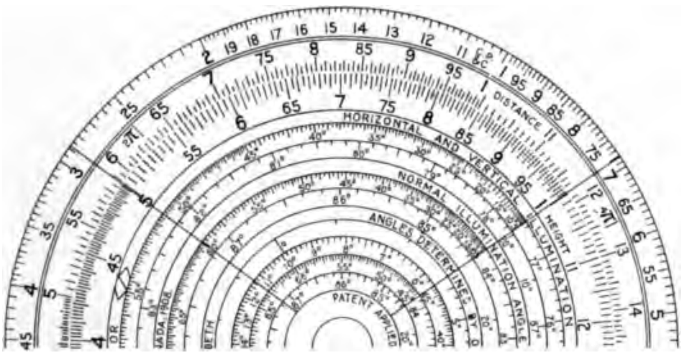

FIG. 102.—Macbeth's calculator.

The two settings illustrated in Fig. 102 show the method of obtaining the normal illumination intensity. In this case a 16 candle-power lamp having a vertical downward candle-power of 7.2 is placed 5 ft. above the plane. The runner is placed over 7.2 and zero on the small disk brought under the hair line. The runner is then moved to 5 on the height scale and the resultant intensity of illumination .288 is read off under the line on the outer scale. As in using a slide-rule the position of the decimal point must be determined.

To determine the horizontal illumination at a point at some location, say 10 ft. from a position directly under the lamp, it first becomes necessary to know the angle on the polar curve at which to read the candle-power of the source in this direction. To obtain this set 5 on the *H* scale over 10 on the *D* scale, Fig. 103, bring the runner to 1 on the *D* scale and under the runner in the second smaller circle *P* scale, read 63° 20′. Assume the candle-power at this angle to be 15. Set the runner at 15 on the candle-power scale, Fig. 104, turn the inner disk to bring 63° 20′ on the scale marked "horizontal and vertical illumination angles" under the line. Hold the two disks, and bring the runner to 5 on the height scale. Under the runner on the outer scale read 542. To determine the position of the decimal point with both disks set, refer back to zero angle, from which we note that the equivalent candle-power at 0° 20′ is 1.35. The decimal point may then be readily determined by I/h^2 or 1.35/25, which equals 0.05 roughly, indicating that the previous reading should be .0540.

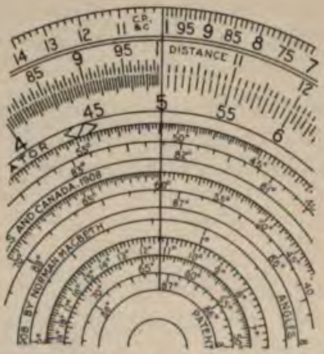

Fig. 103.—Macbeth's calculator.

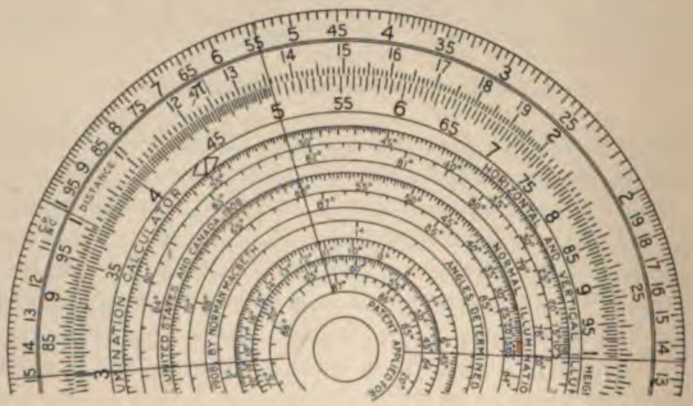

Fig. 104.—Macbeth's calculator.

To determine the lumens, set 1 on the *H* scale on 4π on the *D* scale, bring the runner to the value of the mean spherical candlepower on the *H* scale and read under the runner on the *D* scale the value of the lumens.

There are several other calculations which may be performed by means of this ingenious little device and a set of conversion tables on the back of the calculator will be found very convenient for reference.

Illumination Surveys.—Illumination surveys, or measurements of illumination at points on a reference plane are frequently made to determine the intensity and distribution of the illumination due to a lighting installation. This method is often employed to compare the lighting efficiency of two classes of lighting apparatus. In this latter case too much weight should not be given results so obtained since due to absence of glare and to greater diffusion of the light the system giving the lower average foot-candle intensity may be more effective and more satisfactory.

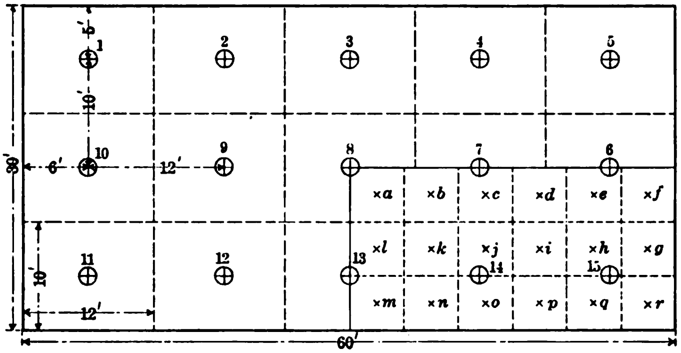

FIG. 105.—Location of test stations for illumination survey.

In making tests of this nature it is customary to divide the test area up into a number of small equal sections, and measure the intensity of the illumination at the center of each section. If the room is very large and the lighting units well distributed it is often sufficient to measure the illumination in a number of equal sections of a part of the total area. Assuming that the result of each measurement represents the average intensity of that section, it follows that the average illumination will be the average of these readings. The total light flux in lumens will equal the average intensity in foot-candles multiplied by the area in square feet.

The method of laying off a room into sections for a test as explained above, is illustrated in Fig. 105. The 15 lamps are located at 1, 2, 3, etc., as shown. It will be seen that the four-fourths of the room are similar, consequently one-fourth of the room may be divided into sections and readings taken at *a*, *b*, *c*, *d*, etc., to obtain the average intensity for the room.

It often happens, when all of the light is received from one lamp or cluster and where the class of service makes it impossible to measure the illumination at the proper places, that a certain direction from beneath the source may be chosen along which the illumination may be measured, and the average intensity calculated from these readings. The locations of test stations for a survey of this nature are shown in Fig. 106 by the points *a*, *b*, *c*, *d*, etc. It is essential in a test of this nature that the distribution of light from the source be equal in azimuth and that the reflected light be proportionately distributed; in other words, the measurement at each station should represent the average intensity along the circumference of a circle drawn through that station with the point beneath the source as a center The representation, interpretation and calculation of the measurements thus obtained are given on the following pages.

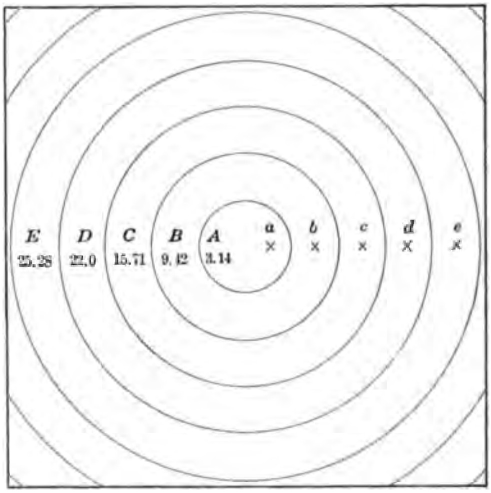

FIG. 106.—Areas and test stations for circular equi-luminous areas.

The Representation of Results.—The results of calculations of illumination intensity by the point-by-point method and the values obtained by illuminometry may be presented as numerical values in tabulated form or they may be represented graphically by curves showing the intensity at different distances from beneath the source or by "equiluminous lines" showing the regions of equal intensity with the value of the intensity given on each line. Still more elaborate methods combining the two just mentioned, and showing one or both in perspective or by projection may be employed.

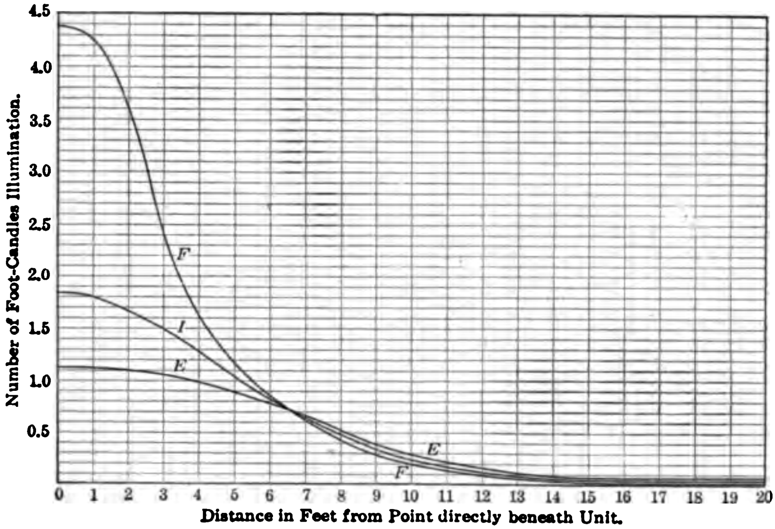

FIG. 107.—Curves showing illumination produced by a lamp with extensive, intensive and focusing types of reflectors. (Lamp eight feet above plane.)

In the case of one lamp the values of illumination intensity may be represented as shown by Fig. 107[1]. These curves represent the distribution of illumination from a 100-watt tungsten lamp when equipped with extensive, intensive and focusing types of high efficiency reflectors and when placed 8 ft. above the illuminated plane.

In the same way that the polar curves showing the distribution of light from a source may lead to erroneous impressions regarding the light flux emitted, the illumination diagram of foot-candle

[1] Bul. 6D, Nat. Elect. Lamp Assoc.

values at various distances may also be misinterpreted in regard to the amount of light flux. It should be remembered that these curves represent the intensity at different distances from a point beneath the lamp and that the intensity along a circle with this point as a center and a radius of, say 4 ft., will be the same as shown at 4 ft. by the curve. If we assume the illuminated area laid off into a number of concentric circular surfaces, centering beneath the source, Fig. 106, it will be readily understood that while the intensity may be less at a distance from beneath the source, it is illuminating a greater area and the total light flux may be more than in smaller areas when the intensity is much greater. If we designate such areas by *A*, *B*, *C*, *D*, etc., each 1 ft. in width, then the area of

$$\begin{aligned}
A &= \pi\, 1^2 &&= \pi = 3.14 \\
B &= \pi\, (2^2 - 1^2) &&= 3\,\pi = 9.42 \\
C &= \pi\, (3^2 - 2^2) &&= 5\,\pi = 15.71 \\
D &= \pi\, (4^2 - 3^2) &&= 7\,\pi = 22.00 \\
\hline
&& \text{Total area} &= 50.27
\end{aligned}$$

and if the average illumination intensity of *A*, *B*, *C* and *D* be 4, 3, 2 and 1 foot-candles, respectively, then the total flux in lumens is

$$\begin{aligned}
A. &= 3.14 \times 4 = 12.56 \\
B. &= 9.42 \times 3 = 28.26 \\
C. &= 15.71 \times 2 = 31.42 \\
D. &= 22. \times 1 = 22. \\
\hline
\text{Total lumens} &= 94.24
\end{aligned}$$

Hence the average illumination will be

$$94.24/50.27 = 1.87 \text{ foot-candles or lumens per square foot,}$$

and not

$$\frac{4+3+2+1}{4} = 2.5 \text{ foot-candles.}$$

The distribution of illumination from two or more lamps may be represented as shown in Fig. 108. These curves show the intensities along a line connecting points beneath the sources when the lamps are equipped with intensive, extensive and focusing types of reflectors. In this case the lamps are placed 14 ft. apart and 12 ft. above the illuminated plane.

From a study of these curves it will be seen that more uniform illumination can be obtained by changing the ratio of the distance between the sources to the height of suspension. In the case of the focusing equipment the ratio d/h must be decreased, *i.e.*, the distance between lamps must be decreased or the height of suspension increased. At the same time it is obvious that the intensity of illumination will change, increasing as the lamps are brought nearer and decreasing as they are elevated. By similar reasoning the other two types must be placed further apart or lower, and the extensive type more than the intensive type.

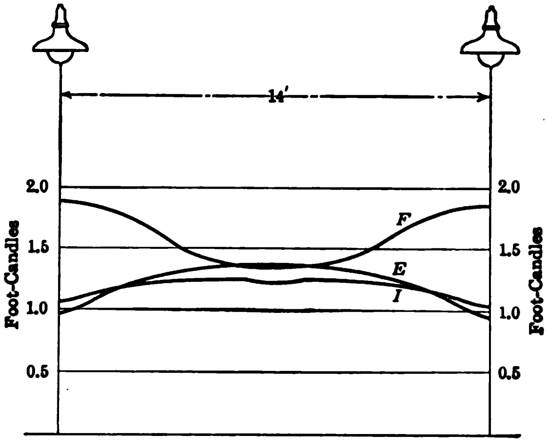

FIG. 108.—Showing illumination due to two units with three types of reflectors. (Lamps spaced 14 feet apart and 12 feet above the floor.)

A very satisfactory method of representing the intensity of illumination over certain areas is to lay off the illuminated surface to some convenient scale and calculate and plot equiluminous lines. Such a diagram is shown in Fig. 109 which illustrates the distribution of illumination from four sources, of rather strong downward intensity, placed over the areas receiving the greatest illumination. The vertical distribution of light through the center of the surface is represented by the broken line at the side of the diagram. See also Figs. 153, 160 and 191.

The Effect of Light Colored Ceiling, Walls and Floor on Illumination.—As stated at the beginning of the chapter, the point-by-point methods are applicable only to those classes of service where the illumination is not increased by reflection from surrounding objects or surfaces such as the ceiling and walls of a room. When the ceiling, walls, or floor of a rôom are light in color, the illumination will be increased by a greater or less amount depending upon the distribution of light from the source or sources and the reflecting power of the different parts of the interior.

Investigations of the effect of reflection and interaction of light colored ceiling, walls, and floor, were carried out by Lansingh and Rolph[1] under actual conditions. The room which they used for the tests was about 24 by 11.5 ft. and 10 ft. high. Dark green burlap and dark green rugs were used for the dark finish, and light cream colored wrapping paper constituted the covering for the light surface. The lamps were placed close to the ceiling. Tests were run with

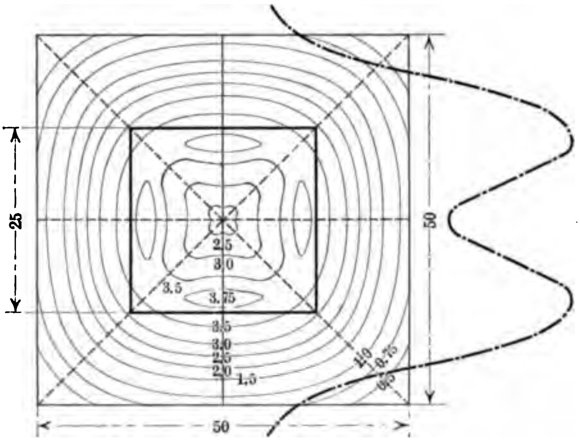

FIG. 109.—Representation of illumination values.

1. 1 bare lamp.
2. 1 lamp with an "extensive" prismatic reflector.
3. 3 bare lamps—distributed.
4. 3 lamps with "extensive" prismatic reflectors.

And eight light surveys were run for each lighting equipment with the following combinations:

1. Ceiling, walls, and floor dark.
2. Ceiling light, walls and floor dark.
3. Ceiling and walls light, floor dark.

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[1] Trans. Illum. Eng. Soc., Vol. 3, p. 584, 1908.

4. Ceiling, walls and floor light.
5. Walls light, ceiling and floor dark.
6. Walls and floor light, ceiling dark.
7. Floor light, ceiling and walls dark.
8. Ceiling and floor light, walls dark.

The increase in illumination due to the reflection and counter-reflection of the different surfaces is surprising. The per cent. increase in illumination over that with dark ceiling, walls, and floor for each of the four equipments and under the different conditions are given in Table 30. Since the values obtained by the "point-by-point" methods would correspond to those obtained in the above tests under the conditions of dark interior surfaces, these percentages increase may be assumed to apply approximately to increases in illumination over those obtained by the point-by-point method under similar conditions.

TABLE 30

Increase in Illumination Due to Light Ceiling, Walls, and Floor

	1 bare lamp, per cent.	1 lamp with reflector, per cent.	3 bare lamps, per cent.	3 lamps with reflectors, per cent.
1. Increase due to light ceiling alone...	79	33	96	29
2. Increase due to light walls alone.....	63	19	77	26
3. Increase due to light floor alone.....	11	5	4	−1
4. Additional increase due to interaction of light ceiling and light walls.	53	23	54	37
5. Additional increase due to interaction of light ceiling and light floor.	5	0	6	4
6. Additional increase due to interaction of light walls and light floor.	5	12	11	10
7. Additional increase due to interaction of light ceiling, light walls and light floor.	42	58	60	45
8. Total increase due to light ceiling, light walls and light floor.	258	150	308	150
9. Increase due to light ceiling, light walls and their interactions, 1 + 2 + 4.	195	75	227	92

From the foregoing discussion and data it is evidently possible to deduce some average figures which can be used with discretion in actual practice in lighting installation problems. Obviously reflection conditions vary widely in practice so that it is impossible to give values universally applicable. However, if high efficiency reflectors of the prismatic or the translucent glass types are used an *increase in illumination* of small and medium size rooms *over that obtained by the point-by-point method* may be expected approximately equal to the figures given in Table 31.

Table 31

Approximate per cent. Increase in Illumination over that Obtained by the Point-by-point Method

Condition of ceiling	Condition of walls	Per cent. increase over calculated values
Very dark	Very dark	0
Medium	Very dark	15
Very light	Very dark	30
Medium	Medium	40
Very light	Medium	50
Very light	Very light	80

In very large rooms the question of reflection from the walls becomes of much less importance since the increase in illumination due to distant lamps will about counter-balance that due to light walls in small and medium size rooms. The reflection from the ceiling remains and the light thus reflected has a tendency to make the illumination more uniform.

Illumination tests of a similar nature by Sharp and Millar[1] in a room approximately 12×12 ft. and 10 ft. high are summarized below. In this room the ceiling was covered with white cloth. The walls were a grayish-white and the windows covered with manila paper. Black cloth coverings for ceiling and walls were provided. Screens, light shades, and test-plates were also provided so that the light or illumination due to the source or any reflecting surface at any point in the room could be measured. The source of light consisted of a 250-watt, bowl-frosted, metallized filament lamp equipped with a satin finish prismatic bowl reflector. It was placed with the lower edge of the reflector 9 ft. 4 in. from the floor. The working plane for the illumination tests was 36 in. from the floor. The results of this series of tests are given in Table 32 and furnish valuable information concerning the action of reflecting surfaces under actual practical conditions.

[1] Trans. Illum. Eng. Soc. Vol. 5, p. 391, 1910.

TABLE 32

Effect of Reflection and Interreflection upon Illumination

Reference Plane Illumination	Lumens
Direct light from source	191
Light from ceiling alone	42
Light from walls by way of ceiling	26
Light from walls alone by way of ceiling	86
Light from ceiling by way of walls	35
Total from different sources on the working plane	380
Total by direct measurement	376
Increase due to light ceiling and walls in per cent	97
Increase due to light ceiling alone in per cent	22
Effective angle light ceiling and medium walls	70°
Effective angle light ceiling, dark walls	50°
Effective angle dark ceiling and walls	45°

Ceiling Illumination	Lumens
Direct from light source	183
Light from ceiling reflected back by walls	20
Wall light reflected by walls	77
Total flux on ceiling	280

Wall Illumination	Lumens
Direct from light source	382
Ceiling light from ceiling	116
Wall light reflected by walls	104
Ceiling *via* walls and walls *via* ceiling	70
Total flux on walls	672

Summation	Lumens
Direct light flux upon ceiling	183
Direct light flux upon walls	382
Direct light flux upon reference plane	191
Total light flux direct from the lamp	756
Total light flux calculated from distribution curve of the light source	779
Total effective light flux upon ceiling	280
Total effective light flux upon walls	672
Total effective light flux upon reference plane	380
Total effective lumens	1332

Total reflected by ceiling (42+26+35+116)	219
Total reflected by walls (86+35+20+77)	218
Reflecting coefficient of ceiling (219/280)	78 per cent.
Reflecting coefficient of walls (218/672)	32 per cent.
Ratio of the effective light flux on the reference plane to the amount received direct from the lamp (380/191)	1.99

In the methods of performing illumination calculations involving the use of constants, discussed on the following pages consideration is given to the effect of reflection from the walls and ceiling in increasing the illumination over that calculated by the point-by-point method from the photometric curves of the unit.

The Flux of Light Method of Performing Illumination Calculations.[1]—For determining approximately the average intensity of illumination obtainable from a given lighting installation or for determining approximately the number of lamps required for a given installation in order to produce a desired illumination intensity, the so-called "flux of light" method, suggested by Dr. Sharp in 1907 and first put into practice by Cravath and Lansingh, will be found very convenient.

It has already been shown that the total flux of light emitted by a source is equal to the *mean spherical candle-power* of the source *multiplied by* 4π.

$$F = 4\pi I_{ms} = 4\pi f I_h. \tag{92}$$

It has also been shown that the light flux illuminating a given surface is equal to the *average incident illumination multiplied by the area of the surface in square feet;* or the effective light flux must be

$$F_e = SE_o \tag{93}$$

Due to absorption of light by various types of reflectors and by different reflecting surfaces it is obvious that all of the light generated by the source is not available for illumination purposes on any reference plane. It will also be evident that there is no fixed ratio between the total amount generated and the amount effective except under identically the same conditions. The per cent. of the total light thus effective will depend upon the distribution of light from the source, the efficiency of the reflector, and the reflecting power of the surroundings. Consequently the flux of light method involves sets of constants for various conditions which must be based on experimental data.

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[1] Trans. Illum. Eng. Soc., Vol. 3, p. 518, 1908; Trans. Illum. Eng. Soc., Vol. 4, p. 310, 1909; *Elect. Rev. and West. Elect*, Vol. 56, pp. 793, 847, April, 1910.

1. These sets of constants may refer to the *"effective angle,"* *i.e.*, the solid angle within which the light flux may be considered effective in producing the desired illumination.

2. They may refer to the *per cent. of the total flux* generated which will be *effective under certain conditions* of light distribution and reflection, since for each solid angle the amount of light flux therein will be a certain per cent. of the total for that lamp and equipment.

3. Again since there is a definite relation, for a particular lamp, between the total light flux emitted and the power consumed, it will be equally practicable to employ a set of constants indicating the *effective lumens per watt or per cubic foot of gas per hour.*

1. *The Effective Angle Method.*—By the effective angle method the light flux within a certain solid angle may be found from the polar curve by means of ~~one of~~ the methods described in the previous chapter. Then the average illumination intensity may be calculated by the equation

$$E_o = NF_e/S \qquad (94)$$

or the number of lamps found by the expression

$$N = SE_o/F_e \qquad (95)$$

where E_o is the average illumination intensity,
N the number of lamps,
F_e the effective lumens per lamp,
and S the area of the surface illuminated in square feet.

The "effective" angle or the angle from the vertical below which the light flux may be considered as effective for illumination purposes depends, of course, upon the size of the room, the location of the lamps, the equipment of the lamps and the color of the walls and ceilings. This may be inferred from the preceding data on the effect of light-colored ceiling and walls upon the value of the illumination. From the same series of tests by Messrs. Lansingh and Rolph[1] in a room 24×11.5 ft. and 10 ft. high the values of the effective angle and effective lumens per watt when using 40-watt tungsten lamps as given in Table 33 were found.

The conclusion which may be drawn from these data are that in a small room with light ceiling 60 degrees may be assumed as

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[1] Trans. Illum. Eng. Soc., Vol. 3, p. 599, 1908.

TABLE 33

Effective Angle under Various Conditions

Conditions	One bare lamp		One lamp with reflector		Three bare lamps		Three bare lamps with reflectors	
	Effective angle, degrees	Lumens per watt	Effective angle, degrees	Lumens per watt	Effective angle, degrees	Lumens per watt	Effective angle, dgerees	Lumens per watt
1. Calculated value—point-by-point method.	50	1.1	50	2.5	47	.94	45	2.04
2. Ceiling, walls and floor dark...........	53	1.3	50	2.5	50	1.1	46	2.08
3. Ceiling light, walls and floor dark......	69	2.32	63	3.33	60	2.12	56	2.7
4. Ceiling and walls light, floor dark.....	86	3.85	82	4.35	83	3.57	80	4.0
5. Ceiling, walls and floor, light.........	90+	4.75	90+	6.25	90+	4.55	90+	5.26
6. Walls light, ceiling and floor dark......	66	2.13	58	2.94	63	1.96	55	2.63
7. Walls and floor light, ceiling dark......	69	2.32	64	3.33	65	2.13	57	2.78
8. Floor light, ceiling and walls dark......	56	1.45	51	2.50	50	1.14	46	2.08
9. Ceiling and floor light, walls dark.....	71	2.56	64	3.33	67	2.27	56	2.78

a fair value of the effective angle. With light ceiling and walls about 80 degrees, while with all the surfaces dark 45 to 50 degrees may be assumed a fair value. The case of a large room with a light ceiling would compare favorably with that of a small room with light ceiling and walls since the light from distant lamps in the former case would counter-balance that received from the light walls of the latter and make the effective angle about 80 degrees. Thus we have for the effective angle:

Small room dark ceiling, walls and floor,	45–50 degrees.
Small room light ceiling,	60 degrees.
Small room light ceiling and walls,	80 degrees.
Large room light ceiling,	80 degrees.

2. *The "Effective Constant" Method.*—Since it has become customary to give the total lumens emitted by a source of light as part of the information concerning its performance, it will be found more convenient where these values are given, to derive a set of constants by which the total lumens may be multiplied to obtain the effective lumens.

In the following table are given the values of the total lumens radiated by each of the incandescent lamps in most common use for lighting purposes. The values are given for lamps operating at high-, medium- and low-rated voltages and are correct for standard makes of lamps of the year 1911.

TABLE 34

Total Lumens for Standard Incandescent Lamps

Rated	Tungsten			Tantalum			Gem			Carbon		
Watts	high	med.	low	high	med.	low	high	med.	low	high	med.	low
25	189	175	162	126	117	109				84	75	68
40	323	302	282	221	206	191	162	148	135			
50				277	257	239	207	189	172	174	158	142
60	505	470	438				249	227	206	208	190	170
80				443	412	383	337	308	278			
100	842	783	731				419	384	346	349	316	284
120										419	379	340
150	1262	1170	1095									
250	2195	2035	1895									
400	3512	3254	3034									
500	4390	4070	3793									

The percentages of the total lumens generated effective in producing useful illumination are given in the following table. It is obvious that these values apply to any of the preceding lamps since they refer to a certain per cent. of the total flux and depend upon the type of reflector or equipment and the surroundings.

TABLE 35[1]

Per cent. of the Total Light-flux which is Effective (k).

Equipment	Color of ceiling	Color of walls	Shortest dimension of room divided by the height	(k) Per cent. efficiency
Direct:				
Prismatic or opal reflectors at ceiling.	Light	Light	3 or more	55 to 65
Prismatic or opal reflectors at ceiling.	Light	Dark	3 or more	50 to 60
Prismatic or opal reflectors at ceiling.	Light	Light	1 to 2.5	50 to 60
Prismatic or opal reflectors at ceiling.	Light	Dark	1 to 2.5	45 to 55
Prismatic or opal reflectors at ceiling.	Dark	Dark	1 to 2.5	30 to 40
Direct:				
Frosted globes at ceiling	Light	Light	3 or more	35 to 45
Frosted globes at ceiling	Light	Dark	3 or more	30 to 40
Frosted globes at ceiling	Light	Light	1 to 2.5	30 to 40
Frosted globes at ceiling	Light	Dark	1 to 2.5	25 to 35
Frosted globes at ceiling	Dark	Dark	1 to 2.5	15 to 25
Direct:				
Bare lamp at ceiling	Light	Light	3 or more	40 to 50
Bare lamp at ceiling	Light	Dark	3 or more	37 to 47
Bare lamp at ceiling	Light	Light	1 to 2.5	38 to 48
Bare lamp at ceiling	Light	Dark	1 to 2.5	30 to 40
Bare lamp at ceiling	Dark	Dark	1 to 2.5	20 to 30
Indirect:				
Cove enameled	Light	Light	3 or more	15 to 25
Cove enameled	Light	Light	1 to 2.5	10 to 20
Efficient conical reflectors on central fixtures.	Light	Light	3 or more	34 to 44
Efficient conical reflectors on central fixtures.	Light	Dark	3 or more	32 to 42
Efficient conical reflectors on central fixtures.	Light	Light	1 to 2.5	30 to 40
Efficient conical reflectors on central fixtures.	Light	Dark	1 to 2.5	25 to 35

It follows then from this method that the illumination intensity may be approximately determined by the equation

$$E_o = NkF_t/S \qquad (97)$$

or the number of lamps by the equation

$$N = SE_o/kF_t \qquad (98)$$

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[1] Compiled by Mr. Cravath.

where E_o is the average intensity in foot-candles.

N, the number of lamps.

k, the constant for the equipment and surroundings or the per cent. of the total flux effective.

F_t, the total lumens radiated by each lamp.

S, the area of the surface illuminated.

3. *The Effective Lumens per Watt or per Cubic Foot of Gas per Hour.*—The third method involves the use of a set of constants giving the effective lumens per watt or per cubic foot of gas per hour for the various lighting equipments and different classes of surroundings. Having given these constants the average illumination intensity will be

$$E_o = k'(W \text{ or } G)/S \tag{99}$$

The total watts or cubic feet of gas required will be

$$W \text{ or } G = SE_o/k' \tag{100}$$

and the number of lamps necessary to give an illumination of E_o foot-candles will be

$$N = \frac{W \text{ or } G}{w \text{ or } g} = \frac{SE_o}{k'w \text{ or } k'g} \tag{101}$$

where E_o is the average illumination intensity.

k' is the effective lumens per watt or per cubic foot of gas per hour.

S is the area illuminated in square feet.

W, the total watts consumed by the installation.

G, the total cubic feet of gas consumed per hour.

w and g, the consumption per lamp.

N, the number of lamps.

To show the manipulation of the flux of light method as given above we will determine the number of lamps required to light a store 30 by 60 ft. or an area of 1800 sq. ft., with an intensity of illumination of 3.75 foot-candles.

If the ceiling and walls of the room are light in color and tungsten lamps with high efficiency reflectors are used the effective lumens per watt should be approximately 4.5. Then by substituting in the equation given above we have

$$N = SE_o/k_w = (1800 \times 3.75)/(100 \times 4.5) = 15 \text{ 100-watt lamps.} \tag{102}$$

Another method of arriving at the same result is to calculate the watts per square foot or cubic feet of gas per hour per square foot, and determine the number of lamps by dividing the total watts or cubic feet of gas required for the installation by the amount taken by one lamp. Thus the

$$\begin{array}{c}\text{watts per sq. ft., or cu. ft.}\\ \text{per hour per sq. ft.}\end{array} = \frac{\text{intensity in foot-candles or effective lumens per square foot}}{\text{effective lumens per watt or per cu. ft. per hour from the luminous source}}$$

$$\text{or } w = \frac{E_o}{K}. \tag{103}$$

In this way the solution of the preceding problem is:

$$w = \frac{E_o}{K} = \frac{3.75}{4.5} = 0.833 \text{ watts per square foot.} \tag{104}$$

The number of lamps will obviously be equal to the product of the area and the watts per square foot divided by the watts per lamp, or

$$N = \frac{\text{area} \times \text{watts per sq. ft.}}{w \text{ per lamp}}. \tag{105}$$

By using 100-watt lamps and substituting we will get as before

$$N = \frac{1800 \times 0.833}{100} = 15 \text{ 100-watt lamps.}$$

By similar reasoning it will be found that 25 60-watt or 10 150-watt tungsten lamps will meet the requirements.

From the foregoing equations it can be shown that the average area illuminated by one lamp will be

$$s = S/N = k'w/E_o \tag{106}$$

No very exhaustive or conclusive data appear to be available concerning the effective lumens per watt or per cubic feet of gas which may be secured from different sources. Of course the effective lumens can be calculated for those lamps and conditions given under the effective constant method, the lumens per watt being the product of the lumens and the constant, divided by the watts. The values of the effective lumens for some of the other types of electrical lamps and different gas lamps are given in Table 36 and 37. The effective lumens per watt were secured by averaging the results obtained from various sources while the effective lumens per cubic foot of gas were compiled by Mr. Macbeth.

TABLE 36

The Effective Lumens per Watt or per Cubic Foot of Gas per Hour

Electric Lamps (Large Room)[1]

Lamp	Reflector	Ceiling	Walls	Effective lumens per watt
Tungsten	Enamel	Light	Light	3.4
Tungsten	Enamel	Light	Dark	3.0
Tungsten	Silvered glass	Light	Light	6.1
West. Nernst	Opaline globe	Light	Light	3.2
West. Nernst	Opaline globe	Light	Dark	2.6
Moore light	None	White	Light	2.7
Moore light	None	Light	Medium	2.1
Arc lamp, 5 amp. D. C. enclosed.	Opal inner, clear outer globes.	Light	Medium	2.0
Mercury arc	Enamel	Medium	Medium	5.5

TABLE 37

Gas Lamps (Small Room, Light Ceiling)[2]

Type of lamp	Glassware or reflector	Consumption cu. ft. per hour	Walls	Effective lumens per cu. ft. per hour
Inverted lamp	Prismatic or opal (concentrating).	3.33	Light	125
			Dark	115
Inverted lamp	Prismatic or opal (distributing).	3.33	Light	110
			Dark	100
Inverted lamp	French roughed ball globe.	3.33	Light	95
			Dark	70
Inverted cluster, four mantles.	Alabaster globe	13.0	Light	85
			Dark	64
Inverted arc, five mantles.	Alabaster globe	16.6	Light	87
			Dark	65
Upright arc, four mantles.	Opal reflector alabaster globe.	20.0	Light	75
			Dark	55
Upright arc, four mantles.	Alabaster globe	20.0	Light	66
			Dark	48

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[1] Trans. Illum. Eng. Soc., Vol. 3, p. 518, 1908; Trans. Illum. Eng. Soc., Vol. 4, pp. 321, 849, 885, 1909.

[2] Proc. Amer. Gas Inst., Vol. 4, p. 305, 1909.

Gas Lamps (Large Room, Light Ceiling)[1]

Type of lamp	Glassware or reflector	Consumption cu. ft. per hour per lamp	Walls	Effective lumens per cu. ft. per hour
Inverted lamp..........	Prismatic reflectors (concentrating).	3.3	Light	140
			Dark	128
Inverted lamp..........	Opal reflectors.......	3.4	Light	120
			Dark	109
Inverted lamp..........	Roughed balls.......	3.3	Light	101
			Dark	75
Inverted lamp..........	None................	3.3	Light	124
			Dark	84
Upright mantle........	Flat fluted cone......	3.7	Light	85
			Dark	50
Upright mantle........	Opal dome..........	3.6	Light	78
			Dark	59
Upright mantle........	Globe..............	5.0	Light	49
			Dark	27
Upright mantle........	Cylinder bare........	3.7	Light	77
			Dark	32
Open burner...........	None................	5.0	Light	22
			Dark	13
Four-burner, upright...	Cluster..............	19.0	Light	82
			Dark	57
Three-burner inverted..	Cluster..............	11.0	Light	123
			Dark	81

In order to use any of these flux of light methods it becomes necessary to have some knowledge of the values of illumination required for different classes of service. To give any fixed value for the illumination for any particular class of work is obviously impossible. The intensity of the illumination required involves the physiological and psychological characteristics of the individual; it depends upon the distribution and diffusion of the light and the location of the source as well as upon the class of service. In the successful application of the flux of light method much depends upon a judicious study of the conditions and the surroundings.

In regard to illumination in general a careful study should be made of the necessary and most satisfactory intensity, quality, and distribution of light concurrent with the general welfare of the people for whose use the system is designed. Too little light is likely to cause a strain on the retina, while, on the other hand, too much light should be avoided since the contraction of the iris is limited and a strain will likewise be imposed upon the optical system.

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[1] Trans. Illum. Eng. Soc., Vol. 4, pp. 96, 804, 1909.

However, values of intensities for different classes of service, which may form the basis of the flux of light method in this respect, are given in Table 38. These should be considered as applying to the general and average cases, for the writer is well aware that either from choice or necessity occasion may arise where values far different from these should be employed.

TABLE 38

Illumination intensity in foot-candles or lumens per square foot which may be used as representing the average requirements for the various classes of service:

Armory or drill hall	1.5–2
Art gallery walls	3–5
Auditorium	1–3
Automobile showroom	3–5
Baggage-room	0.5–1.5
Ballroom	2–3
Bank	3–4
Barber shop	2–4
Billboard	4–6
Billiard-room	0.5–1
Billiard-table	4–5
Book-keeping	3–5
Bowling- alley	1–1.5
Bowling-pins	4–5
Cafe	2–4
Carpenter shop	2–4
Cars, baggage	1–1.5
day coach	2–3
dining	2–3
mail	4–6
Pullman	2–3
street	2–3
Courts, handball	5–8
tennis	5–8
Courtroom	2–3
Church	2–3
Depot	0.5–2
waiting-room	2–3
Desk	3–5
Drafting-room	5–8
Factory, general illumination	1–2
bench illumination	4–6
complete illumination	4–5
fine work	5–6
Fire station, at time of alarm	2–3
at other times	1–1.5
Foundry	2–3
Garage	1.5–2
Gymnasium	2–4
Hall, concert and entertainment	2–4
Hospital corridor	0.5–1.5
wards, general	1.5–2
wards with local illumination	0.5–1
operating-table	8–10
Hotel, corridor	0.5–1
dining-room	2–3
bedroom	1–2
lobby	1.5–2
parlor	2–3
writing-room	2–3
Laboratory	3–5
Laundry	1–2
Library, stack-room	1–2
reading-room, no local illumination	3–4
reading-room, with local illumination	0.5–1
Lodge-room	2–3
Lunch-room	2–3
Machine-shop, general	1–1.5
local	3–4
Market	2–3
Moving-picture theater	1–2
Museum	3–4
Office desks	3–5
general illumination	1.5–2

Office, private	1–3
Pattern shop	3–4
Pool (see Billiard).	
Power-house	2–3
Postal service	4–6
Press-room	3–5
Public square	0.5–1
Reading, good print	2–3
fine print	3–5
Residence, porch	0.2–0.5
hall	0.5–1
reception-room	1–2
sitting-room	2–3
parlor	2–3
library	2–3
dining-room	1–2
music-room	2–3
kitchen	2–3
pantry	2–3
laundry	1–2
bedroom	1–2
bathroom	2–3
furnace-room	0.5–1
store-room	0.5–1
Restaurant	2–4
Rink, skating	2–3
Rug rack	10–15
School, class-room	2–3
study-room	2–3
assembly-room	1.5–2.5
office	2–3
cloak room	0.5–1
corridor	0.5–1
manual training	3–5
laboratory	3–5
drawing	4–6
drafting	5–7
Sewing, light goods	4–5
dark goods	8–10
Shipping room	1–2
Show window, light goods	5–20
medium goods	10–30
dark goods	20–50
Sign	4–6
Stable	0.5–1.5
Spinning mills	1.5–2
Station, railroad	1–2
Stereotyping	3–5
Stock room	1–2

Store, art	4–5
baker	2–4
book	2–4
butcher	2–4
china	2–3
cigar	2–4
clothing	4–6
cloak and suit	4–6
confectionery	2–4
decorator	2–4
drug	3–5
dry goods	4–6
Store, florist	3–5
furniture	4–6
furrier	4–6
grocery	2–4
harberdasher	3–5
hardware	3–5
hat	4–5
jewelry	4–5
lace	3–4
leather	4–5
meat	2–4
men's furnishings	3–5
millinery	4–6
music	3–4
notions	3–4
piano	4–5
post cards	3–4
shoe	3–4
stationery	3–4
tailor	4–6
tobacco	2–3
Street, business	0.4–0.6
residence	0.1–0.2
prominent residence	0.2–0.4
Studio	4–5
Telephone exchange	3–4
Theater, lobby	2–3
auditorium	2–3
moving picture	1–2
Train shed	0.5–1.5
Type setting	5–10
Warehouse	1–1.5
Weaving, light colors	2–3
dark colors	3–5
Wharf	1–1.5
Windows (see Show windows).	

McAllister's Absorption of Light Method[1].—Theoretical consideration and experimental observation show that when a lighting unit radiating equally in all directions is placed at the center of a sphere whose inner surface possesses a light-reflecting coefficient, k, the effective illumination over the surface of the sphere has a value of $1 \div (1-k)$ as compared with the value of 1 for a surface whose reflecting coefficient is zero. The relation here expressed has been found convenient in determining the characteristics of the integrating sphere already described, but it is claimed that it cannot be applied in determining the inter-reflections from the wall, ceiling and floor of a room not spherical in shape and having a non-uniform coefficient of reflection. Dr. McAllister believes that a more useful relation is that connected with the *absorption of light* rather than the reflection. That is to say, quite independent of reflection and counter-reflection, the lighting units within a room must produce the *sum of the lumens absorbed* by the various surfaces. From this relation one can readily determine the total lumens to be generated in order to produce a certain incident illumination, because for each surface there is a direct ratio between the absorbed and the incident lumens. The above relation can be expressed as follows:

$$F = aE_oS \qquad (107)$$

where

$F =$ total lumens absorbed by the surface,
$a =$ light-absorption coefficient $= (1-k)$,
$E_o =$ average incident light flux density in foot-candles,
$S =$ area of surface in square feet.

The application of this method may be more clearly shown by means of an example.

Assume a room 15 ft. in width, 20 ft. in length, and 10 ft. in height, having a white ceiling, light walls and a dark floor to be so lighted as to produce an average illumination intensity of 2 foot-candles on the ceiling, 1 foot-candle on the walls and 4 foot-candles upon the floor. Assume also that the light absorption coefficient of the ceiling is 0.20, of the walls 0.40, and of the floor 0.90, and determine the candle-power necessary to produce the desired results.

Then by means of the above formula the lumens absorbed by the various surfaces will be

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[1] *Elect. World,* Vol. 52, p. 1128, Nov. 21, 1908; *Elect. World,* Vol. 55, p. 1388, May 16, 1910.

$$0.20 \times 2 \times 300 = 120 \text{ for the ceiling,}$$
$$0.40 \times 1 \times 700 = 280 \text{ for the walls,}$$
$$0.90 \times 4 \times 300 = 1080 \text{ for the floor.}$$

This gives a total of 1480 lumens which would require a luminous source of 118 mean spherical candle-power.

It will be interesting to note in the above example that while only 1480 lumens are generated the total effective lumens on the various surfaces due to reflection and counter-reflection will be

$$2 \times 300 = 600 \text{ for the ceiling,}$$
$$1 \times 700 = 700 \text{ for the walls,}$$
$$4 \times 300 = 1200 \text{ for the floor,}$$

or 2500 effective lumens.

If it be assumed that the light received by the walls from the ceiling and floor is equal to that reflected from the walls to the ceiling and floor and the amount absorbed by the walls is supplied entirely by the source, then by means of the absorption of light method one may very easily calculate the amounts of light which must be directed toward the floor and ceiling in order to give the desired intensities on these two surfaces. In the above example the total lumens effective at the ceiling is 600. Of these 120 lumens are absorbed and 480 lumens are reflected to the floor. Of the total incident illumination of 1200 lumens on the floor 1080 lumens are absorbed and 120 lumens reflected. Of the 600 lumens at the ceiling 120 come from the floor, leaving 480 to be supplied by the illuminants. Similarly of the 1200 lumens at the floor, 480 come from the ceiling, leaving 720 lumens to be supplied by the lamps. Thus:

	Area sq. ft.	Intensity	Effective or incident lumens	Coefficient of absorption	Lumens absorbed
Ceiling....	300	2	600	0.20	120
Walls.....	700	1	700	0.40	280
Floor.....	300	4	1200	0.90	1080
			2500		1480

	Lumens reflected	Lumens received by reflection	Lumens supplied by lamps
Ceiling.....	480	120	480
Walls......	420	420[1]	280[1]
Floor......	120	480	720
		1020	1480

An analysis of the above values shows some interesting relations. The sums of the fluxes absorbed by the surfaces is equal to the sum of the fluxes sent to these surfaces from the lamps, but there is no immediate relation between the lumens directed toward each surface from the lamps and the lumens absorbed by that surface. For example, the lamps send 720 lumens to the floor, which absorbs 1080 lumens from a total of 1200 received by it. It is noteworthy in this connection that the familiar point-by-point method applied to the lighting installation here assumed would indicate an incident flux on the floor and ceiling of 720 and 480, respectively, whereas the true values are 1200 and 600, the increase being due solely to the inter-reflection between the two surfaces. For the purpose of investigating such inter-reflection and of determining the amount of light which the lamps should produce on each surface the absorption-of-light method is highly advantageous.

An objection which may be urged against the absorption-of-light method is that it ignores any lack of uniformity in the distribution of illumination over the individual surfaces considered. Such a criticism is fully justified by the facts, but it has no bearing on the value of the method both as a check on other methods which take into account the variation in light distribution and as a direct final method to be used in problems where the variation in light density is of no importance. For purpose of comparison it may be likened to a method which would permit one to ascertain at once the required horse-power rating of a motor to drive a certain machine tool, but did not specify whether the motor should be a series-wound direct-current machine equipped with interpoles or an induction motor with a coil-wound secondary.

For solving problems connected with the illuminating of reflecting surfaces, investigating quantitatively the effect of interreflection between surfaces, and ascertaining the limits in the distribution of light flux between illuminated surfaces, the absorption-of-light method should prove of much value to the illuminating engineer.

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[1] Assumed.

Although the above example, which is based on certain arbitrarily chosen constants, does not represent any actual installation, and the method here outlined cannot always be applied with the degree of simplicity indicated above, yet it possesses merit from a theoretical point of view, and it may prove of value in practical application.

Incidentally it may be mentioned that the only assumptions affecting the accuracy of the results obtained by the above method are those relating to the absorption of light by ceiling, wall and floor coverings of various fabrics and color combinations, and to the distribution of light flux. Knowledge concerning light absorption is not complete, but much is known about the absorption of light by the fabrics usually employed in rooms. The distribution of light flux depends both upon the arrangement of the illuminants and upon the reflecting surfaces, and hence difficulty would be encountered in assigning exact values to the distribution. However, the calculations are based on the average flux density over only three different areas in each room and the conditions are such that a large variation in distribution of illumination from the assumed average values produce only a moderate error in the final results.

A word of caution should here be interposed. The method allows one readily to determine the value of lumens required to produce a certain desired illumination on chosen surfaces, but it should not be assumed that any lighting equipment which affords the required lumens will produce the desired illumination, because the distribution of light flux may be such as to supply an excess of illumination at some points and a deficiency at others. The method shows merely the value of lumens that must be generated by the lamps when the lighting equipment is so designed as to afford the specific distribution of flux. It gives no clue as to the distribution, which distribution must be ascertained from other considerations.

It is instructive to apply the above method to the calculation of the illumination produced within an integrating photometric sphere. Let the internal area be 10 sq. ft., the incident light flux density be 25 foot-candles and the absorption coefficient be 0.20. The absorbed lumens will be $0.20\times 25\times 10$, thus requiring about 4 candle-power. The total incident light flux being 250 lumens, the multiplication factor is 5 – a value which would be obtained from substituting 0.80 for k in the expression $1\div(1-k)$. Thus, the method is applicable to homogeneous spherical surfaces as well as to non-homogeneous plane surfaces.

Being based on nature's law of conservation, this method is absolutely exact, and should not be considered as a mere approximation; any error resulting from applying the method in practice is attributable to errors in assigning the proper values to the "constants" involved, and does not indicate inexactness with reference to the method itself. The method gives no clue as to the distribution of illumination over each surface, but takes fully into account all of the inter-reflections between the various surfaces, and permits one to ascertain the total amount of light which the lamps must supply directly to each surface involved. Moreover, it discloses in a convincing manner certain limitations encountered in illuminating installations which have usually been ignored in applying other methods.

The significance of the remarks just made can best be appreciated by applying the method under certain assumed conditions, which are within the range of possibility, but somewhat unusual. For the purpose of avoiding all unnecessary complications, consider first the relations in an extremely narrow, long hall of great height, against one wall of which a total light of, say, 2000 lumens (160 mean spherical candles) is sent directly from the lamps, the other wall being illuminated exclusively by the light reflected from the former wall; assume that the light-absorption coefficient for each wall is 0.20. The problem is to determine the total incident flux on each wall and the amount absorbed by each. Assign arbitrarily a value of 100 units to the total light incident upon the first wall. Eighty per cent., or 80 units of this light, are reflected to the other wall where 20 per cent., or 16 units, are absorbed, 64 units being returned to the first wall. Thus, of the 100 units reaching the first wall 64 are reflected from the second wall and 36 come directly from the light sources. The 36 units of light are equal to 2000 lumens, so that each unit has a value of 55.55 lumens. It is seen, therefore, that on the first wall the incident lumens have a value of 5555, of which 2000 come directly from the lamps and 3555 are reflected from the second wall; the second wall receives 5555×0.80 = 4444 lumens, and reflects 4444×0.80 = 3555 lumens to the first wall. The former wall absorbs 5555 ×0.20 = 1111 lumens and the latter 4444×0.20 = 889 lumens, the total being the 2000 lumens produced by the lamps.

As one should have anticipated, the calculations noted above show that, quite independent of the wall upon which the light falls directly from the lamps, the illumination upon one wall cannot differ from that upon the other by more than 20 per cent. when the light-absorption coefficient has a value of 0.20, provided only that the two walls are so situated that all of the light reflected from the one falls upon the other.

It is well to investigate also the limits of inter-reflection between two oppositely facing walls having different coefficients of absorption. Assume, therefore, conditions to be the same as above, but assign to one wall an absorption coefficient of 0.10 and to the other a coefficient of 0.50, and determine the effect of directing 2000 lumens toward first the one and then the other wall. Proceeding as above, assign arbitrarily a value of 100 units to the total light incident upon the wall having the 10 per cent. coefficient toward which the light from the lamp is first directed. Ninety units are reflected to the second wall, where 45 are absorbed and 45 returned to the first wall. Of the 100 units on the first wall 45 come from the second wall and 55 from the lamps. The 55 units equal 2000 lumens and hence each unit is 36.36 lumens. Therefore, 3636 lumens fall on the first wall, 2000 coming from the lamps and 1636 from the second wall. The second wall receives 3636×0.90 = 3272 lumens and returns 3272×0.50 = 1636. The former wall absorbs 3636×0.10 = 363.6, and the latter 3272×0.50 = 1636, the total light absorbed being 2000 lumens.

Consider now that the light from the lamps falls first on the wall having a coefficient of 0.50, and assign an arbitrary value of 100 units to the total light incident upon this wall. Fifty units are reflected to the other wall where 50×0.90 = 45 are returned to the first wall. Hence, of the 100 units reaching the first wall, 100 − 45 = 55 come from the lamps directly, so that each unit equals 36.36 lumens. The first wall receives 3636 lumens, 2000 of which come from the lamps and 1636 from the second wall, which receives 3636×0.50 = 1818 from the first wall. The first wall absorbs 3636×0.50 = 1818, and the second 1818× 0.10 = 181.8, the total absorption being 2000 lumens.

A striking result is shown by the example just given, namely, that, quite independent of the wall upon which the 2000 lumens from the lamps fall directly, that wall receives an incident flux of 3636 lumens, while the other wall receives a value less than this amount by the light absorbed by that wall upon which the rays from the lamp are directed. With the rays directed toward the wall having a low absorption coefficient the other wall received 3272 lumens, while with the rays in the opposite direction the indirectly lighted wall received 1818 lumens. It should be evident from the above that reflection may play a highly important part in illumination, and that the direction of the light from lamps becomes of less and less importance as the coefficient of absorption decreases. In any event, the absorption-of-light method of calculation affords a valuable insight into the limitations upon the distribution of illumination between inter-reflecting surfaces, and represents a convenient simple method for solving certain of the problems in illumination usually considered complex.

Theoretical Curves for Uniform Illumination.[1]—The discussion thus far on calculations has dwelled on the determinations of illumination intensities due to the luminous source. It is often desirable to know the distribution of light from a luminous source which will give an approximately uniform illumination.

In order to obtain uniform illumination from one lamp recourse is made to the expression $E_h = \frac{I_a}{h^2} \cos^3 a$, which is transposed into the form $I_a = \frac{E_h h^2}{\cos^3 a}$ where E_h is the horizontal illumination in foot candles, a the angle which the luminous rays make with a vertical through the source; I_a the candle-power of the source of light a degrees from the vertical; and h is the height of the lamp above the working plane.

For uniform illumination E_h and h^2 will be constant and I_a at various values of a must vary inversely as $\cos^3 a$. A polar curve showing this relation is given in Fig. 110, which represents the distribution of candle-power intensity of a lamp in a vertical plane which will uniformly illuminate the area beneath it. Obviously, the area thus illuminated by one lamp is limited. In order, therefore, to illuminate larger areas uniformly use must be made of a number of lamps so arranged and with the distribution of light such as to produce the desired effect. Obviously a number of lamps having a distribution of light similar to that shown in Fig. 110 would not suffice since each lamp would uniformly illuminate a circular area as indicated in Fig. 111, and the areas between the circles shown shaded in the figure would receive no light.

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[1] Acknowledgement is made of an excellent article on this subject by Mr. A. A. Wohlauer, *Elect. World*, Vol. 50, p. 1207, Dec. 21, 1907.

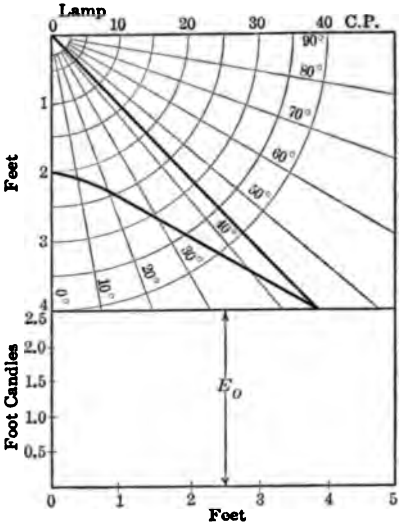


FIG. 110.—Distribution curve for uniform illumination by means of one source.

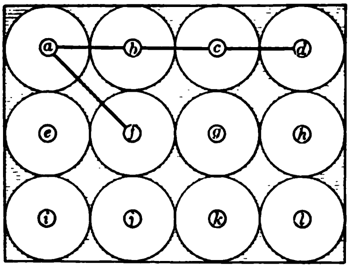


FIG. 111.—Sporadical uniform illumination.

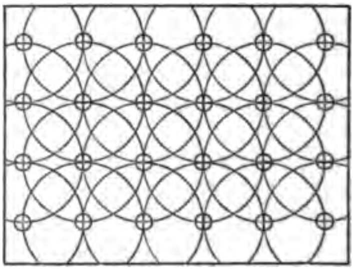
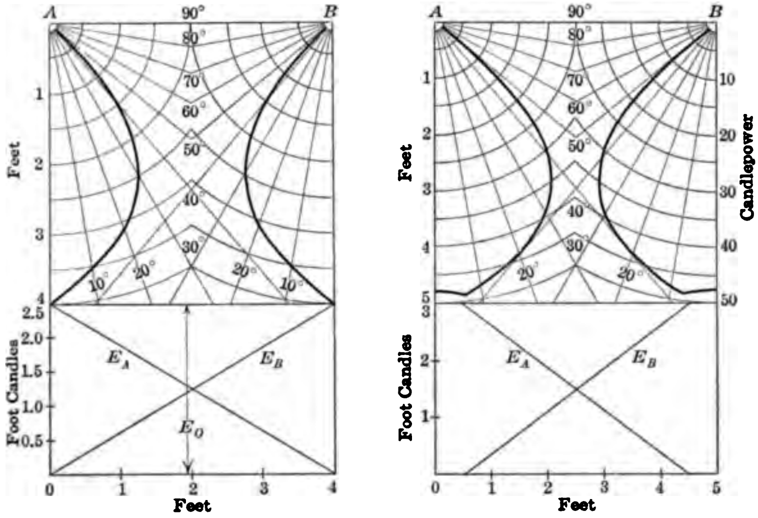


FIG. 112.—Uniform illumination by overlapping straight illumination curves.

It follows, then, that the polar candle-power distribution must be modified and the lamps so placed that the illumination due to lamps near one another will overlap. Such a condition is represented by Fig. 112 where the small circles represent the location of the lamps and the large circles concentric with them represent the areas illuminated by the respective lamps.

It will be readily understood that in order to secure approximately uniform illumination with a number of lamps the illumination due to each lamp must decrease with departure from beneath the source. Polar candle-power curves of different shapes whereby these conditions may be realized are shown in the following illustrations. To simplify matters consideration will be given to only the area beneath and between two lamps



FIGS. 113 and 114.—Polar curves for uniform illumination.

A and B and a study will be made of the distribution of light in a vertical plane through the centers of the lamps which will uniformly illuminate this area. The simplest way of effecting this is indicated in Fig. 113. There the vertical candle-power of each lamp is of sufficient value to give the desired intensity E_0 beneath the source, and of sufficient intensity in other directions that the intensity of illumination decreases in a straight line to zero beneath the other lamp.

If d = distance between lamps,

d' = distance from beneath lamp to the point in question,

h = height of lamp,

E_o = desired illumination,
a = angle of rays with the vertical,
I_a = candle-power a degrees from vertical,

then the intensity of illumination at d' due to the source, may be expressed as follows:

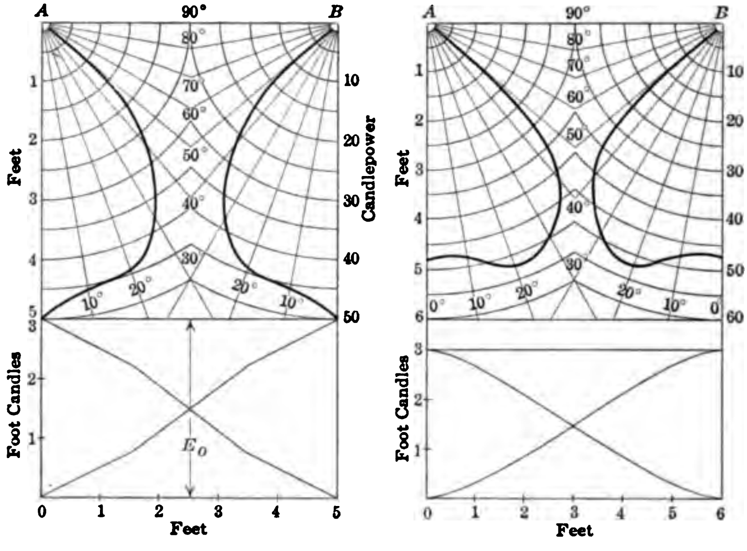
$$\text{since } \frac{E_o}{E_d}=\frac{d}{d-d'}, \text{ and } d=h\tan a, \tag{109}$$

$$E_d=\frac{E_o}{d'}(d'-h\tan a)=\frac{I_a\cos^3 a}{h^2}, \tag{110}$$

therefore, the equation for this curve is

$$I_a=\frac{E_o h^2}{\cos^3 a}\left(\frac{d'-h\tan a}{d'}\right). \tag{111}$$

This is the simplest form of curve for uniform illumination with a number of lamps. It is evident, however, that with four lamps



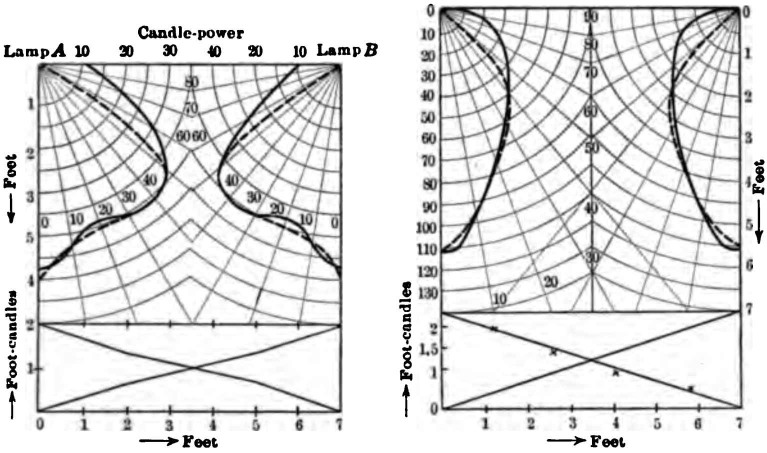
FIGS. 115 and 116.—Polar curves for uniform illumination.

placed at the corners of a square the illumination along the sides of the square will be uniform, but not so at the intersection of the diagonals of the square. The illumination of this point is four times the intensity at a distance equal to $0.5\sqrt{2d^2}$ from a point beneath one lamp, or $1.17E_o$. In this figure it may be seen that the ratio

$$d/h=1. \tag{112}$$

It often happens that it is impractical to locate lamps according to this relation. If the ceilings happen to be low and the illuminants of high candle-power the lamps must be placed further apart.

Under these conditions the polar curve must be made up of a combination of those shown in Figs. 112 and 113, *i.e.*, each lamp must uniformly illuminate a section of the area beneath itself from which the illumination may then assume a constant decline reaching zero value where the uniform illumination due to the next lamp begins. The curves of this nature are shown in Fig. 114. Another form of polar curve for uniform illumination is



FIGS. 117 and 118.—Comparison between theoretical and actual curves.

shown in Fig. 115. The equations of these curves are too complicated for practical purposes.

The general case of polar curves yielding uniform illumination is indicated by Fig. 116. The equation for curves of this nature is

$$I_a = E_o\left(\frac{d-h\tan a}{d} + c\sin\frac{4\pi h\tan a}{d}\right), \qquad (113)$$

where c is a constant which must be determined for each particular case.

We have demonstrated that in order to illuminate uniformly large horizontal planes a number of lamps must be employed and their light so distributed that the lamps in their combination produce uniform illumination. The different possibilities have been theoretically discussed and a conception has been formed as to the shape of the polar curves.

It now will be interesting and useful to observe to what extent these theoretical requirements are fulfilled in practice. To this end a number of tests were made by the Electrical Testing Laboratories under Mr. Wohlauer's personal direction.[1]

A few of the results of these tests are reproduced in Figs. 117, 118, and 119. These results represented by curves are supplemented by curves obtained from theoretical considerations. The experimental curves are in full line, while the theoretical curves are dotted.

The comparison reveals that it is reasonable to expect that uniform illumination of horizontal planes can be realized in practice, in accordance with the laws derived above. The deviations are slight, the greatest occurring at the upper end, which is due to the fact that the illumination curve of one lamp passes through the zero mark where the maximum illumination of the next lamp sets in. This necessitates that the polar curve must return to zero also. In other words, no light should be emitted in the horizontal direction, or in the case of incandescent lamps, reflectors must be used so designed that all the light is utilized below the lamp in accordance with the theoretical requirements discussed above.

It is interesting to note further that the distance, *d*, between two lamps can be made the same in all three cases which have been selected in order to compare theory with practice. It shows that the same number of lamps are used to illuminate the same area, the variation being only in the height of the suspension and in the intensity of illumination. Or, in other words, it is possible in practice also, to vary the intensity of uniform illumination by simply changing the reflectors and the height of suspension. It verifies also the recognized fact that the higher the lamp is suspended above the surface, the less the illumination obtained thereon; concentrating reflectors are required for a higher suspension, while for low-ceiling rooms more distributing reflectors are necessary.

For high suspension—high being a relative term with regard to the candle-power of the lamp — and for concentrating reflectors, the illumination curve is always a straight line, while for rooms with low ceilings one often is compelled to introduce the more complicated illumination curves if small lamp units are not available or too expensive in their total cost.

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[1] *Elect. World*, Vol. 51, p. 1376, June 27, 1908.

Otherwise the tests show that it is possible to meet the theoretical requirements with the existing sources of light and equipment fairly well. Of course, the reflector will play the most important part in realizing the theoretical requirements.

It will be interesting to call attention to the uniform illumination of very large areas, such as streets, public squares, etc., by arc lamps especially adapted for that purpose. The enclosed arc lamps, for instance, curves of which are shown in Fig. 119, are not very far from realizing the theory in an ideal way. Fig. 119 shows that with the lamps suspended 20 ft. from the ground and placed 64 ft. apart, an almost ideal uniform illumination will be obtained at a height of 4 ft. above the ground.

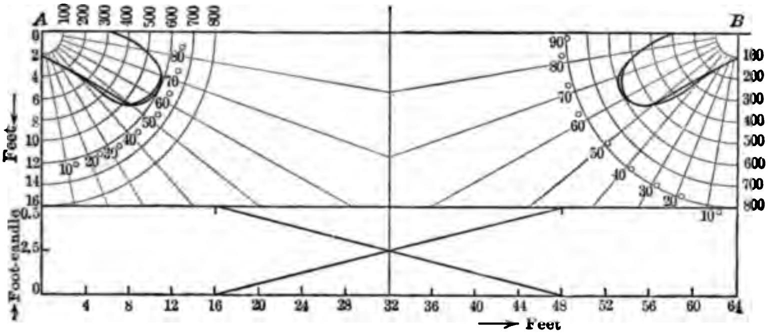

FIG. 119.—Uniform illumination by two enclosed arc lamps, d/h = 4.

CHAPTER X

PHOTOMETRICAL AND ILLUMINATION CALCULATIONS—SURFACE SOURCES[1]

In the previous chapter we considered the light flux from a source assumed to be a point and defined illumination and intensity in terms of the flux and candle-power, respectively. In this chapter we will assume the light distributed over a luminous area of brightness b and the total quantity (Q) distributed over the area equal to

$$Q = \int b dS \tag{114}$$

with a luminous flux filling the surrounding space and producing an illumination (E) on a surface in that space equal to the flux per unit of area, or

$$E = F/S \tag{115}$$

Unit Disk.[2]—(Rosa Ref. Cit.) Concerning a body charged with electricity, we have the two ideas: (1) The electricity of density σ and total quantity $Q = \int \sigma dS$ on the surface of the charged body, and (2) the flux of force throughout the surrounding space, there being $4\pi Q$ lines of force for a quantity Q of electricity. We do not believe in the fluid theory of electricity in the same way that Franklin did, but we nevertheless find the idea of a surface density of electricity very useful. In the corresponding case with light we may have similarly two distinct ideas:

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[1] This chapter consists essentially of the papers, with slight modifications by the author, of Dr. E. B. Rosa, *Bull. Bur. Stand.* Vol. 6, p. 543, 1910, in which he acknowledges the writings of Blondel, Palaz, Liebenthal, Hering, Kennelly, Sharp, Hyde and Jones. Dr. A. G. McAllister, *Elect. World*, Vol. 56, p. 1356, 1910. Mr. M. D. Cooper, *Elect. World*, Vol. 54, p. 718, 1909.

[2] By unit disk and unit sphere is meant a disk or sphere whose linear dimensions are negligible compared with the distance from source to receiver.

(1) A surface distribution of light over a luminous area of *brightness* or *specific quantity* b, and total quantity $Q=\int bdS$, and (2) a luminous flux filling the surrounding space and producing an illumination E on any body equal to the flux per unit of area,

$$\text{or } E=\frac{F}{S}$$

We have so far defined illumination and intensity in terms of the flux. Let us now obtain their values in terms of the *quantity of light on the surface of the luminous source.*

The *illumination from a very small source* is inversely proportional to the square of the distance from the source and directly proportional to the brightness of the source. Hence for a luminous plane of area dS (Fig. 120) we may write

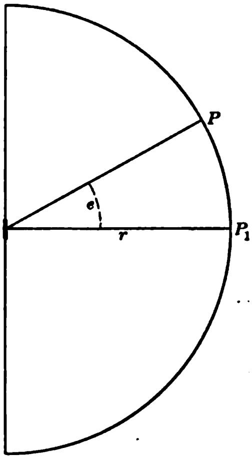

FIG. 120.—A small surface source.

$$E_{P_1}=\frac{Q}{r^2}=\frac{bdS}{r^2} \tag{116}$$

where Q is the total quantity of light on the disk, and the radiation to P_1 at a distance r is normal. For a point P at an angle e from the normal the illumination would be

$$E_P=\frac{bdS\cos e}{r^2}=\frac{Q\cos e}{r^2} \tag{117}$$

The *total flux over the hemisphere illuminated by the disk* is

$$\mathrm{F}=\int_0^{\frac{\pi}{2}} E_P 2\pi r^2 \sin e\,de=Q\int\frac{2\pi r^2 \sin e\cos e\,de}{r^2}$$

$$=\Big[\pi Q\sin^2 e\Big]_0^{\frac{\pi}{2}}=\pi Q \tag{118}$$

Thus the total luminous flux F from a small plane disk is π times the quantity of light Q on the disk.[1]

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[1] In electrostatics the total flux is 4π times the quantity Q. The difference is due, first, to the fact that the luminous disk is supposed luminous on one side only and hence there is radiation only on one side, whereas the electric flux would be on both sides; secondly, the cosine law makes the average flux only half what it would be if the factor $\cos e$ were omitted.

The *average illumination over the hemisphere* of radius r is

$$\frac{F}{2\pi r^2}=\frac{Q}{2r^2} \tag{119}$$

whereas the maximum illumination E_n normal to the disk is $\frac{Q}{r^2}$. Thus the mean is half the maximum. The intensity I has been defined as the *angular rate of flux* in any particular direction. It is, therefore, proportional to the illumination produced in the given direction. Thus in the case of the luminous disk we have

$$I_n = \text{maximum intensity, (normal)} = Q \tag{120}$$

$$I_{hs} = \text{mean hemispherical intensity} = \frac{Q}{2} \tag{121}$$

$$I_s = \text{mean spherical intensity} = \frac{Q}{4} \tag{122}$$

$$\text{Thus } F=\pi I_n = 4\pi I_s \tag{123}$$

That is, the intensity is numerically equal to the total quantity of light on the small disk *for all points on the normal.* It *decreases to zero* as we pass 90° away from the normal, having a mean value of half the maximum for the whole hemisphere, and is on the average only one-fourth the maximum for the whole sphere. We may, therefore, say that the *hemispherical reduction factor* for the disk is one-half, and the mean *spherical reduction factor* is one-fourth, the disk being supposed luminous on one side only.

Since the total flux F from an area is πQ, where Q is the quantity of light on the area, the flux from a unit of area is πb. This is the radiation E'. Hence, in general

$$E' = \pi b \tag{124}$$

For a small sphere of radius a, the total flux is

$$\begin{aligned} F &= E' \times \text{surface} \\ &= \pi b \times 4\pi a^2 = \pi Q \end{aligned} \tag{125}$$

$$\text{also, } F = 4\pi I$$

$$\therefore I = \frac{Q}{4} \tag{126}$$

That is, for a unit sphere the intensity is one-fourth the quantity of light on the sphere. If the distribution of light over the sphere is not uniform, the *mean spherical intensity* is still one-fourth the total quantity of light on the sphere, as it is also for a disk. In other words, a sphere produces the same illumination at a given point as a disk of the same diameter and same brightness placed so that the radiation from the disk to the point is normal.

Extended Source.—(Rosa, Ref. Cit.) If we let dS be an element of a plane radiating surface of brightness b, defined by the equation

$$Q = bS \tag{127}$$

that is, the quantity of light Q is equal to the product of b into the surface S; b is the value of the quantity Q when the surface is unity, and is the quantity of light per unit of area measured in candles, then the intensity I of such a source (or of any source) would be measured by comparing it experimentally with a standard light source, and it is equal to the intensity of a point source or unit sphere which produces the same illumination on a given test screen(of a photometer). Thus, while we *define* the intensity of a light source as the luminous flux per unit solid angle, we *determine* it by comparison with a concrete standard by means of the illumination produced on a test screen at a convenient distance, using a photometer and employing the law of inverse squares.

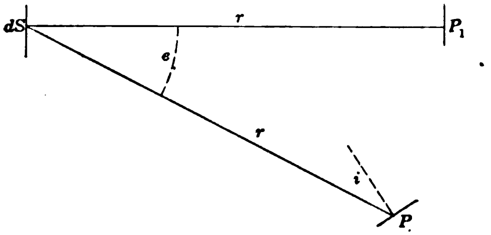

FIG. 121.—A small circular source.

In Fig. 121 the illumination at P_1 in the normal to dS is

$$E_1 = \frac{bdS}{r^2} \tag{128}$$

while the illumination at P, the angles of emergence and incidence being e and i respectively is

$$E_2 = \frac{bdS \cos e \cos i}{r^2} \tag{129}$$

The cosine law is assumed to hold exactly for both surfaces.

To calculate the illumination due to a large circular disk of brightness b and any radius a on a small plane area P_1, normal to the axis of the disk and situated on the axis at distance r from the disk (see Fig. 122) we integrate the effect of each elementary circular ring of the disk. Thus, in equation (129), putting $dS = 2\pi x dx$,

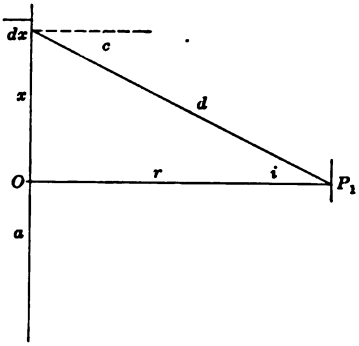

FIG. 122.—A large circular source.

$$E = b\int_0^a \frac{2\pi x dx \cos e \cos i}{r^2 + x^2} \tag{130}$$

$$\text{Since } \cos e = \cos i = \frac{r}{\sqrt{r^2 + x^2}} \tag{131}$$

$$E = \pi b \int_0^a \frac{2\, x dx r^2}{(r^2 + x^2)^2}$$

$$= \pi b \left[\frac{-r^2}{r^2 + x^2}\right]_0^a = \pi b\left[1 - \frac{r^2}{r^2 + a^2}\right] \tag{132}$$

$$\text{or}\quad E = \frac{\pi b a^2}{r^2 + a^2} = \frac{bS}{r^2 + a^2} = \frac{Q}{r^2 + a^2} \tag{133}$$

where Q is the product of the surface of the disk into the brightness b, and is the total quantity of light upon the disk measured in candles. If the disk were very small Q would be the same as the maximum intensity I_n of the source; but for an extended source we must distinguish between the equivalent *intensity* I_o and the surface integral of the *brightness* b, which is Q. The latter we have called the quantity of light upon the disk; it is proportional to the total luminous flux F coming from the extended source, and is equal to F/π, equation (118). Q and F really measure the same thing, except that Q is located on the source and is measured in candles, while F is located in the surrounding space and is measured in lumens; their ratio is constant as $F=\pi Q$ always.[1]

In the case of the disk above mentioned, the illumination E on a small plane normal to the axis is proportional to the total quantity of light Q on the extended source (the circular disk) and inversely proportional to the square of the distance d from P_1 to the *edge of disk.* This holds true for all distances r from zero to infinity. Thus the *law of inverse squares holds generally* for the illumination along its axis due to a circular disk of any size, but the distance is measured, not to the center of the disk, but to the edge.

Thus we have

$$E=\frac{I}{r^2} \text{ for a point source or a unit disk,} \qquad (134)$$

and

$$E=\frac{Q}{d^2} \text{ for an extended disk.} \qquad (135)$$

To illustrate the rate of variation of the illumination with the distance, let $a=I$, $r_1=1$, $r_2=5$ (Fig. 123).

In the first case for the point P_1, $E_1=\frac{Q}{d_1^2}=\frac{\pi b}{2}$

In the second case for the point P_2, $E_2=\frac{Q}{d_2^2}=\frac{\pi b}{26}$

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[1] The total quantity of electricity on a disk of area S is equal to the integral of the surface density σ over the area.

$$Q=\int \sigma dS$$
$$=\sigma S \text{ when } \sigma \text{ is uniform.}$$

The brightness b of a source corresponds to the surface density of electricity σ, and the total quantity of light over a surface is, in the same way, the surface integral of b. Thus

$$Q=\int bdS$$
$$=bS \text{ when } b \text{ is uniform over the area } S.$$

In the case of a sphere, the surface $S=4\pi a^2$. Therefore, for a spherical source $Q=4\pi a^2b$, whereas the intensity $I=\pi a^2b$. That is, the intensity I of a spherical source is one-fourth of Q, and is equal to the light on a disk of radius a and brightness b. That is, the intensity of the sphere is equivalent to that of a disk of the same diameter and the same brightness for points at a great distance.

Thus in the first case the distance is five times less and the illumination is 13 times more instead of 25 times more as it would be if the light Q were all concentrated at the center of the disk. If $r=o$, the illumination is πb or twice as much as at P_1, and not infinite as it would be at zero distance from a point source.

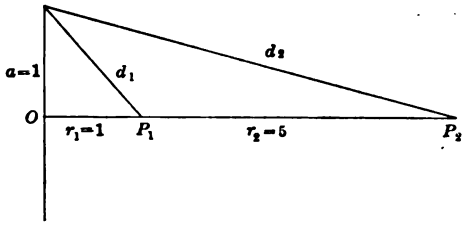

FIG. 123.—A large circular source.

This theorem is useful in measuring the radiation from walls, as the radiating area may be quite large and the photometer relatively near.

Infinite Plane.—(Rosa, Ref. Cit.) The *radiation from an infinite plane* S (Fig. 124) upon a unit area of a parallel plane T is found by integrating equation (132) to infinity. Thus

$$E=\pi b\int_{o}^{\infty}\frac{2xdx\cdot r^2}{(r^2+x^2)^2}=\pi b\left[\frac{-r^2}{r^2+x^2}\right]_{o}^{\infty}=\pi b \qquad (136)$$

Thus the flux density or *illumination* at any point P on the T plane is π times the brightness b on the radiating plane S and is independent of the distance r.

From each unit of area of S having a brightness b the total flux is πb, as shown above. The resultant flux at all points is the same as though the total flux πb from each unit of area of S was confined to a cylindrical tube of unit area perpendicular to S, in which case the flux density would, of course, be constant at all sections—that is, at all distances (see Fig. 125).

There is presented below a graphical solution of this problem for the case of a flat circular source which possesses the maximum of simplicity.[1]

Although the solution of the problem in any practical case involves the use of only straight lines, circles and the lower branches of mathematics, it is not claimed that the proof of the accuracy of the method outlined can be given in such simple terms. However, even the proof is not complicated. It seems well to give first the simplest possible solution and subsequently to outline the proof of the accuracy.

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[1] Dr. A. S. McAllister, *Elect. World*, Vol. 56, 1356, 1910.

Equilux Spheres.—(McAllister, Ref. Cit.) In Fig. 126 let *ACB* represent an edgewise view of a flat circular surface lighting source, assumed to be in the ceiling of a room, having any given value of uniform surface illuminating density and any desired radius. If through the edges *A* and *B* there be passed an imaginary sphere of any chosen size—such as *ADB*—with its center at some point on a vertical line passing through the center *C*, the inner surface of this imaginary sphere below the lighting source will receive an illumination which will be uniform in intensity normal to the surface throughout the whole interior of the imaginary sphere. Evidently the density of the normal illumination against the inner surface of the sphere will bear to the illumination of the circular surface source the inverse ratio of the interior area of the exposed zone of the sphere to the area of the lighting source, since the lumens produced must equal those utilized. This ratio equals the square of the ratio of the radius *AC* to the diagonal *AD;* when the radius of the circular source is taken as the unit of length for the measurement of all distances,

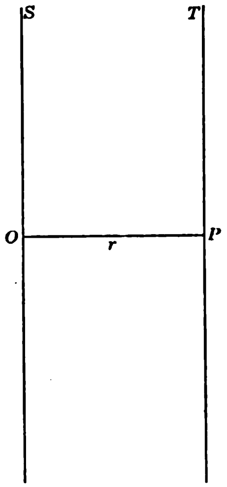


FIG. 124.—Radiation from an infinite on a unit area.

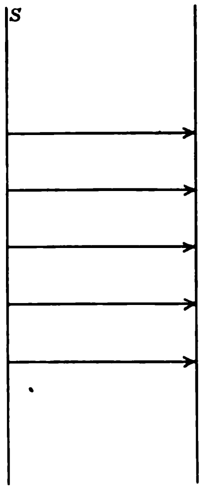


FIG. 125.—Radiation from an infinite plane on an infinite area.

and the unit of illumination is taken as the surface brilliancy of the source, then the percentage value of the illumination on the interior of the sphere is equal to 100, divided by the square of the diagonal *AD*.

For convenience any sphere passing through the edges *A* and *B*, as just indicated, can be referred to as an "equilux" sphere. Equilux spheres of the proper sizes being employed, one is enabled "to explore the whole region" illuminated by the source and to ascertain immediately for any desired point within the space explored the exact value of that component of the light flux which is normal to the particular equilux sphere passing through that point.

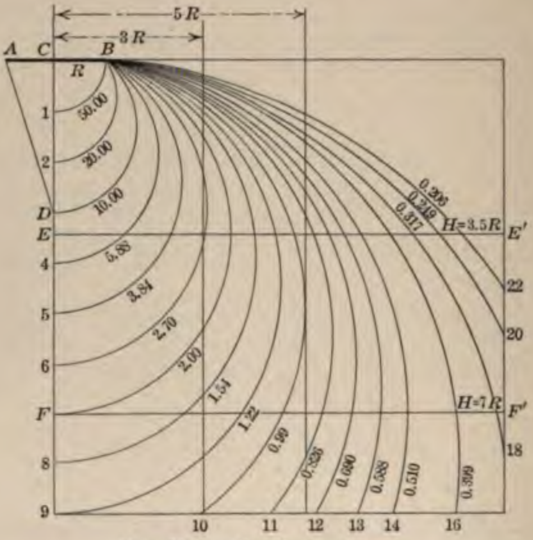
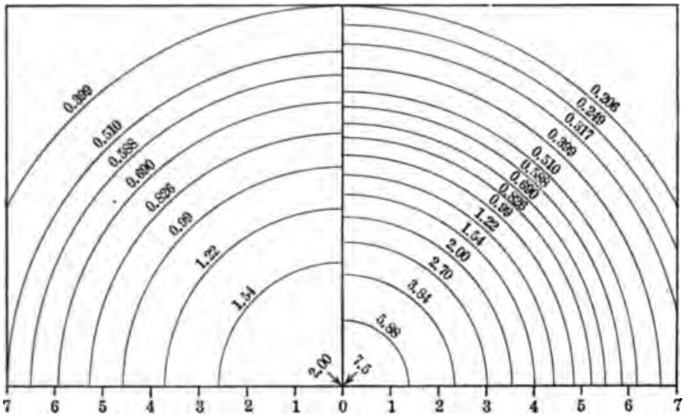


FIG. 126.—Equilux spheres.

In Fig. 126 are indicated numerous equilux spheres and the points of intersection of these spheres with horizontal planes (floors) at distances of 3.5 units and 7 units of length (radii) below the light source, and with vertical planes 3 and 5 length-units distant from the center of the source.

Illumination on Horizontal Planes.—(McAllister, Ref. Cit.) Intersections of the two assumed horizontal planes (floors) with the equilux spheres are shown in Figs. 127 and 128; evidently the points of equal illumination lie on circles having as the common center the point on the floor immediately below the center of the circular ceiling lighting source. It is an interesting fact that at any point on the floor the component of the flux normal to the floor is equal to the component normal to the equilux sphere at that point, so that the values of equilux density are simultaneously the values of light flux density normal to the horizontal plane (floor). Expressed in other words, the illumination along the floor at any point is known at once when one has determined the value of light flux density on the equilux sphere passing through that point; the whole problem of floor illumination density and distribution is completely solved when the equilux spheres in Fig. 126 and intersections in Figs. 127 and 128 have



FIGS. 127 and 128.—Floor illumination for two ceiling heights.

been constructed. One could not well wish for a simpler solution. It is especially worthy of note that the square of any diagonal such as *AD*—which is used in calculating the value of the illumination density on each equilux sphere—is in each case equal to the constant quantity 1 plus the square of the height *CD*, so that by using whole numbers for assumed height values all of the numerical calculations are reduced to the simple process of mental arithmetic; as noted above, the graphical constructions involve the use of merely straight lines and circles.

Example of Floor Illumination.—(McAllister, Ref. Cit.) In Fig. 129 are shown results obtained directly from Figs. 127 and 128. It will be noted that with a ceiling height equal to 3.5 times the radius of the lighting source the light flux density at a point on the floor immediately below the center of the source reaches a value of about 7.55 per cent. of that of the source, while the density along the floor decreases rapidly with increase in the distance from the point of maximum density. With a ceiling twice as high as formerly the maximum light flux density is reduced to 2 per cent., but the rate of decrease with increase of distance from the point below the center of the source is much less; in fact, at distances greater than 5 units of length the illumination from the high source is greater than from the low source. This fact will be appreciated when it is recalled that the solid angle subtended by the source when viewed from the floor at a great distance from the center is larger with the high ceiling than with the low ceiling.

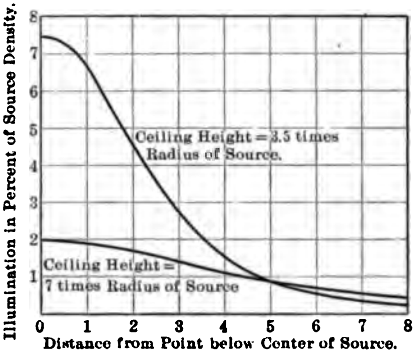
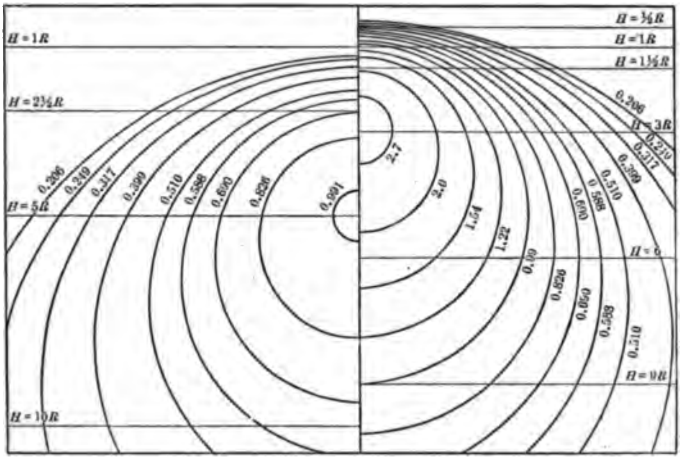


FIG. 129.—Illumination on floor with two ceiling heights.

Illumination on Vertical Planes.—(McAllister, Ref. Cit.) In Figs. 130 and 131 are shown intersections of the equilux spheres with vertical planes (side walls) at distances of 3 units and 5 units (radii) from the center of the lighting source. The points of equal illumination—normal to the equilux spheres—lie on circles which are not concentric, but which have their centers on a straight vertical line. The method of locating these circles will be appreciated immediately from a brief study of Fig. 126. Figs. 130 and 131 show directly the value of the equilux illumination at each point indicated but they do not give the value of the flux density normal to the vertical plane at that point. However, this value can be readily calculated from each illustration by means of mental arithmetic. The process of calculation involves merely the multiplication of the equilux value at the point considered by the (constant) normal horizontal distance of the vertical plane from the center of the source and the division of this constant quantity by the vertical distance of the point below the ceiling.

At all points along a side wall at a distance below the ceiling equal to the normal distance of this wall from the center of the lighting source the vertical component of the flux density is equal to the equilux sphere value, being equal obviously to the horizontal component at these points; at points nearer the ceiling the illumination on the vertical plane is correspondingly greater than the equilux value at each point, while at lower points it is correspondingly less. For example, with a distance of 3 radii between the wall and the source (Fig. 131) the normal illumination on the wall is equal to the equilux values at a distance of 3 radii below the ceiling; at 1 radius it is three times as great as the equilux value at each point; at 1.5 radii it is twice as great as the equilux value at each point; at 6 radii it is one-half as great; at 9 radii it is one-third as great, etc. From these relations one can easily determine the flux density values over the whole wall area by means of equilux circles such as are shown on Figs. 130 and 131.



FIGS. 130 and 131.—Wall illumination.

Theory of the Equilux Method.—(McAllister, Ref. Cit.) The method outlined above is based primarily on the "solid angle" theory of illumination. In other words, it is assumed that when a surface source is viewed from two points so located that the "solid angle" subtended by the source in one case is equal to that in the other the light flux densities at the two points are equal; when the two angles are unequal the flux densities differ in the exact ratio of the angles. Referring now to Fig. 132, let AB represent a surface source having a plane circular edge through which passes the imaginary sphere $PP'P''$. Evidently the "solid angle" subtended by the source when viewed from any point within this sphere would be the same whether the source be a plane or the internal zonal section of an imaginary sphere. The source would be seen as a circular plane when viewed from any point along a line passing through the center of the source and perpendicular to a plane passing through the edge of the source; when viewed from any point not on this line it may be assumed that the source would appear as an ellipse—the error involved in this assumption is of no importance so far as practical results are concerned. It is with the two axes of this ellipse that the present problem deals. In Fig. 132 one of the axes of the ellipse is the straight line AB, the other axis being perpendicular to this line. At a point, such at P', below the center of the light source the separate plane angles subtended by these two axes are individually equal to the angle a; the "solid angle" is proportional directly to the product of these two separate plane angles, so that the relative value of the solid angle is known when the values of the two plane angles are known.

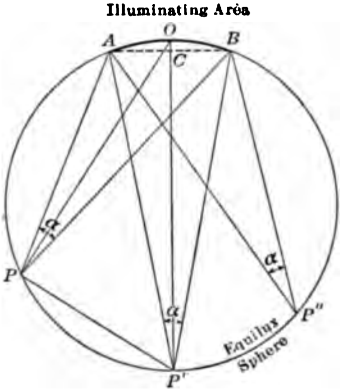


FIG. 132.—Theory of equilux sphere.

At any point, such as P, on a circle passing through the points A, B, and P' the plane angle APB subtended by the axis AB is equal to the plane angle $AP'B = a$, and hence is constant without respect to the position of P along the circle $APP'P''B$. The plane angle subtended by the axis of the ellipse at right angles to the axis AB is greater for the point P than for the point P' because the distance from which its constant length $(= AB)$ is viewed is less at P than at P'. The ratio of the two distances indicated, PO/OP', is the cosine of the angle POP'. Since the solid angle subtended by the lighting source when viewed at point P is proportional to the product of the two plane angles subtended by the axes of the source it follows that the solid angle at P is greater than the solid angle at P' in the ratio of unity to the cosine of angle POP'. Now the flux density normal to the sphere at P bears to the maximum flux density along the line PO the ratio of the cosine of angle POP' to unity. Hence the normal flux density at P is equal to that at P'; or the normal flux density is uniform over the whole inner surface of the sphere below the lighting source. The slight errors involved in the simplifying assumption used above are not cumulative; in fact, they cancel in the final results. The only error that can rightly be so designated follows from assigning a definite "direction" to the flux which is emitted by an extended surface source. Even this error is zero in so far as the "cosine" law is applicable. It would seem, therefore, that the equality of normal flux density along the inner surface of the imaginary equilux spheres can be treated as sufficiently well established without resort to the complications that a more complete demonstration would involve.

Value of the Equilux Illumination.—(McAllister, Ref. Cit.) In determining the numerical value of the light flux density along each equilux sphere resort is had to the law of conservation. That is to say, it is assumed merely that all of the flux emitted by the source is utilized in illuminating the interior of the imaginary sphere below the source. Since the flux is constant over the lighting source and that over the equilux sphere is also constant, it follows at once that the two densities bear to each other the inverse ratio of the two areas. It remains merely to determine the ratio of the areas. Referring to Fig. 132 the superficial area of the zone of the sphere below the plane ACB bears to the whole area of the sphere the ratio of CP' to OP'. The whole area is $\pi(OP')^2$, and hence the lower zonal area is $\pi(OP' \times$

CP'). Now from similar triangles $OP'/AP' = AP'/CP'$ so that $OP' \times CP' = (AP')^2$, therefore the lower zonal area is equal to $\pi(AP')^2$. The area of the plane circular source is $\pi(AC)^2$, so that the ratio of the two areas is $(AC)^2/(AP')^2$. When the length AC is used as the unit of length the ratio of the areas becomes $1/(AP')^2$ as has been used above. It will be noted that this ratio is absolutely accurate. When use is made of this ratio in connection with the law of conservation relative to the production and utilization of flux, the apparent inaccuracies in the simplified proofs of the uniformity of illumination disappear.

Illumination on Horizontal Planes.—(McAllister, Ref. Cit.) In the solution of the practical problems given above it was assumed that at any point below the lighting source the component of the flux (and hence the flux density) normal to a horizontal plane is equal to the equilux sphere value at that point. Since such an assumption involves the conception of definite direction of flux coming from an extended surface source it is open to the objection noted above under "uniformity." However, in so far as any direction can be assigned to the flux from such a source the method here outlined is as accurate as any that can be employed. In Fig. 133 the line showing the mean direction of the flux reaching the point P is indicated by the line OP; this line bisects the angle QPN and hence the component of the flux normal to QP is equal to the component normal to NP. The former component produces the equilux illumination at the point P, while the latter produces the floor illumination at the same point;

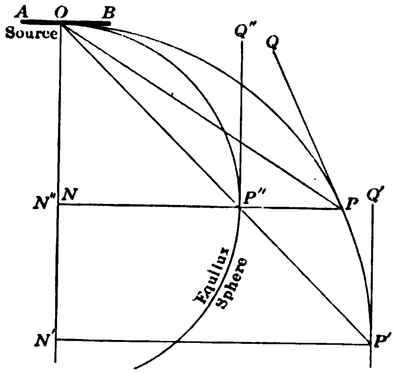


FIG. 133.—Proof of equality of equilux sphere and floor illuminations.

hence the floor illumination at any point such as P, P' or P'' is equal to the equilux illumination at this same point. The inaccuracies involved in the simplified construction used in Fig. 133 are of no practical importance.

Illumination on Vertical Planes.—(McAllister, Ref. Cit.). Having determined the horizontal or vertical component of a light flux at any one point and knowing the horizontal and vertical distances from the source to that point one can easily calculate the vertical or horizontal component by the ratio of these two distances. The component on the vertical plane thus found would be the value which would exist normal to a vertical plane passing through the point and perpendicular to a line joining the point and the source. On any other vertical plane passing through the same point the normal component at that point would be less in the ratio to unity of the cosine of the angle of deviation between the two planes. Since the constant horizontal distance from the source to any plane is in each case equal to the product of the cosine just noted and the horizontal distance to any chosen point on any plane, it follows that at any point whatsoever the flux density normal to any vertical plane bears to the equilux illumination at that point the inverse ratio of the vertical distance of that point from the ceiling to the horizontal perpendicular (shortest) distance between the vertical plane and the center of the ceiling lighting source.

Infinite Cylinder.—(Rosa, Ref. Cit.) In a manner similar to our treatment of infinite surface sources, we may consider the flux from an infinite circular cylinder of uniform brightness b and radius a.

The flux coming from unit length of the cylinder is πb times the area. Hence $F = 2\pi^2 ab$ whereas the flux falling on the inner surface of a concentric cylinder of radius r is E times the area, E being the illumination. Hence for a unit of length of the cylinder $F = 2\pi r E$. Therefore,

$$E = \frac{\pi ab}{r} = \frac{1}{2}\frac{Q}{r} \qquad (137)$$

Thus the illumination due to an infinite cylinder varies *inversely as the distance*. This is intermediate between the case of the point source, for which E varies inversely as r^2, and the infinite plane, where E is independent of the distance—that is, proportional to r_0.

The quantity Q for the luminous cylinder is b times the surface. Therefore the *quantity per unit of length* is

$$Q_1 = 2\pi ab \tag{138}$$

The total luminous flux F, as stated above, is $2\pi^2 ab$. Hence the total flux per unit of length F_1 is π times the quantity, or

$$F_1 = \pi \, Q_1 \tag{139}$$

or, for any portion (or the whole) of an infinite cylinder of uniform brightness, the total flux is π times the quantity; that is,

$$F = \pi Q \tag{140}$$

as shown above for a circular disk.

Unit Length of Cylinder.—(Rosa, Ref. Cit.) Suppose a light source in the form of a very long cylinder of radius a and uniform brightness b. It is desired to determine experimentally its total luminous flux F. Suppose one has measured by means of a photometer the equivalent intensity I_1 of unit length of the cylinder (screening the photometer from all but a short section of the cylinder). We are to calculate the total flux F_1 from I_1. The unit length of cylinder will produce the same illumination at a distance as a rectangular plane of breadth $2a$ and height unity of brightness b equal to that of the surface of the cylinder. Hence the equivalent intensity I_1 is equal to $2ab$ and the illumination produced on a photometer screen at distance r is

$$E = \frac{2ab}{r^2} = \frac{I_1}{r^2} \tag{141}$$

The quantity of light on the cylinder per unit of length is b times the surface, or $2\pi ab$, and the total flux F_1 is π times the quantity. Thus we have

$$F_1 = 2\pi^2 ab \tag{142}$$

$$I_1 = 2ab \tag{143}$$

$$F_1 = \pi^2 I_1 \tag{144}$$

Thus, to obtain the total luminous flux F_1 from the measured value of the equivalent intensity of a unit of length of the luminous cylinder we multiply this intensity I_1 by π^2 instead of multiplying by 4π, as in the case of a sphere.

The spherical reduction factor of a short cylinder (the convex surface only being luminous) is therefore $\pi^2/4\pi = \pi/4 = 0.785$.

This would be nearly true for an incandescent lamp having one or more straight filaments. The value for a hairpin filament would be only slightly larger. Tantalum and tungsten lamps have reduction factors of about this value.

If the cylinder is long, we should then get the total flux F by multiplying F_1 by the length of the cylinder. This demonstration is of course based on the assumption that the cosine law holds for the cylinder. If the source is a long tube, like the Moore light, the result would be subject to any modification dependent on its departure from the cosine law.

Thus while the total flux F is always π times the quantity Q of the source, it is not always 4π times the intensity. It is 4π times the intensity I for a point source or sphere, π^2L times the equivalent intensity I_1 (measured at a relatively great distance) of unit length of a long cylinder, L being the length, and πS times the equivalent intensity I_1 of unit of area of a plane, S being the area of the plane.

It is, however, always 4π times the *mean spherical intensity* of the given source. The illumination produced by a short cylinder is approximately inversely proportional to the square of the distance. For all distances greater than five times the length, the departures are not greater than 0.2 per cent. in a particular case worked out by Hyde; the diameter of the cylinder in this case was one-tenth the length. The exact expression for the illumination due to a finite cylinder is not simple as will be seen by the following deduction.[1]

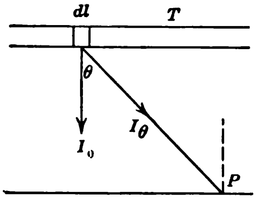


Fig. 134.—Section of luminous tube.

In Fig. 134 let T represent a section of a luminous tube and P be a point at which it is desired to find the illumination. If the candle-power of the element dl in the direction $dl-P$ is known it will be a simple matter to write an expression for the illumination produced at P by the element dl. Integration over the length of the tube would then give the total illumination at the point P.

Theoretical Considerations.—At first glance it would appear that the cosine law could be applied to the cylindrical element, or that $I_\theta = I_0 \cos \theta$, I_θ being the candle-power in the direction $dl-P$, and I_0 the normal candle-power of the cylindrical element.

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[1] M. D. Cooper, *Elect. World*, Vol. 54, p. 718, Sept. 23, 1909.

The assumption of the cosine law would be warranted if the tube could be considered equivalent to a flat radiating strip of a width equal in diameter of the tube, but the curvature of the tube surface tends to make such an assumption inaccurate.

If, however, the tube is considered as a solid radiating cylinder, the relations are more involved. Inspection of Fig. 135 will show what results can be anticipated in this case. To calculate the normal candle-power of an element dl at B, a summation would be made of the luminous intensities in the direction of P of elementary areas of the strip dl. As the summation would be carried over only the arc HBC, the computed normal candle-power, I_o, would be less than when considering the tube equivalent to a flat strip. For an element dl at F, I_θ will be less than $I_o \cos \theta$, because at all points other than F, θ_1 will be greater than θ. This consideration leads to the conclusion that I_θ, Fig. 134, will be somewhat less than $I_o \cos \theta$.

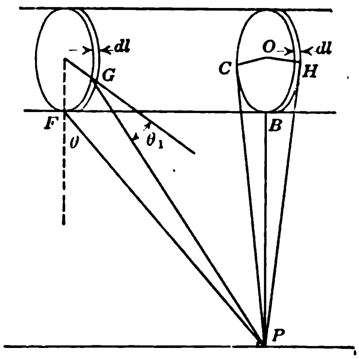

Fig. 135.—Sections of tube illuminating point P.

The tube is not, however, a solid radiating body, but gives light by the luminosity of a gas, hence consideration on this latter basis will lead to more correct conclusions. Experiment proves that an open gas flame gives nearly the same candle-power in all horizontal directions, indicating that the luminous intensity of a radiating gas depends primarily on the number of luminous particles contributing light, and only in a very secondary degree upon the alignment of these particles. This fact would seem to indicate that the intermolecular spaces are so great compared to the size of the molecules that there is practically no interference with free radiation from each and every molecule.

This line of reasoning affords another method of ascertaining I_θ. The candle-power along any line can be assumed to be proportional to the length of this line inside the tube. Refraction need not be considered, as the rays of light will suffer merely a slight lateral displacement and will not be changed in direction on passing the walls of the tube. With these facts in mind, inspection of Fig. 135 will now show that I_θ will be at least equal to $I_0 \cos \theta$, if not actually greater.

Assuming, as a fair approximation, that the cosine law can be applied, the integration for the total illumination at P, Fig. 134, will be performed.

Application of Cosine Law.—Let Fig. 136 represent a length of tube and P be the point at which it is desired to determine the illumination. Furthermore, let

I = total candle-power of tube, interpreted physically as the candle-power measured at an infinite distance.

L = length of tube, in feet.

h = height above reference plane, in feet.

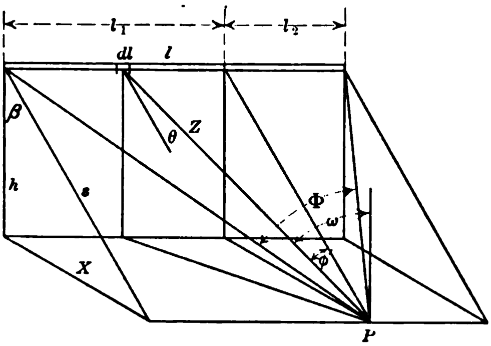

FIG. 136.—Space diagram of positions.

s = slant distance from P to tube, in feet.

l = distance (parallel to tube) from P to element dl.

ω = angle between ray P-dl and normal to reference plane.

θ = angle between ray P-dl and normal to tube.

ϕ = angle subtended at P by the tube.

β = angle between s and h.

Assuming the cosine relation, $I_\theta = I_0 \cos \theta$, to be correct

$$I_0 = \frac{I}{L}\,dl \tag{145}$$

$$I_\theta = \frac{I}{L}\,dl \cos \theta \tag{146}$$

The element dl of the tube will contribute to the illumination at P an element

$$dE=\left(\frac{I}{L}dl\cos\theta\right)\left(\frac{1}{z^2}\right)(\cos\omega)$$
$$=\frac{I}{L}dl\frac{s}{z}\frac{1}{z^2}\frac{h}{z}$$
$$=\frac{I}{L}\frac{sh}{z^4}dl \tag{147}$$

Noting that $z=\sqrt{s^2+l^2}$, the illumination at P due to the part l_1 of the tube becomes

$$E_1=\frac{Ish}{L}\int_0^h\frac{dl}{(s^2+l^2)^2}$$
$$=\frac{Ish}{L}\left[{}_0^{l_1}\frac{l}{2s^2(s^2+l^2)}+\frac{1}{2s^3}\tan^{-1}\frac{l}{s}\right]$$
$$=\frac{Ih}{2Ls^2}\left[\frac{sl_1}{s^2+l_1^2}+\phi_1\right] \tag{148}$$

Putting l_2 for l_1 there would result Y_2 due to the part l_2 of the tube, whence addition gives the total illumination at P as

$$E=\frac{Ih}{2Ls^2}\left[\phi_1+\phi_2+\frac{sl_1}{s^2+l_1^2}+\frac{sl_2}{s^2+l_2^2}\right]$$
$$=\frac{Ih}{2Ls^2}\left[\phi+\frac{s}{L}\left[\frac{\frac{l_1}{L}}{\left[\frac{s}{L}\right]^2+\left[\frac{l_1}{L}\right]^2}+\frac{\frac{l_2}{L}}{\left[\frac{s}{L}\right]^2+\left[\frac{l_2}{L}\right]^2}\right]\right] \tag{149}$$

Let $m=\frac{s}{L}$, the ratio of the slant height from P to the total length of the tube, and let $p=\frac{l_1}{L}$, the fractional distance from the end of the tube. It will be well at this point to emphasize the significiance of these two ratios, as they appear throughout all the later work and furnish a key to its complete understanding. Substituting in these ratios,

$$E=\frac{Ih}{2Ls^2}\left[\phi+m\left(\frac{p}{p^2+m^2}-\frac{1-p}{(1-p)^2+{}^2m}\right)\right] \tag{150}$$

Then

$$E_o=\frac{Ih}{2Ls^2}K=\frac{I}{2L}\frac{h}{s^2}K \tag{151}$$

in which expression the constant K depends for its value only on the relative, rather than the actual, dimensions of the tube installation. $\frac{I}{2L}$ is half of the candle-power per foot, $\frac{h}{s^2}$ is constant for any line on the reference plane parallel to the tube, and K varies for different positions along such a line.

It becomes then a simple matter to explore the reference plane in investigating the illumination produced by the tube. A line can be taken parallel to the tube $\frac{I}{2L} \times \frac{h}{s^2}$ determined for this line, and the values of K can be obtained from Fig. 137 for points along the line; multiplication of these two quantities will give the value of the illumination at the points selected. The curves of Fig. 137 are plotted from data computed from equation (150). Different values of m were assumed, ϕ was ascertained from a graphical construction, and the remainder of the expression for K was computed by slide-rule. As K is the only variable in equation (151) when a line parallel to the tube is under consideration, the curves of Fig. 137 show the variation of illumination along such a line. The curves of Fig. 138 are derived from those of Fig. 137, p being the variable parameter instead of m. With the two sets of curves, double interpolation is made possible, and K can be determined for any and all values of p and m. In order to facilitate extended computations, values of the

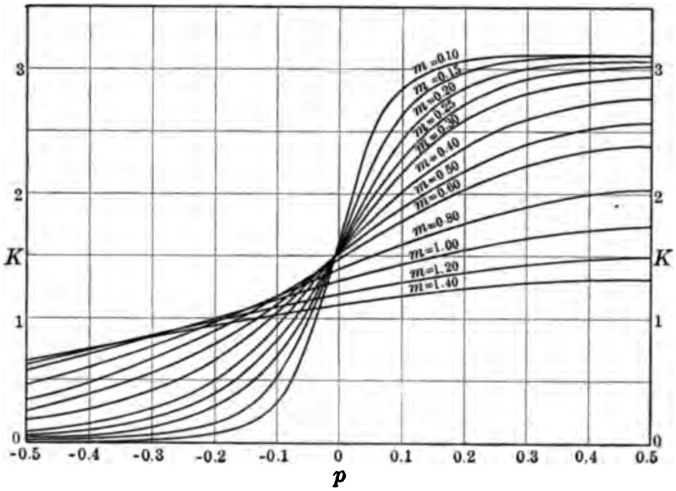

Fig. 137.—Graphs of K in equation 151.

expression h/s^2 may be derived for various heights of tube h and various distances x out from a line directly beneath the tube. These values can be arranged for reference in the form of a table.

It is interesting to consider the case of the illumination on the line directly beneath the tube. For this condition $s=h$, and equation (150) becomes

$$E = \frac{I}{2Lh}\left[\phi + m\left(\frac{p}{p^2+m^2} + \frac{1-p}{(1-p)^2+m^2}\right)\right] \qquad (152)$$

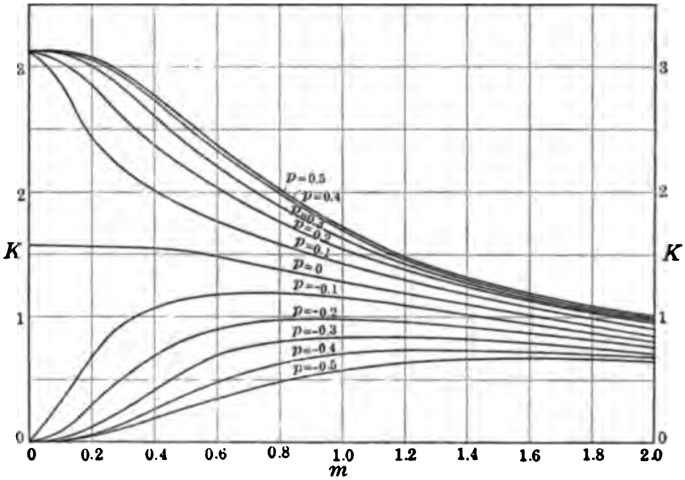

FIG. 138.—Values of K in equation 151.

Apparently the general formula (150) can be derived from the above by multiplication by $\frac{h^2}{s^2}$, or $\cos^2 \beta$. As the point P moves out from a position directly underneath the tube, however, m and ϕ change and the corresponding curve of K becomes of lower value (beneath the tube; perhaps higher beyond the tube ends), and more nearly horizontal. Thus, by moving out from positions beneath the tube the illumination is decreased somewhat more than by the multiplier $\cos^2 \beta$.

Photometry of Tubular Lamps.—In order to apply formula (150) it is necessary to know the candle-power per foot of the tube to be considered. To determine this quantity experimentally the tube might be covered with black cloth or paper, leaving exposed a length of a foot, say. Any ordinary photometer could then be used to determine the candle-power of the exposed section.

If the tube is at right angles to the photometer bar, p in equation (152) becomes 0.5 and the equation reduces to

$$E_a = \frac{I}{2Lh}\left[\phi + m\left(\frac{1}{m^2+.25}\right)\right]$$

$$= \frac{I}{2Lh}\left[2\tan^{-1}\frac{L}{2h} + \frac{h}{L}\left(\frac{1}{m^2+.25}\right)\right] \qquad (153)$$

the subscript $_a$ under E signifying that the actual illumination produced is meant.

Applying the inverse-square law to the same case, the theoretical illumination E_t at a distance h is given by the expression

$$E_t = I/h^2 \qquad (154)$$

For this equation to hold it is necessary that all the light strike the photometer normally, but since with a tube source this condition is not realized, it is evident that E_a will be less than E_t. The percentage error in E_t, *i.e.*, the percentage by which E_t must be decreased to give E_a, is

$$\text{Percentage error} = 100\left(\frac{E_t - E_a}{E_t}\right) \qquad (155)$$

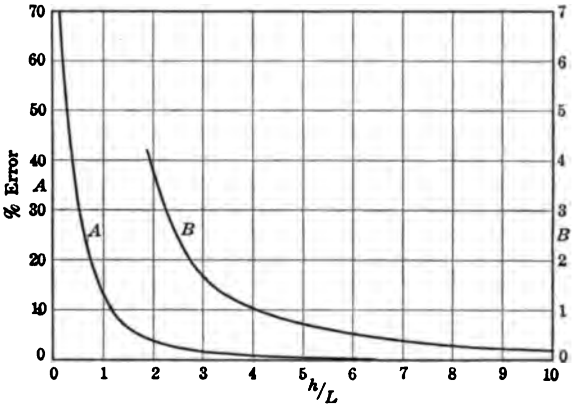

FIG. 139.—Errors in calculations.

Fig. 139 shows the values of this percentage error for various values of h/L. A practical application of this curve would be its use in the derivation of a correction factor to be applied to candle-power measurements of tube sources. The candle-power could be determined in the ordinary manner by the application of the inverse-square relation, then by multiplying by the correction factor would reduce the erroneous result to the correct value.

In the photometrical equation, $I = I'h^2/h'^2$, I'/h'^2 is the illumination on the photometer screen produced by the comparison standard, which at a balance is the same as the illumination produced by the unknown source. With the same notation as before, I'/h'^2 is E_t, the subscript meaning that the inverse square relation has been assumed, then the equation for I above could be written $I = E_t h^2$. An illumination E_t would not be produced on the screen by a tubular source of candle-power I at a distance h, for it has been shown that E_a is less than E_t. In order to make the above equation usable for computation it will be necessary to introduce a factor changing E_t to E_a.

Now from equation (155), letting e represent the error (decimal, not per cent.)

$$e = \frac{E_t - E_a}{E_t}, \text{ or } E_t = \frac{E_a}{1-e},$$

and by substituting above

$$I_a = \frac{E_a h^2}{1-e} \tag{156}$$

As I_a/h^2 gives E_t, the erroneous illumination computed on the assumption of the inverse square relation, so $E_a h^2$ gives I_t, the erroneous candle-power computed to the same assumption. Thus, substituting I_t for $E_a h^2$, there results

$$I_a = \frac{I_t}{1-e} \tag{157}$$

In other words, the candle-power computed by the inverse-square method will, when divided by $(1-e)$, give the actual candle-power.

Conditions for Uniform Illumination.—(Cooper, Ref. Cit.) It has been shown that, aside from the variations of K, the illumination at any point as the point moves out from beneath the tube, varies as $\cos^2 B$ where B is the angle between the slant line from the point to the tube and the normal to the reference plane. If the length of the tube is large compared to its height above the reference plane (m being, therefore, small), and only points near the center of the tube are considered, K will remain nearly constant. Then if in Fig. 140 T_1 and T_2 are two similar tubes spaced a distance d apart, and E_o denotes the illumination produced by one tube directly beneath its center, it can be written that

$$E_p = E_o \cos^2 B_1 + E_o \cos^2 B_2 \qquad (158)$$

Dividing by E_o,

$$\begin{aligned} E_p/E_o &= \cos^2 B_1 + \cos^2 B_2 \\ &= \frac{h^2}{h^2+x^2} + \frac{h^2}{h^2+(d-x)^2} \\ &= \frac{1}{1+\left[\frac{x}{h}\right]^2} + \frac{1}{1+\left[\frac{d}{h}-\frac{x}{h}\right]^2} \end{aligned} \qquad (159)$$

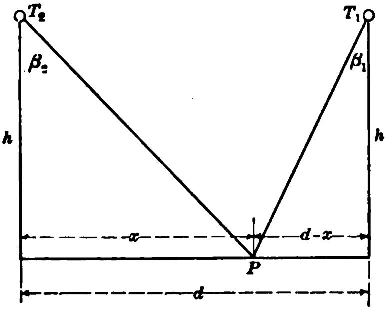

FIG. 140.—Illumination from two tubes.

If now a value of $d \div h$ can be found that will give a constant value of $E_p \div E_o$ for all values of $x \div h$, the problem of so spacing the tubes as to secure uniform illumination is solved. The exact solution for this condition by calculus is rather complicated, hence a solution by trial will be given. The accompanying table shows what variation in resultant illumination is encountered when $d \div h$ is 2, 1.5 and 1.0. This table shows that when $d = 2h$ the illumination half-way between the two tubes is only 83.3 per cent. of that directly beneath a tube; when $d = 1.5h$ there is a variation of 2 per cent. above and below the mean; and when $d = h$ there is an increase of 6.7 per cent. It is seen that with a separation of tubes of 1.5 times their height above the reference plane the most uniform illumination is secured, and also that under this condition the average illumination between tubes is about 1.3 times that produced by one tube directly beneath its center.

TABLE 39

VARIATION IN RESULTANT ILLUMINATION

	$d+h=2$			$d\div h=1.5$			$d+h=1.0$	
$x+h$	E_p+E_o	Per cent.	$x+h$	E_p+E_o	Per cent.	$x+h$	$E_p\div E_o$	Per cent.
.0	1.200	100.0	.0	1.308	100.0	.0	1.500	100.0
.2	1.198	99.8	.2	1.333	102.0	.1	1.553	103.5
.4	1.143	95.3	.4	1.315	100.5	.2	1.572	104.7
.6	1.073	89.8	.6	1.288	98.5	.3	1.589	105.9
.8	1.020	85.0	.75	1.282	98.1	.4	1.597	106.4
1.0	1.000	83.3	......	....	....	.5	1.600	106.7

Method of Design.—(Cooper, Ref. Cit.) The conclusions of the preceding paragraph lead to a very simple method of designing installations of lighting tubes. In most cases the room to be illuminated will be rectangular in shape, so that the parts of the installation which are hung along the length of the room will contribute most of the required illumination. The correct placing of the tubes on the longer sides of the room is then the main point to consider.

The side tubes must be near enough to the sides of the room to give a reasonable illumination at the edges of the reference plane. If the distance from the tube to the wall is from one-quarter to one-third of the distance between tubes, this condition will generally be amply fulfilled. Approximating, then, the distance apart the tubes are to be spaced, the condition for uniform illumination is that the height shall equal the spacing divided by from 1.5 to 1.2, the former being the preferable divisor. Spaced in this manner, the average illumination will be very nearly 1.3 times that given by one tube alone; hence, dividing 1.3 into the required foot-candles of illumination will give that required of one tube beneath its center. Substitution in equation (151) will give the candle-power per foot required of the tube to furnish this illumination, and the voltage of supply can be adjusted to give the necessary candle-power. The luminosity of a tube lamp can be varied from nothing to 30 candle-power per foot by changing the applied voltage, hence adjustment over a wide range is possible. For best operating conditions, the luminosity should be about 10 candle-power per foot. Other luminosities are obtained at a slight sacrifice of efficiency.

Specifications.—Room, 35 ft. × 25 ft.; 2 foot-candles illumination required on reference plane 2.5 ft. above the floor.

Calculations.—$2 \div 1.3 = 1.54$ foot-candles required of one tube. The first trial spacing will be 15 ft. Height of tubes above reference plane $= 15/1.5 = 10$ ft. Under the center of the tube

$$E_o = \frac{IK}{L\,2h} \qquad (160)$$

Let $L = 30$, $h = 10$, then $m = 0.33$ and at $p = 0.5$, $K = 2.88$

$$1.54 = \frac{I}{L}\,\frac{2.88}{2 \times 10} \text{ or } \frac{I}{L} = 10.7, \text{ a satisfactory value.}$$

With this spacing the necessary candle-power per foot proves satisfactory, but the tubes will be 5 ft. from the edges of the room, and the illumination there might prove insufficient. Accordingly, the spacing will be increased to 18 ft. This value will leave only 3.5 ft. between the side tubes and the wall, and will undoubtedly prove more effective. Leaving the length unchanged, $L = 30$ ft., $h = 12$ ft., $m = 0.4$, and at $p = 0.5$, $K = 2.77$, $1.54 = \frac{I}{L}\frac{2.77}{2 \times 12}$, or $\frac{I}{L} = 13.35$, which is higher than customary, but easily securable.

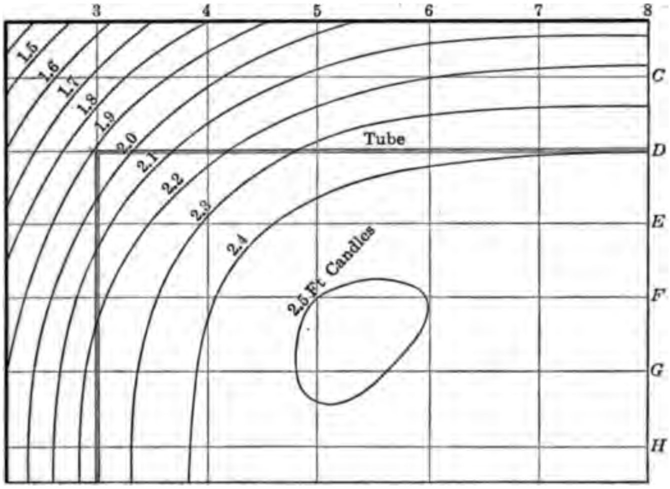

FIG. 141.—Illumination contours.

An exploration of the reference plane may now be made on lines perpendicular to the tubes. For assigned valves of x, s is obtained from the equation $s = \sqrt{h^2 + x^2}$. The value of $\frac{h}{s^2}$ is computed and multiplied by $I \div 2L = \frac{13.35}{2}$ for use in formula No. (151). The value of K is read from Fig. 138 for various values of P, whence $E = K\frac{6.67h}{s^2}$ gives the foot-candle illumination. A similar calculation may be made for the same points on lines perpendicular to the end tubes, whence addition of the illuminations produced at a point by the tubes on the four sides of the room will give values from which illumination contours can be plotted, as shown in Fig. 141. It is necessary to consider only one-quarter of the room, because the illumination at symmetrical points is the same. The lower right-hand corner in Fig. 141 represents the center of the room.

Inspection of Fig. 141 will show that the tube system of lighting can be designed to yield excellent results as regards the uniformity of illumination. In the case of this trial installation the results are better than the contour plat shows, because multiple reflection from the walls and ceiling will increase the illumination at the edges of the room more than in the middle; furthermore, near the ends of the tube, or beyond, there is actually more illumination than is computed on the assumption of the validity of the cosine law.

A Large Spherical Source.[1]—If a surface S_1 (supposed a portion of a spherical surface of radius r_1) has a brightness b and subtends a small solid angle ω the illumination which it produces at P (Fig. 142) is

$$E = \frac{bS_1}{r_1^2} = \frac{b\omega r_1^2}{r_1^2} = b\omega \qquad (161)$$

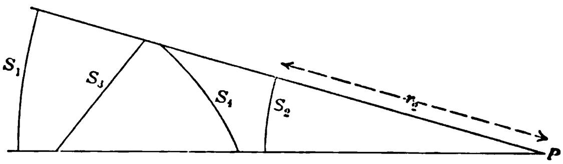

Fig. 142.—Section of large spherical source.

A second surface S_2 of the same brightness will produce the same illumination at P provided it subtends the same angle ω. A third surface S_3 at any angle will also produce the same illumination at P if it has the same brightness b and subtends the same solid angle ω.

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[1] Dr. E. B. Rosa, *Bull. Bur., Stand.* Vol. 6, p. 543, 1910.

For the radiation of each element dS_3 is

$$\frac{bdS_3}{r_3^2}\cos e=\frac{b\omega r_3^2}{r_3^2}=b\omega \qquad (161a)$$

as before. So also with the curved surface S_4. In every case the greater distance from P or the inclination of the angular position is compensated by the greater area included within the given (small) solid angle.

Let us calculate the illumination at P due to a large luminous sphere of radius a and brightness b, r being the distance from P (Fig. 143) to the center of the sphere. Let the solid angle APB subtended at P by the sphere be subdivided into a large number of elementary solid angles. Each of the latter incloses an area, as S_1, on the surface of the sphere, and also a corresponding area S_1', on the circular disk AB. As we have just seen, the illumination produced at P by each spherical area S_1, S_2, etc., is exactly the same as that produced by the corresponding plane areas S_1' S_2', etc., of the disk, if the brightness b is the same for the disk as for the sphere. Therefore the illumination at P due to the entire sphere is the same as that due to the disk AB, and we can calculate the latter by formula (134). That is,

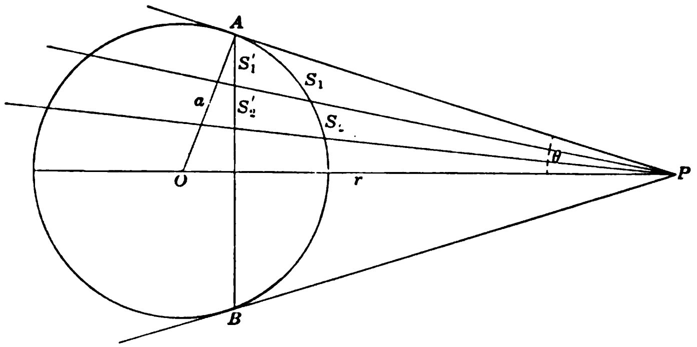

FIG. 143.—A large spherical source.

$$E=\frac{Q}{d^2}$$

where Q is the quantity of light on the disk and d is the distance AP from the point P to the edge of the disk. Q is equal to b times the area of the disk, or

$$Q=\pi(a \cos \theta)^2 b \qquad (162)$$

$$d=r \cos \theta \qquad (163)$$

$$\therefore E=\frac{Q}{d^2}=\frac{\pi a^2 b}{r^2}$$

$$=1/4\frac{Q_s}{r^2}=\frac{I_s}{r^2} \qquad (164)$$

where Q_s is the quantity of light on the sphere $=4\pi a^2 b$ and is constant for all distances, and I_s is the intensity of the equivalent point source. Therefore the illumination produced by a *sphere of any size* is inversely proportional to the square of the distance measured *from its center*, and is equal to the intensity of a point source (or unit sphere) having the same total amount of light divided by the square of the distance. In other words, the inverse square law holds just as rigorously for large spheres as for points (always of course assuming the cosine law to hold for the spherical surfaces, and the brightness b to be uniform over the sphere). When P comes very near to the surface the area AB of the sphere (Fig. 144) available for illuminating P is very small, but the distance is just enough less to counterbalance. When P comes up to the surface, $r=a$, and

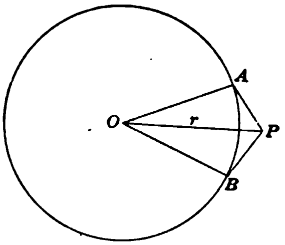

FIG. 144.—A large spherical source (P near).

$$E=\pi b$$

the same as for an infinite plane, to which the sphere is equivalent when the distance from the surface is reduced to zero.

The same result is reached more simply as follows:

A luminous sphere of radius a and uniform brightness b gives off a total flux $F=4\pi a^2 \times \pi b=4\pi^2 a^2 b$. This produces an illumination on the inner surface of any concentric sphere, which by symmetry will be everywhere the same, and $F=4\pi r^2 E$.

$$\therefore E=\frac{\pi a^2 b}{r^2}=\frac{I}{r^2} \qquad (165)$$

Therefore the *illumination produced by a sphere* of uniform brightness is inversely proportional to the square of the distance from the center for all distances from the surface of the sphere to infinity.

A Hollow Spherical Source.—(Rosa, Ref. Cit.) From what precedes we see that the illumination at any point P due to the hollow hemisphere ACB (Fig. 145) is the same as that due to the circular disk AOB. The latter is

$$E = \frac{\pi a^2 b}{AP^2} \tag{166}$$

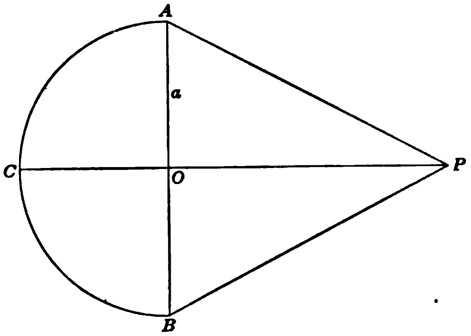

FIG. 145.—A hollow spherical source.

When OP is reduced to zero the illumination due to the disk is πb, and hence the illumination at O on an elementary plane area in the diametral plane is π times the brightness b of the surface of the sphere. We have already seen that the total flux from a unit of surface of brightness b is πb. Hence the total flux through unit area S at O, due to the hemisphere, is equal to the total flux through the hemisphere due to the luminous unit area S, the brightness b being the same in each case.

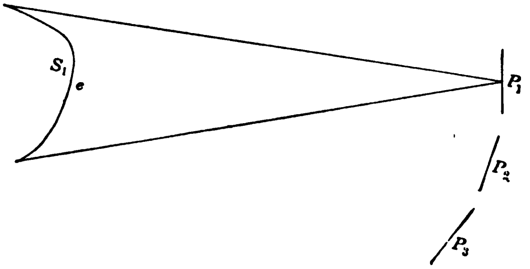

FIG. 146.—Illumination independent of shape of source.

This is a particular case of a more general proposition, namely, *the flux due to any surface S passing through an element dS is equal to the flux due to the latter passing through the former*, the brightness being the same in each case.

As shown above, the illumination E at P_1, due to S_1 (Fig. 146) is equal to $b\omega$, where b is the brightness of S_1 and ω is the (small) solid angle subtended at P_1 by S_1; this is independent of the shape of S_1 or its distance from P_1. The flux F passing through dS at P_1 is therefore

$$F = bd\omega dS \cos\theta, \text{ over the area of } S_1 \tag{167}$$

Or,

$$F = bdS \int \cos\theta \, d\omega \tag{168}$$

Similarly, the flux due to dS at P_1 passing through S_1 is

$$F = \int bdS \cos\theta \, d\omega$$

$$= bdS \int \cos\theta \, d\omega \tag{169}$$

In the integration every element $d\omega$ of the solid angle is to be multiplied by the cosine of the angle it makes with the normal to the area dS.

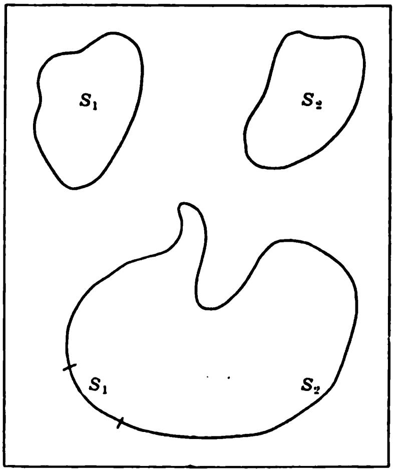

FIG. 147.—Surface source.

As the same theorem holds for the elementary areas P_2 and P_3, etc., it holds for their sum, and hence for a finite surface, S_2 (Fig. 147). Hence we see generally that *the luminous flux due to a surface* S_1 *passing through* S_2 *is equal to the luminous flux due to* S_2 *passing through* S_1, the brightness being the same in each case. This is analogous to the theorem that the magnetic flux due to a magnetic shell S_1, which passes through a second shell S_2, is equal to that part of the magnetic flux S_2 which passes through S_1, the strength of the shells being supposed the same. Or, again, the number of lines of force due to unit current in an electric circuit S_1 passing through S_2 is equal to the number of lines of force due to unit current in S_2 passing through S_1. It follows from the above that in any closed surface of uniform brightness the flux passing out from any portion S_1 is equal to that received from the remainder of the surface S_2.

A Hollow Sphere.[1]—(Rosa, Ref. Cit.) Suppose a hollow sphere (Fig. 148) of uniform surface having a coefficient of diffuse reflection m,

$$1-m=\text{absorption.}$$

Let $E=$ illumination at S.

$E'=mE=$ radiation from S.

$b=\frac{mE}{\pi}=$ brightness of S.

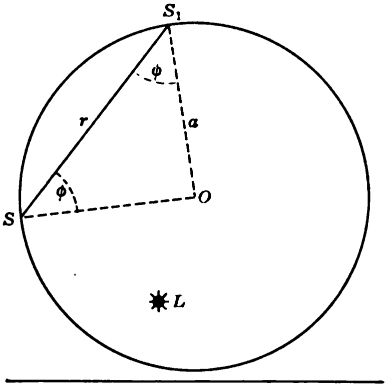

FIG. 148.—The hollow sphere.

The flux falling on S_1 due to S is

$$S_1dE_1=\frac{eSS_1\cos^2\varphi}{r^2}=\frac{mE}{\pi}\cdot\frac{SS_1\cos^2\varphi}{r^2} \tag{170}$$

But

$$r=2a\cos\varphi$$
$$r^2=4a^2\cos^2\varphi$$
$$\frac{\cos^2\varphi}{r^2}=\frac{1}{4a^2}$$
$$\therefore dE_1=\frac{mE}{\pi}\cdot\frac{S}{4a^2} \tag{171}$$

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[1] See Liebenthal, "Praktische Photometrie," p. 301.

and this is the same for every element of the sphere. Hence *every element* illuminates all other elements equally. Therefore the *indirect* illumination of the sphere must be the same everywhere, no matter how unequal the direct illumination may be. That is, a light at L illuminates the sphere unequally, directly. But that part of the total illumination due to diffuse reflection, is, notwithstanding, everywhere equal.

A light of mean spherical intensity I sends out $4\pi I$ lumens.

Of this there is reflected, first, $4\pi mI$ lumens.

Of this there is reflected, second, $4\pi m^2 I$ lumens.

Of this there is reflected, third, $4\pi m^3 I$ lumens, and so on.

Therefore the total amount of flux reflected is

$$4\pi Im\left[1+m+m^2+m^3+\ .\ .\ .\ .\ .\right]=4\pi I\frac{m}{1-m}=F_2 \quad (172)$$

Hence the secondary illumination everywhere equal on the surface of the sphere is

$$E_2=\frac{F_2}{4\pi a^2}=\frac{mI}{(1-m)a^2} \quad (173)$$

Thus the indirect illumination is proportional to I, and the lamp of intensity I may be anywhere in the sphere. It is equal to $\frac{m}{1-m}$ of what the direct illumination would be if the source were placed at the center of the sphere. For example, let a 16 candle-power lamp be placed within a sphere having a radius of 1 meter and a coefficient of diffuse reflection of 0.8.

Then $I=16$

$a=1$ meter

$m=0.8$

$E_2=\frac{0.8}{0.2}\times\frac{16}{1}=64$ meter-candles

$E_1=\frac{I}{a^2}=16$ meter-candles, if lamp is in the center

$E=E_1+E_2=80$

Thus the total illumination is five times what it would be if the walls were perfectly black. We can put this in another way; of the total illumination of 80 meter-candles 20 per cent. is absorbed by the walls. Therefore the lamp or source must supply only one-fifth of the total, just enough to make good the constant loss.

Thus the source is analogous to an exciter of electric waves that must supply just enough energy to make good the resistance losses in the circuit.

Luminous Flux within an Inclosure.—(Rosa, Ref. Cit.) If the inner surface of the hollow sphere has a brightness b and a specific radiation $E' = \pi b$, a unit disk at the center of the sphere will receive an illumination $E = \pi b$. The same will be true wherever the unit disk is placed within the sphere and whatever the orientation of the disk; that is, the flux falling on the disk will be everywhere the same. The flux density within the hollow sphere is therefore everywhere uniform and equal to πb. The flux from a point source is thought of as in straight lines, and a disk can be placed normal to the direction of the flux. But within the sphere the flux has a uniform value, but no resultant direction.

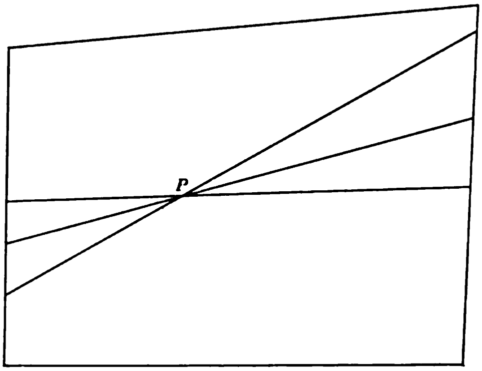

FIG. 149.—Illumination within an enclosure.

Within a cube or inclosure of any shape, of which the walls have a uniform brightness b or uniform specific radiation E', the same condition obtains as in the sphere—namely, the luminous flux is everywhere the same, and a small area will have the same illumination no matter where it is placed or how it is oriented. This is seen by dividing up the space about any point P (Fig. 149) into elementary solid angles. The illuminations due to the surface subtending an angle ω is independent of the distance from P, and hence it will be πb for the total angle 2π on either side of the surface at P, no matter where the surface is placed.

The same is true, therefore, for the space between two infinite planes of brightness b. The illumination is πb on a small plane at P_1, P_2, or P_3 (Fig. 150), anywhere between the two radiating planes S and T no matter how they may be placed. Evidently we can not think of the flux as normal to the planes, as with the lines of force due to electrostatic charges on the planes S and T. The luminous flux normal to P_3 is the same as normal to P_1. On the other hand, the electric force normal to P_3 would be zero.

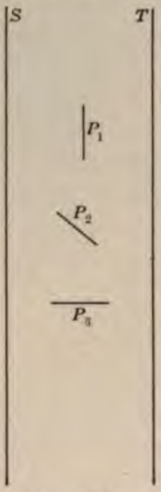

FIG. 150.—Illumination between infinite planes.

Summary of Photometric Relations.—(Rosa, Ref. Cit.) The preceding discussion has shown the necessity of distinguishing several different photometric quantities which are sometimes confused. In order to fix our ideas more clearly, it will be advantageous now to state as concisely as possible the definitions of the several quantities and distinctions between them.

Luminous flux, or light as the term is used in photometry, is the usual physical stimulus which excites vision. It is propagated by means of the vibratory motion in the ether, and the frequency of the vibrations or the combination of frequencies present in any given case determines the color. The total quantity of flux F flowing away from a monochromatic luminous source is proportional to the total radiant energy (per second) and to a stimulus coefficient, the latter being the luminous efficiency K_λ for the particular frequency or wave length of the given radiation. Thus the equations:

$$F = K_\lambda W \tag{174}$$

$$K_\lambda = F_\lambda / W_\lambda \tag{175}$$

express the luminous flux as the power W multiplied by the luminous efficiency K_λ and if flux is expressed in *lumens* and the power in *watts*, the luminous efficiency is the number of lumens per watt of radiation of the wave-length λ. For white or chromatic light K will have a value depending on the distribution of the energy in the spectrum. It is a maximum in the yellow-green region and falls off rapidly in either direction, reaching zero at the limits of the visible spectrum. The luminous efficiency of most light sources is greatly reduced by the amount of radiation outside the visible spectrum, chiefly of longer wave-length than that of visible radiation, and the total efficiency of such a source

$$K=F/W \tag{176}$$

is the quotient of the total luminous flux divided by the total radiant power.

For the purposes of definition and of expressing the mathematical relations involved in photometry, it is permissible to confine ourselves to monochromatic light and to consider K a constant, although it does in fact vary somewhat with the magnitude of the flux density. We also assume that all surfaces are perfectly diffusing and obey the cosine law and that there is no absorption in the atmosphere.

The intensity of a point source or uniform luminous sphere is measured by the luminous flux flowing through a unit solid angle whose apex is the given point or center of the given sphere. Thus from a source of intensity I, light is flowing away at a rate of I lumens per unit solid angle or a total of $4\pi I$ lumens for the point source or uniform sphere. If the source is not uniform and light is flowing away at unequal rates in different directions, the intensity I in any direction is equal to the flux dF in an elementary solid angle $d\omega$ taken in the given direction. Thus

$$I=\frac{dF}{d\omega} \tag{177}$$

is a general expression applying to all point sources whether radiating equally or unequally in different directions. If the unsymmetrical source is extended—as, for example, an incandescent lamp or a diffusing globe—the same holds true if the distance at which the measurements are made is sufficiently great so that the distribution of light is practically the same as from an unsymmetrical point source. For less distances than this the intensity is not a constant in a given direction, but varies with r. In this case the equivalent intensity at any point is equal to that of a point source which gives the same flux density, or lumens per square centimeter, at the point that the given source does. The mean spherical intensity I_{ms} is the average value of the intensity and is equal to the total flux F divided by 4π.

The total flux from a given extended source is therefore a constant independent of distance, as is also the mean spherical intensity I_{ms}. The intensity I in a particular direction, however, in the case of extended sources other than spheres varies with the distance; but at relatively great distances the variation is inappreciable. Thus the luminous flux is the fundamental quantity. But while we *define* I as the flux per unit solid angle, or *rate of flux with respect to solid angle,* we *determine* I by comparison with a concrete standard. *Thus photometric standards of intensity are standards of light flux, their values being expressed in candles.* If f is the spherical reduction factor with respect to any particular direction, and I is the intensity of a source in that direction,

$$I_{ms} = fI \tag{178}$$

For a unit disk—that is, a small circular disk of uniform brightness—the total flux is π times the normal intensity I_n, whereas the mean spherical reduction factor with respect to the normal is one-fourth. Hence, the total flux is

$$\begin{aligned} F &= \pi I_1 \\ &\doteq 4\pi I_{ms} \text{ as for a sphere.} \end{aligned} \tag{179}$$

In general, for any light source, $F_s = 4\pi I_{ms} = 4\pi f I$, but for extended sources other than spheres, the value of f as well as I varies with the distance from the source for points relatively near the source.

The *specific flux* or *flux density* is the luminous flux per unit of area, or lumens per square centimeter. When the flux falls upon a material surface, we call the specific flux the *illumination, E.* When we speak of the flux coming *from* a surface, whether it be a self-luminous source at high temperature or a reflecting or radiating surface at low temperature, we call the specific flux the specific radiation, or simply the *radiation, E'.*

Thus the illumination E is

$$E = \frac{F_i}{S} = \frac{dF_i}{dS} = \frac{I}{r^2} \tag{180}$$

The radiation E' is

$$E' = \frac{F_e}{S} = \frac{dF_e}{dS} \tag{181}$$

F_i is the incident flux, F_e is the emitted or radiated flux. If m is the coefficient of diffuse reflection or transmission, $(1-m)$ being the absorption,

$$F_e = mF_i \tag{182}$$

$$E' = mE \tag{183}$$

where the radiation consists in the diffuse reflection or transmission of a portion of the incident flux or illumination.

The radiation or illumination when large may be expressed in lumens per square centimeter and is nearly equivalent to the foot-candle.

1 lumen per square centimeter = 10,000 lumens per square meter.
= 10,000 meter-candles.
1 milli-lumen per squar ecentimeter = 10 meter-candles = 10 lux.
$= \frac{1}{1.0765}$ foot-candles.

The *brightness* b of a source is the intensity in candles per square centimeter of area, taken normally. Thus

$$b=\frac{I}{S}=\frac{dI}{dS}=\frac{dQ}{dS} \tag{185}$$

Brightness, or *specific quantity*, refers to the quantity of light per unit of area of a source, and is measured in candles per square centimeter. Brightness can refer equally to luminous sources of relatively high specific intensity or to reflecting and radiating sources of low intensities. The latter may be conveniently expressed in milli-lumens per square centimeter. Thus we may say a flame has a specific radiation of 10 lumens per square centimeter or a brightness of 0.8 candles per square centimeter; and a wall has a specific radiation of 10 milli-lumens per square centimeter or a brightness of 0.8 milli-candles per square centimeter or of 8 candles per square meter.

The quantity Q is proportional to the total amount of light emitted by the source, and is equal to the surface integral of the brightness b. Thus

$$Q=\int b\,dS \tag{186}$$

The quantity for a small luminous circular disk of radius a and uniform *brightness* b is

$$Q=\pi a^2 b=I_n \tag{187}$$

That is, the quantity is equal to the maximum intensity. In this case the whole surface is equally effective in producing the illumination on the test screen by which the intensity I_n is measured. But for an extended disk, the quantity and the normal intensity, as we have seen above, are not the same. Thus, the quantity is b times the surface, or

$$Q=\pi a^2 b$$

$$E_n=\frac{Q}{a^2+r^2}=\frac{I_n}{r^2}$$

$$\therefore \frac{I_n}{Q_1}=\frac{r^2}{a^2+r^2}=\frac{r^2}{d^2}=\cos^2\theta \qquad (188)$$

That is, the *normal equivalent intensity* I_n of the disk (Fig. 151) with respect to the point P on the axis of the disk is Q times $\cos^2\theta$. When the distance is equal to the radius of the disk, the quantity Q is twice the normal intensity I_n. The total luminous flux is πbS or π times the quantity, and the *mean hemispherical intensity* is $\frac{Q}{2}$ or half the quantity.

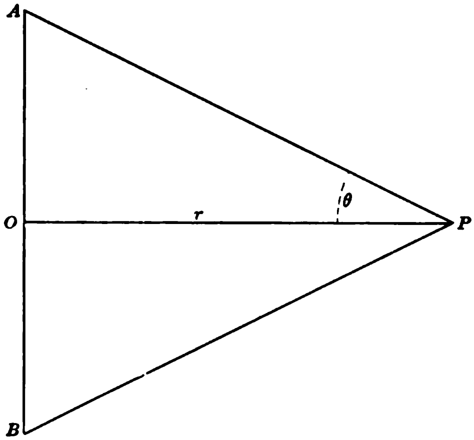

FIG. 151.—Luminous circular disk.

In the case of a sphere of uniform brightness b the quantity is $\int bdS=4\pi a^2 b$. The intensity $I=\pi a^2 b$. Hence the intensity is one-fourth the quantity. In other words, the total radiation from the sphere is four times as great as from a unit disk of the same normal intensity. The relations between quantity and intensity for a few simple cases are as follows:

For a unit disk $I_n=Q$ (189)

For an extended circular disk $I_n=Q\cos^2\theta=Q\frac{r^2}{d^2}$ (190)

For a sphere $I = \frac{1}{4}Q$

For a unit cylinder $I_1 = \frac{1}{\pi}Q$

The total luminous flux delivered in a given time—that is, the time integral of the luminous flux—may be expressed in lumen-seconds or lumen-hours, according to circumstances. Thus putting LH for the total lighting in the time T

$$LH = FT$$

$$LH = \int FdT, \text{ if } F \text{ is variable,} \tag{191}$$

where F is in lumens and the time is expressed in the most convenient unit. The flash of a firefly may be expressed in lumen-seconds. The total lighting per gram of an illuminant, or the total lighting given during the life of an incandescent lamp, is better expressed in lumen-hours.

Since flux of light may also be expressed in spherical candles ($1/4\pi$ times the lumens) we may also express the time integral or total lighting in terms of spherical candles and hours. Thus $CH = I_s T = \int IdT$, if the spherical candle-power is a variable with respect to T, the value of CH being here given in candle-hours.

TABLE 40

Collected Definitions, Symbols, Units and Equations

Photometric definition	Symbol	Unit	Equation
1. Intensity of light....	I	Candle	$I = F/\omega$
2. Luminous flux	F	Lumen	$F = I\omega = Is/r^2 = ES = \pi Q$
3. Illumination.........	E	Foot-candle Lumens/ft.²	$E = I/r^2 = F_i/S$
4. Radiation...........	E'	Lumens/ft.²	$E' = F_e/S = mE$
5. Brightness..........	b	$\frac{\text{Candles}}{\text{cm}^2 \text{ or in.}^2}$	$b = I/s \cos e$
6. Quantity............	Q	Candles	$Q = b\ s$
7. Lighting............	LH	Lumen-hours	$LH = FT$
	CH	Candle-hours	$CH = I_{ms}T$

I, b, and Q are expressed in candles.

F, E, E' are expressed in lumens.

m = the coefficient of diffuse reflection or transmission.

$(1 - m)$ = coefficient of absorption.

F_i = incident flux.

F_e = emergent flux.

Collection of Formulae

1. $E=\frac{I}{r^2}$ for point source, unit sphere or sphere of any size.

$I=\pi a^2b$ where a = radius of sphere and b = brightness of surface.

2. $E=\frac{\pi a^2 b}{r^2}$ for sphere of radius a,

$=\pi b$ when $r=a$, that is, at the surface of the sphere, same as for an infinite plane.

3. $E=\frac{\pi a^2 b}{a^2+r^2}=\frac{Q}{d^2}$ for disk of radius a, at distance r on axis, and distance d of point on axis to edge of disk.

4. $E=\frac{\pi ab}{r}$ for infinite cylinder, b = brightness, a = radius;

$=Q_1/2r$ where Q_1 = quantity of light per unit of length;

$=\pi b$ at surface. $I_1=\frac{2ab}{r^2}$ intensity per unit of length.

5. $E=\pi b$ for infinite plane, *at all distances.*

6. $E=\frac{b\,dS\cos e}{r^2}=cbd\omega$ for any small surface dS subtending a small angle $d\omega$, at any distance.

7. $E=\frac{2b}{r}\cos e$ for infinitely long, very narrow strip of b units of light per unit of length,

$=2\frac{Q_1}{r}$

8. $Q=\int bdS$ over sphere, cylinder, disk or other surface, where b = normal intensity = quantity of light per unit of area.

9. $F_s=\pi Q$ for sphere or other extended source.

10. $E=\frac{Q}{d^2}=\frac{I_0}{r^2}\ \therefore\ \frac{I_0}{Q}=\frac{r^2}{d^2}$ for a disk.

I_0 = equivalent point source, Q = quantity of light over disk
$I\ =Q/4$ for a sphere.

CHAPTER XI

THE PRINCIPLES OF INTERIOR ILLUMINATION

The object of the preceding ten chapters has been to lead up to the subject of interior and exterior lighting and to aid the reader in solving the numerous problems arising in these classes of illumination. We have discussed the physical and physiological characteristics of light; its colors and luminous values; its measurement and the apparatus involved; the calculation of the amount available from a source; and the calculations of illumination due to point and surface sources.

In analyzing the subject of lighting Mr. Sweet[1] arranges the salient features of illumination as shown in Fig. 152 and very properly considers *illumination a complex problem,* made up of a number of subordinate problems such as intensity, distribution, diffusion and the like.

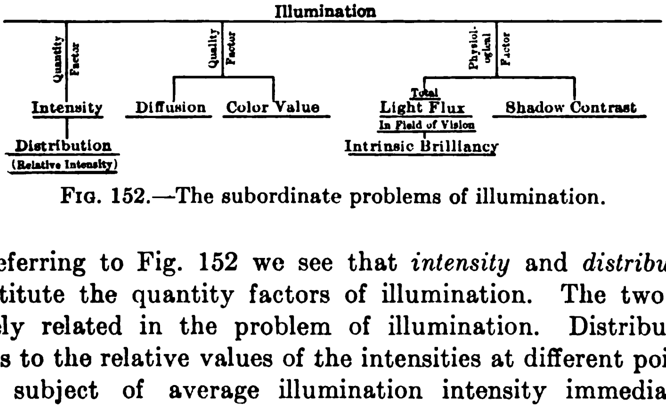

FIG. 152.—The subordinate problems of illumination.

Referring to Fig. 152 we see that *intensity* and *distribution* constitute the quantity factors of illumination. The two are closely related in the problem of illumination. Distribution refers to the relative values of the intensities at different points. The subject of average illumination intensity immediately brings up the question of distribution. The quality factors are the *diffusion* and *color* values of the light. These are independent of each other. The total light flux in the field of vision and shadow contrast are two closely related yet separate factors affecting the efficiency of the eye. Closely associated with the subject of *total light flux* and at the same time of vital importance is the factor of *intrinsic brightness* of the light or light reflecting objects in the field of vision.

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[1] Trans. Illum. Eng. Soc., Vol. 4, p. 745, 1909.

With these various factors in mind the question arises as to what constitutes the essentials of a *good lighting installation*. Following the order given above we may formulate a rule which will apply to all but exceptional cases. The *intensity* should be ample to see clearly and distinctly, and the *distribution* should be such that the illumination over part of the room at least will be nearly uniform. The *light* should be soft and well diffused. The *color* of the light preferable depends upon the class of service and the tastes of the individual. A light approaching daylight in quality or inclined toward the yellow will in general be found satisfactory. The *sources of light* should be placed well above the range of vision, its intrinsic brightness reduced by diffusing glassware and objects capable of high specular reflection removed from the range of vision. *Shadows* are necessary for distinguishing outlines but such shadows should be toned down and be not too abrupt or dense.

The foregoing is a brief summary of the preceding chapters. Much pertaining to interior illumination is incorporated in Chapter IX. The different methods of calculating illumination, the different intensities of various services, the efficiency constants for different equipments and interiors, and the methods of securing uniform illumination have already been discussed. This chapter then will be devoted to the realization of those conditions which go to make good illumination.

It is an established fact that *bright walls* in a room contract the pupil and make a certain value of illumination less effective than if the walls are dark in color. The *illumination intensity* for any particular service is by no means a constant quantity. Some people require several times as much light as others for the same class of work. It is also a function of the brightness of other objects in the field of vision. Consequently, best results will be obtained if light is on the subject of observation and not on surrounding objects if those objects are light in color or have glossy surfaces.

In regard to the distribution of light, there are three general classes of interiors:

1. A *small room* as those of a residence and a small office where there is only one lamp or one lighting fixture.

2. A *long room* comparatively narrow where the lamps can be placed in a row running lengthwise of the room, as in the case of hallways or small stores.

3. *Large rooms* such as are found in large stores, and general offices, theatres, churches, auditoriums, etc.

There are also three general systems of illuminating any of these interiors:

1. The *"direct" system*, where the light from the lamp passes direct to the working plane. The lamp may or may not be equipped with a reflector, shade, or diffusing media.

2. The *"semi-direct" system*, where the lamp is equipped with an inverted translucent reflector which lets some of the light pass directly to the working plane while the remainder is thrown to the ceiling or walls, from which part is reflected and in this way reaches the reference plane.

3. The *"indirect" system*, where efficient opaque reflectors are used and the entire available light thrown on the ceiling from which part is reflected to the reference plane.

Any of these lighting systems may be used for any of the three classes of interiors provided conditions are favorable. The semi-direct and the indirect systems obviously require very light-colored ceilings in order that their use be permissible.

The Direct Lighting System.—The problem in the case of the small room is to uniformly illuminate a certain portion of it at the required intensity. The solution of this problem with the direct system was indicated in Chapter IX. The relation between the illumination results desired and the distribution of light from the source required to produce those results depend upon:

(a) The size of the room.

(b) The height of the source above the reference plane.

For any given size of room and height of light center one can easily determine the distribution curve which will give the desired distribution. It is obvious that any given distribution curve will give the same proportionate illumination results so long as the relation between the size of the room and the height of the light center is kept constant.

If now we let

$$J = \frac{\text{mean dimension of room}}{\text{height of light center above reference plane}}$$

then for any value of J there is a given distribution curve which will produce the desired distribution of illumination.

Small Rooms.—For lighting a residence room, or small office, it is usually satisfactory to have the central half of the room uniformly illuminated and allow the intensity near the walls to drop to 1/3 the central value.

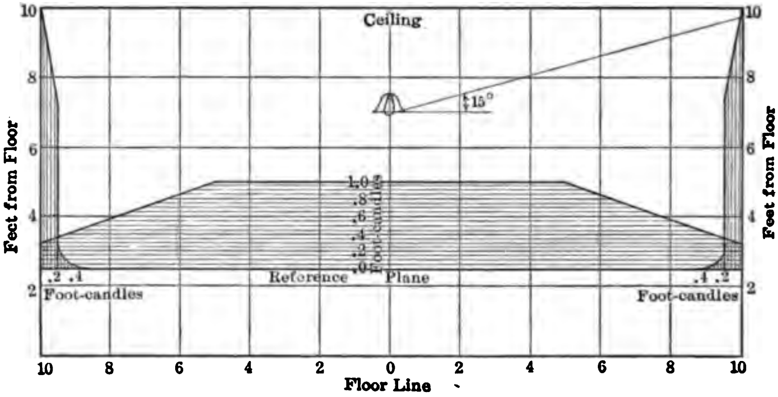

FIG. 153.—Desirable distribution of illumination in a room.

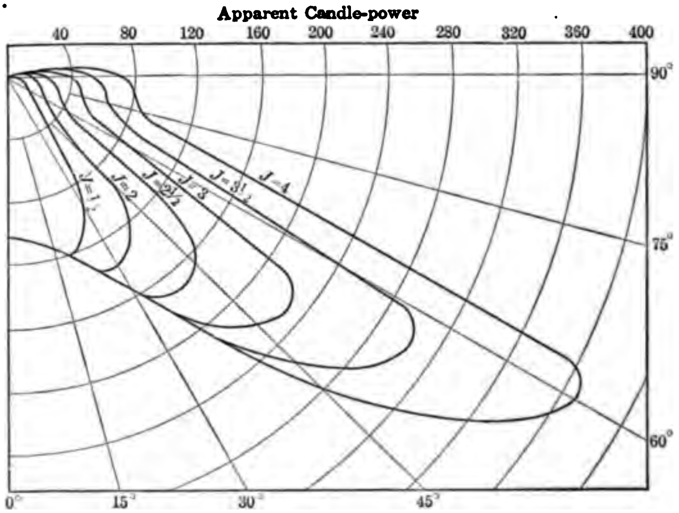

FIG. 154.—Curves for uniform illumination.

One-fifth the central intensity should be sufficient for the walls to illuminate the wall furnishings. Assuming these conditions, Mr. Sweet (Ref. Cit.) represented the distribution of illumination as shown in Fig. 153, and to meet these conditions he derived the curves shown in Fig. 154 for different values of J from 1.5 to 4.

Hence for properly lighting a small room having one central fixture a type of distribution such as is shown in Fig. 154 is required. The mounting height of the fixtures of average existing installations would require a distribution indicated by $J=3$. The tendency of the future is to place the lamps higher corresponding to values for J of 2 or 2.5.

There are now available reflectors designed to meet the requirements of this class of service. The curves for 100, 150, and 250 watt bowl-frosted tungsten lamps with "extensive" prismatic reflectors are shown in Fig. 155.[1] By comparing these curves with those of Fig. 154 we find that they approximate a curve for J having a value of 2. The value of the required candle-power of the lamp may be found by some of the methods described in Chapter IX.

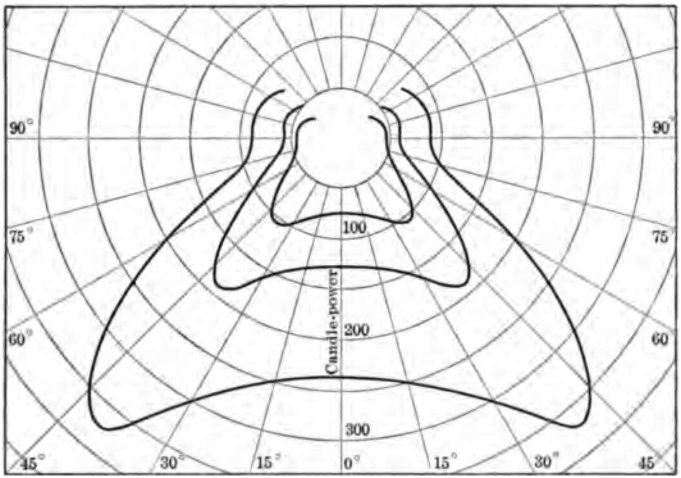

FIG. 155.—Characteristic curves "Mazda" 100, 150, and 250-watt, 100–125-volt, bowl frosted lamps with extensive reflectors.

Long Narrow Rooms.—In the lighting of interiors of the second class or long comparatively narrow rooms, the area may be divided up into squares and each square treated similar to the small room just described. As will be seen by referring to the polar curves of the last figure each lighting unit emits a cone of light which will illuminate a certain area or circle. Placing these units in a row lengthwise of the room spaced as indicated by Fig. 156 will satisfactorily illuminate this class of interiors.

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[1] *Bull.* 7B, Nat. Elect. Lamp Assoc.

Large Rooms.—The illumination of large interiors offers no particular difficulties. The practice is to determine the style of lighting equipment best suited for the class of service and then from a study of the conditions, surroundings, etc., calculate the number of lamps required by some one of the methods taken up in Chapter IX. What concerns us now is the location of the lamps—the height above the reference plane, and the distance between the different units. It has become the practice to divide the area of the room into as many equal rectangular sections as there are lamps required, making the sections as near squares as possible, and place an outlet over the center of each section. If the room is of such dimensions that it can be divided into N equal squares, than the distance d between lamps located above the centers of the squares will be

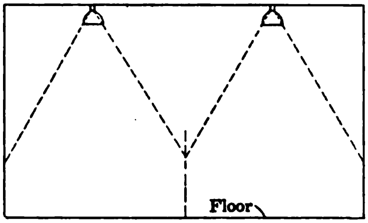

FIG. 156.—Cones of light.

$$d=\sqrt{S/N} \tag{192}$$

where S is the area of the room in square feet, N the number of lamps or squares, and d the distance between lamps in feet.

If the area must be divided into equal rectangles, having the dimensions b and c ft., and a lamp placed above the center of each rectangle, then $b=\frac{S}{cN}$, the distance between lamps in one direction, and $c=\frac{S}{bN}$ the distance in the other direction. The proper methods of spacing the lamps in a room are shown in Figs. 105 and 161. Having settled the question of spacing the light sources, the next item is the height of suspension. Uniform illumination is a good criterion to work for, and with the distance between lamps established it follows that the minimum height of suspension above the reference plane will be determined by the distribution of light from the unit chosen. At this point it would be well to refer to the polar curves near the end of Chapter IX and from the values of d/h given for each polar curve one can easily determine the height if the chosen unit has one of those types of distribution.

Analyzing this phase of the subject Mr. Sweet (Ref. Cit.) has derived a set of curves, Fig. 157, showing the different distributions for $d/h = K = 1/2$, 3/4, 1, 1 1/4, 1 1/2, and 2. In other words, for uniform illumination, the maximum distance between lamps must not be more than 1/2, 3/4, etc., of the height of suspension above the reference plane.

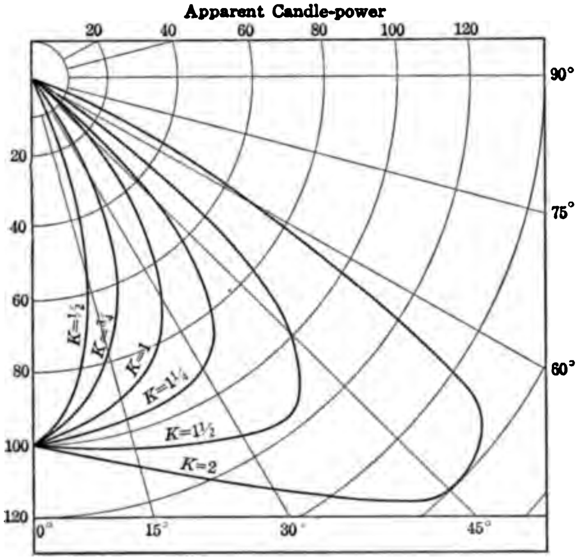

FIG. 157.—Curves to give uniform illumination. (Two or more lamps.)

Reflectors have been designed to meet the requirements of this class of service. The curves for 100, 150, and 250 watt lamps equipped with "intensive" and "focusing" types of prismatic reflectors are shown in Figs. 158[1] and 159[1] respectively. Those of the "extensive" type shown in Fig. 155 may also be used for lighting interiors where it is desirable to place the lamps either lower or farther apart.

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[1] *Bul.* 7 B, Nat. Elect. Lamp Assoc.

For these three types of curves the relation of the distance between lamp centers to the height above the reference plane for approximately uniform illumination is:

Extensive type curve, $k=d/h=2.0$.
Intensive type curve, $k=d/h=1.25$.
Focusing type curve, $k=d/h=0.8$.

It should not be forgotton that the value of h used in this relationship is the minimum height at which uniform illumination can be secured with the lamps a distance d apart. The lamps can be placed at a greater height than h without impairing the uniformity of the illumination. It is obvious from the shapes of these polar curves that the extensive type of distribution would be used in rooms having low ceilings or low fixtures or where it is desirable to light a large area. The intensive distribution is used for medium and average conditions where more than one lamp or outlet are required. The focusing distribution is used where the lamps are placed comparatively high above the reference plane.

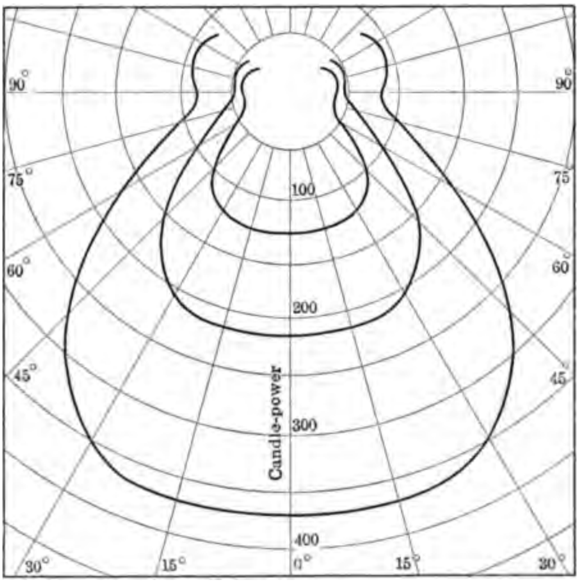

Fig. 158.—Characteristic curves "Mazda" 100, 150, and 250-watt, 100-125-volt, bowl-frosted lamps with intensive reflectors.

In Fig. 160 are shown the illumination curves for one and for two 250-watt tungsten lamps equipped with extensive, intensive and focusing types of reflectors. The details are given in the following table:

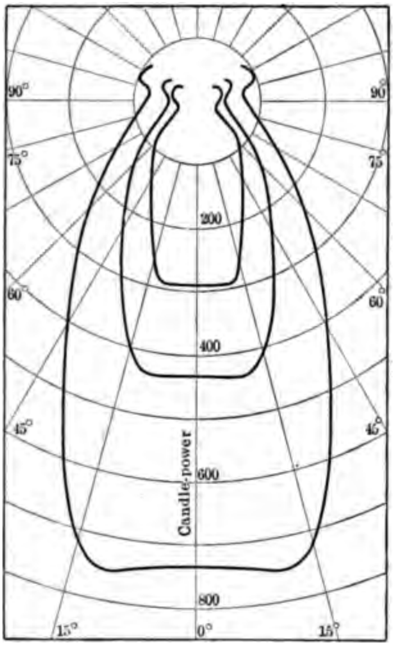

FIG. 159.—Characteristic curves "Mazda" 100, 150, 250-watt, 100–125-volt, bowl-frosted lamps with focusing reflectors.

TABLE 41

Relative Location of Lamps with Different Types of Reflectors.

Curve	Reflector	Height above reference plane (feet)	Distance between lamps (feet)	Distance / Height	Number of lamps
D_1	**Distributive**	9			1
D_2	**Distributive**	9	18	2	2
I_1	**Intensive**	12			1
I_2	**Intensive**	12	15	1.25	2
F_1	**Focusing**	15			1
F_2	**Focusing**	15	12	0.8	2

The relative distributions of illumination from a lamp equipped with these three types of reflectors when placed at the same heights are shown in Fig. 107, and from two lamps equipped in the same way are shown in Fig. 108, the location of the lamps being the same in each three cases.

The False Ceiling.—Another class of direct lighting consists in placing the lamps above false ceilings of milk or opal glass, the object being to secure a well-diffused illumination, and in this respect it is remarkably successful. Its use is obviously restricted due to the necessity of the false ceiling, the inconveniences pertaining to renewals or trimming and to the fact that other systems of either the direct, semi-direct, or indirect types are found quite as satisfactory.

The Effect of Height of Suspension of Lamps upon the Intensity of Illumination.—The fact that the intensity of illumination due to a point source of light varies inversely as the square of the distance between the source of light and the surface illuminated is familiar to all, but the significance and the practical application of the relationship existing between the distance from an infinite linear source or an infinite surface source and the intensity of illumination resulting therefrom are realized only by those most vitally interested in or intimately associated with the theory or uses of light.

It has been demonstrated and shown mathematically in the preceding chapter that the illumination at a given point in a normal plane due to a tubular source of infinite length varies inversely as the distance from the source to the point illuminated and not inversely as the square of the distance as in the case of a point source. It is also well established that the illumination at a point on a normal plane due to an infinite surface source is quite independent of the distance between the point and the source.

It is obvious that luminous sources having infinite dimensions are merely theoretical, but it is interesting to assume certain practical conditions and learn how approximately the infinite dimension relationship may apply in practice. For this purpose the writer has chosen the 100-watt tungsten unit equipped with the intensive type of high efficiency reflector. This type of reflector is so designed that when used with the proper lamp and the units are properly placed approximately uniform illumination on the working plane will be obtained.

The distribution of light in a vertical plane through the axis of this type of lighting equipment is shown in Fig. 158. This polar curve shows only the intensity of the light in different directions in a vertical plane and gives no clue as to the resulting illumination. The illumination due to one lamp placed 12 ft. above the reference plane is indicated by curve I_1 of Fig. 160. If two lamps are placed 15 ft. apart and 12 ft. above the plane the illumination along a line beneath the two lamps will be approximately uniform as indicated by curve I_2 of Fig. 160.

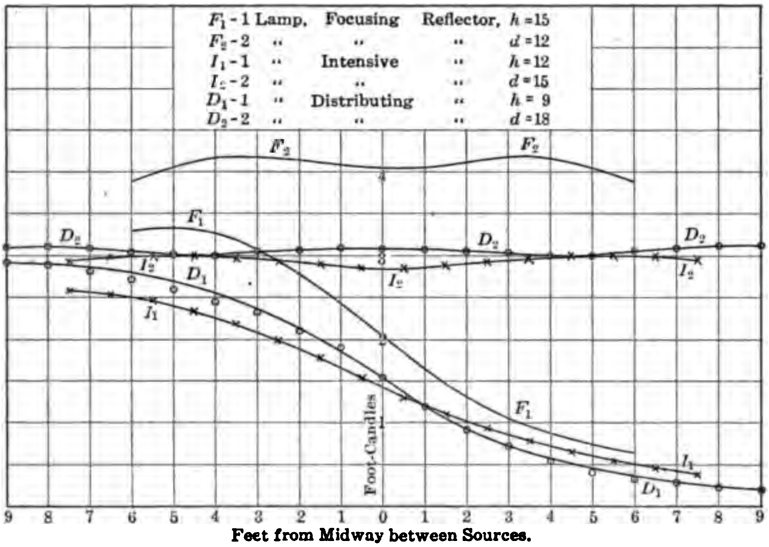

FIG. 160.—Illumination due to one and two 250-watt Tungsten lamps equipped with intensive, extensive and focusing types of reflectors.

With a distance of 15 ft. between lamps, 12 ft. is the minimum height of suspension which will secure approximately uniform illumination. The two lamps can be placed at greater heights with the same spacing without impairing the uniformity of illumination, although the intensity of illumination will be lower.

To investigate the effect of the height of suspension of lamps above the plane illuminated the writer has assumed a room 75 by 75 ft. in size and lighted by the equipment just described. The lamps are placed, according to common practice, at the centers of equal areas each 15 ft. square in this case, as shown in Fig. 161. The reflection from walls and ceiling is neglected, and the results are calculated from the polar curves of Fig. 158, by the point-by-point method employing the well-known equation:

$$E_h = \frac{I_a \cos^3 a}{h^2} \tag{77}$$

where E_h is the intensity of illumination on the reference plane.
I_a, the candle-power a degrees from the vertical.
h, the height of the lamps above the reference plane.

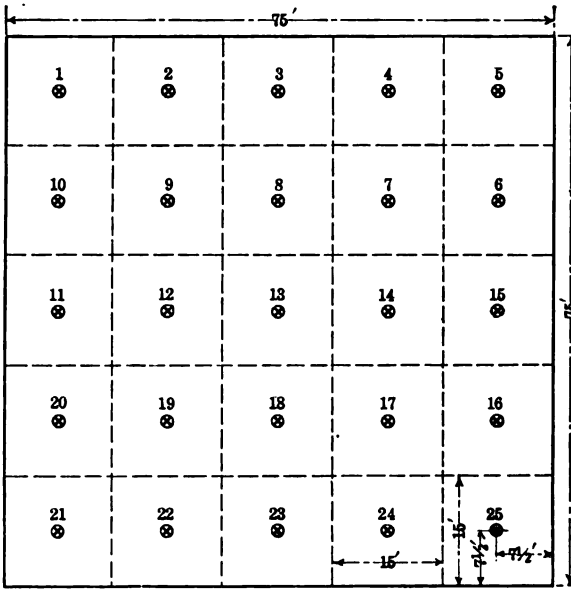

FIG. 161.—Location of 25 lamps in room 75 ft. square.

The illumination at the center of the room was determined for: first, due to one lamp (No. 13), which may be considered a point source also; second, due to a row of lamps (No. 11 to No. 15, inclusive); and third, due to all of the 25 lamps.

The results so calculated with the lamps at different heights are represented graphically in Fig. 162, together with those for the theoretical infinite linear and surface sources. The minimum height is 8 ft., the least height for uniform illumination with a spacing of 10 ft. The maximum height is taken as 30 ft. The uniformity of illumination is approximately the same for all heights from 12 to 30 ft. In this figure the heights at which the around the sides of the room, and where the fixtures or reflectors are not visible.

The former was made practical by the development of the tungsten lamp, although inverted arc lamps giving pure indirect lighting had been used in Europe (especially Germany and England) for some time. The latter system may employ any of the existing types of lamps. All things being equal it is obvious that the former method is the more efficient and it is more extensively used.

In order to use this system, the first requisite is a light colored ceiling. Pure white is the most efficient reflector and the nearer white the color of the ceiling, the more efficient will it be as a reflector. For decorative reasons it is often desirable to introduce some tint to change the color from a pure white. The most efficient tints for this purpose are. cream or light pink. Reds, browns, blues and greens are very inefficient. Even light gray, which is a mixture of black and white, is a poor reflector.

Next after the color of the ceiling comes the question of its surface and contour. For the cove or recess system an arch or dome-shaped ceiling is preferable. The light from the lamps is then thrown from the coves onto these concave surfaces and a better illumination is obtained on the reference plane than if the ceiling be flat.

The most universally applicable indirect system is the one where inverted reflectors in pendent fixtures are placed above the centers of the areas to be lighted. The engineering features of this system have been very clearly set forth by Mr. Cravath.[1] This system can be installed in small rooms where the ceiling outlet is located in the center of the ceiling, or in large rooms in the center of bays, the outlets in general being located as for direct lighting. In general the best conditions for this system are the ordinary flat ceilings with a smooth surface. Corrugations in the ceiling tend to lower the efficiency of the system, as do also arches and beams in some cases.

Fig. 163 shows a typical case of indirect lighting from a fixture hung in the center of a small room with a flat ceiling. It represents a cross-section of such a room. An inverted reflector with a lamp in it, located at *A*, throws a cone of light to the ceiling. Most of the flux of light strikes the ceiling between the points *B* and *C*. From *B* and *C* to the walls there should be a gradual difference between curves *d* and *e*. The same line of reasoning will apply to some extent to the other cases as well. Moreover, a little thought will make it clear that as the lamps are raised the rays passing to the walls and ceiling strike those surfaces at angles which are more effective in throwing this light onto the working plane.

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[1] *Elect. World,* Vol. 57, p. 1172, May, 1911.

Many mistakes are continually being made in the lighting of large rooms by locating the lamps too low. There appears to be an insistency among those "who know not, but think they know," that the proper height at which to locate the lamps of an installation is 10 or 12 ft. above the floor, irrespective of the size of the room, height of ceiling, class of service, or condition of the surroundings. In a large room with the lamps placed at this height the sources of light are but little above the line of vision, and shine into the eyes of the individual when observing objects around the room. In this way the pupil of the eye is contracted and the illumination is less effective and less satisfactory than if its intensity be perhaps several per cent. less but with the lamps placed higher and out of the range of vision.

The Semi-Direct Lighting System.—The semi-direct system is a combination of the direct and indirect systems as the name indicates. It may employ some of the translucent reflectors inverted so that the greater part of the light is thrown to the ceiling while more or less passes directly through the reflectors to the reference plane, or translucent reflectors of special design, and possessing artistic and ornamental features, which produce the same results as the inverted reflectors.

These may be of either prismatic or opal glass. The efficiency of this system depends upon the per cent. of the light from the source which passes directly through the reflector to the reference plane. With light opal reflectors the efficiency may be as high as those of the bare lamp given in Table 35, while on the other hand if the reflector be surrounded by a translucent holder for æsthetic purposes the per cent. of the light flux effective may be less than for the indirect system under the same conditions of ceiling and surroundings.

The Indirect Lighting System.—The indirect systems may be sub-divided into

(*a*) Those where the source of light is located in fixtures supported or suspended in different parts of the room.

(*b*) Those where the lamps are placed in coves or recesses around the sides of the room, and where the fixtures or reflectors are not visible.

The former was made practical by the development of the tungsten lamp, although inverted arc lamps giving pure indirect lighting had been used in Europe (especially Germany and England) for some time. The latter system may employ any of the existing types of lamps. All things being equal it is obvious that the former method is the more efficient and it is more extensively used.

In order to use this system, the first requisite is a light colored ceiling. Pure white is the most efficient reflector and the nearer white the color of the ceiling, the more efficient will it be as a reflector. For decorative reasons it is often desirable to introduce some tint to change the color from a pure white. The most efficient tints for this purpose are cream or light pink. Reds, browns, blues and greens are very inefficient. Even light gray, which is a mixture of black and white, is a poor reflector.

Next after the color of the ceiling comes the question of its surface and contour. For the cove or recess system an arch or dome-shaped ceiling is preferable. The light from the lamps is then thrown from the coves onto these concave surfaces and a better illumination is obtained on the reference plane than if the ceiling be flat.

The most universally applicable indirect system is the one where inverted reflectors in pendent fixtures are placed above the centers of the areas to be lighted. The engineering features of this system have been very clearly set forth by Mr. Cravath.[1] This system can be installed in small rooms where the ceiling outlet is located in the center of the ceiling, or in large rooms in the center of bays, the outlets in general being located as for direct lighting. In general the best conditions for this system are the ordinary flat ceilings with a smooth surface. Corrugations in the ceiling tend to lower the efficiency of the system, as do also arches and beams in some cases.

Fig. 163 shows a typical case of indirect lighting from a fixture hung in the center of a small room with a flat ceiling. It represents a cross-section of such a room. An inverted reflector with a lamp in it, located at A, throws a cone of light to the ceiling. Most of the flux of light strikes the ceiling between the points B and C. From B and C to the walls there should be a gradual tapering off of the illumination on the ceiling if a pleasant effect is to be produced. The reason for confining most of the flux of light to the center of the ceiling is to secure the best efficiency possible. For example, the ray AX in Fig. 163, if reflected from the ceiling at the same angle as its incidence, will take the path XZ and reach the working plane in the room directly from the ceiling with only one reflection. On the other hand, rays given off from the lamp and reflector at angles more oblique than the angle of the ray AB would strike the ceiling outside of the area between B and C, and if regularly reflected—that is, reflected at the same angle as that of incidence—would strike the wall and would necessarily undergo another reflection before reaching the working plane.

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[1] *Elect. World,* Vol. 57, p. 1172, May, 1911.

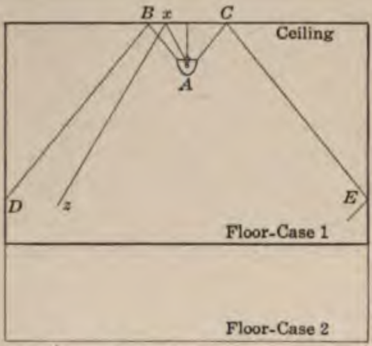

FIG. 163.—Section of room.

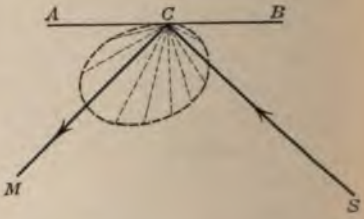

FIG. 164.—Diffuse and regular reflection.

In these two cases just noted the assumption was made that the ceiling gives "regular" reflection like a mirror, with the angle of incidence equal to the angle of reflection. This assumption is necessarily made for the purpose of arriving approximately at the correct principles to follow in the design of indirect illumination. As a matter of fact, however, it is known that every calcimined or painted ceiling is not a regular reflector, but an irregular or diffuse reflector. Such ceilings are slightly rough. Each minute portion of the ceiling must be considered as a reflector by itself, and the minute portions are at an infinite number of different angles with reference to the general plane of the ceiling.

The effect of diffuse reflection is shown in Fig. 164, where AB represents the plane of the ceiling and SC a beam of light striking the ceiling. If the ceiling were a regular or plane reflector the beam would be reflected from the ceiling at the same angle as the angle of incidence and would take the path *CM*. As a matter of fact it is diffused in various directions as indicated by the dotted lines. The subject of diffuse reflection has been investigated by Mr. Gilpin,[1] from whose work this curve and others of a similar nature shown in Chapter I, were obtained. From these curves it will be seen that while the ceiling may be far from a regular reflector, still the intensity of the reflected light is a maximum at the angle equal to the angle of incidence. Hence in the theory of indirect illumination design it is proper to consider primarily the regular reflection from the ceiling.

The absence of shadow beneath the fixture is obviously due to the diffusing properties of the ceiling. If the ceiling were a perfectly plain mirror it is evident that there would be such a shadow. However, from an inspection of Fig. 164 it is seen that it is impossible for such a shadow to exist, because of the diffuse reflection of the light from all parts of the ceiling surrounding the fixture outlet. As a matter of fact the lightest place in the room is the point directly underneath the fixture, except in rare cases, where a very large bowl type of fixture is placed too close to the ceiling.

In the lighting of small rooms, say from 10 to 20 ft. square, the common practice for the majority of conditions is to locate an outlet at the center of the ceiling. When planning the lighting of large interiors the space to be lighted should if possible be divided up into squares with an outlet at the center of each square, or rectangle, just as would be the case with direct lighting. When planning direct lighting each lamp is assumed to throw a cone of useful light which will cover a certain area or circle. For example, the room which is shown in cross-section in Fig. 156 has two direct-lighting units. The cone of light from each covers a certain area. If the reflectors are translucent considerable light will be given off outside of the cones indicated. The object of the designer of efficient direct illumination, however, is to confine a considerable portion of the flux of light within a certain cone, and to locate the outlets so that the cones will cover the area to be lighted. In practice in a large interior the cones of light overlap each other and, in fact, considerable overlapping is always desirable on account of the reduction of objectionable shadows.

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 854, 1910.

In the planning of indirect-lighting installations one must deal also with the fact that properly designed indirect-lighting units give cones of light just as do direct-lighting units. Thus in Fig. 163 *BCED* represents the lower part of a cone of light obtained by reflection from the cone *ABC*. The diameter of the base of the cone in Fig. 163 can be increased by lowering the fixture *A* or decreased by raising the fixture nearer the ceiling. To be sure, there is no well-defined cone in either the direct or indirect installations referred to, but there are certain limits within which one must work if good results are desired.

Referring again to Fig. 163, the cone of light of which *DE* is the base is just sufficient to cover the working plane with the floor located as in case 1, indicated by the solid line. If the height of the room be so increased that the floor is located as in case 2, Fig. 163, indicated by the dotted lines, and if the fixture be left in the same position relative to the ceiling, it is evident that much more light will be directed to the walls than in case 1. The flux of light striking the wall represents a certain loss, which may be considerably greater than in case 1. Thus it is impossible to light high rooms as efficiently as low rooms by indirect lighting. In case 2, Fig. 163, better results could be obtained by using a more concentrating reflector, or raising the reflector *A* nearer the ceiling, so as to reduce the diameter of the base of the cone of light that less may strike the wall.

On account of the diffuse reflection from ceiling and walls it is impossible to control the flux of light very accurately with indirect lighting, and in high narrow rooms the efficiency must necessarily be lower than in low wide rooms. The same thing is true with direct lighting, but the light can be more nearly controlled when one is dealing with the reflection from reflectors over lamps than when dealing with diffuse reflection from ceilings.

In a small room better results will be obtained by using a concentrating reflector, while in large rooms the distributing type of reflector will give better satisfaction. The difference due to placing the reflectors at various distances from the ceiling is so small that the general appearance of the installation due to the position of the fixtures and the distribution of the light on the ceiling carry more weight than the difference in light distribution on the working plane due to the several heights of fixture suspensions.

In Fig. 165 is given the polar candle-power curve from a type of reflector designed for indirect lighting of this character and classed as of the distributing type. With this reflector the lamp filament is about even with the top of the reflector and considerable light is given off to light the far corners of small rooms, in which it may be hung in the center. In Fig. 166 is given the polar candle-power curve of a reflector for similar uses, classed as of the concentrating type. The distribution of light obtained on the working plane with the reflector shown in Fig. 165, under given conditions, is shown in Fig. 167. This is a curve showing the foot-candle illumination along a radial line, beginning at a point directly underneath the reflector. The test was made in a very large room with a ceiling 13.5 ft. high and the reflector 36 in. from the ceiling. Reflection from surrounding walls, etc., enters into the curve, Fig. 167, to some extent. For the lighting of a large general office with reflectors of this kind, at the ceiling height mentioned, remarkably uniform illumination can manifestly be obtained by placing lamps at outlets 20 ft. apart.

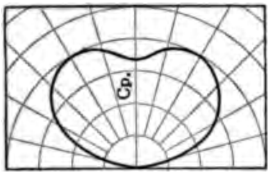

Fig. 165.—Relative distribution of light from a distributing-type indirect reflector.

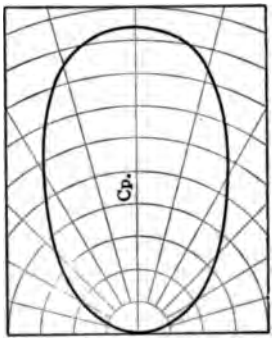

Fig. 166.—Relative distribution of light from a concentrating-type reflector for indirect lighting.

With types of reflectors the candle-power distribution of which is illustrated in Figs. 165 and 166 the lamp filament is placed vertically within the reflector. It has not been found feasible to design reflectors which will permit spacing of outlets

In the planning of indirect-lighting installations one must deal also with the fact that properly designed indirect-lighting units give cones of light just as do direct-lighting units. Thus in Fig. 163 *BCED* represents the lower part of a cone of light obtained by reflection from the cone *ABC*. The diameter of the base of the cone in Fig. 163 can be increased by lowering the fixture *A* or decreased by raising the fixture nearer the ceiling. To be sure, there is no well-defined cone in either the direct or indirect installations referred to, but there are certain limits within which one must work if good results are desired.

Referring again to Fig. 163, the cone of light of which *DE* is the base is just sufficient to cover the working plane with the floor located as in case 1, indicated by the solid line. If the height of the room be so increased that the floor is located as in case 2, Fig. 163, indicated by the dotted lines, and if the fixture be left in the same position relative to the ceiling, it is evident that much more light will be directed to the walls than in case 1. The flux of light striking the wall represents a certain loss, which may be considerably greater than in case 1. Thus it is impossible to light high rooms as efficiently as low rooms by indirect lighting. In case 2, Fig. 163, better results could be obtained by using a more concentrating reflector, or raising the reflector *A* nearer the ceiling, so as to reduce the diameter of the base of the cone of light that less may strike the wall.

On account of the diffuse reflection from ceiling and walls it is impossible to control the flux of light very accurately with indirect lighting, and in high narrow rooms the efficiency must necessarily be lower than in low wide rooms. The same thing is true with direct lighting, but the light can be more nearly controlled when one is dealing with the reflection from reflectors over lamps than when dealing with diffuse reflection from ceilings.

In a small room better results will be obtained by using a concentrating reflector, while in large rooms the distributing type of reflector will give better satisfaction. The difference due to placing the reflectors at various distances from the ceiling is so small that the general appearance of the installation due to the position of the fixtures and the distribution of the light on the ceiling carry more weight than the difference in light distribution on the working plane due to the several heights of fixture suspensions.

In Fig. 165 is given the polar candle-power curve from a type of reflector designed for indirect lighting of this character and classed as of the distributing type. With this reflector the lamp filament is about even with the top of the reflector and considerable light is given off to light the far corners of small rooms, in which it may be hung in the center. In Fig. 166 is given the polar candle-power curve of a reflector for similar uses, classed as of the concentrating type. The distribution of light obtained on the working plane with the reflector shown in Fig. 165, under given conditions, is shown in Fig. 167. This is a curve showing the foot-candle illumination along a radial line, beginning at a point directly underneath the reflector. The test was made in a very large room with a ceiling 13.5 ft. high and the reflector 36 in. from the ceiling. Reflection from surrounding walls, etc., enters into the curve, Fig. 167, to some extent. For the lighting of a large general office with reflectors of this kind, at the ceiling height mentioned, remarkably uniform illumination can manifestly be obtained by placing lamps at outlets 20 ft. apart.

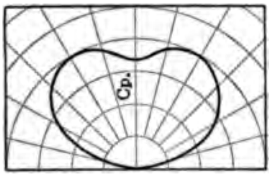

FIG. 165.—Relative distribution of light from a distributing-type indirect reflector.

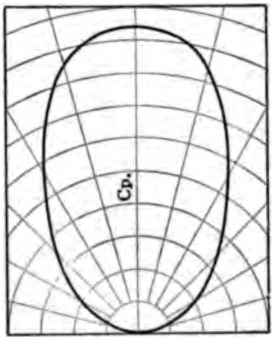

FIG. 166.—Relative distribution of light from a concentrating-type reflector for indirect lighting.

With types of reflectors the candle-power distribution of which is illustrated in Figs. 165 and 166 the lamp filament is placed vertically within the reflector. It has not been found feasible to design reflectors which will permit spacing of outlets more than 20 ft. apart when very uniform illumination is desired in a large area with ceilings 14 ft. high, although very good results would be obtained with 24 ft. spacing. This is because of the limitations of the cone of light given from a reflector or group of reflectors at a single outlet. For example, the portion of a cone indicated by *DBCE* in Fig. 163 represents what might be called the cone of useful light from a reflector of the type of distribution shown in Fig. 165. The cone for Fig. 166 would have a narrower angle. The area of the base of this cone, as before explained, will be dependent on the distance the reflector is hung from the ceiling and on the ceiling height. In order to serve as a rough guide in determining distances of reflectors from ceiling and probable wall losses in various sizes of room it is well to prepare for each type of reflector used a piece of tracing cloth or transparent celluloid having the angle *ACE* marked upon it. A cross-section of the room should be drawn, for example, as in Fig. 163. Then by laying the angle *ACE* marked on tracing cloth over the cross-section of the room the amount of area that can be covered and the suspension height of the fixture can be determined at once. As before stated, however, considerable latitude can be allowed in fixture height. It is, however, manifestly uneconomical to hang a fixture with a distributing type of reflector very low in a high narrow room.

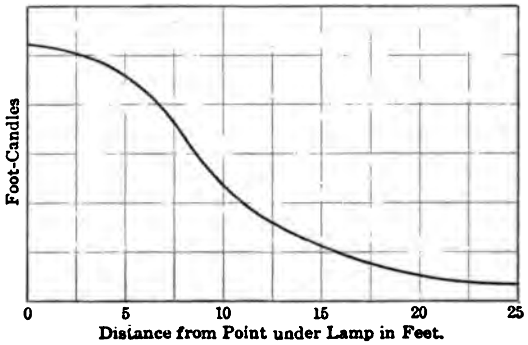

FIG. 167.—Illumination from one lamp (reflector shown in Fig. 165).

In the lighting of large interiors with flat ceilings with indirect illumination of this kind a safe general rule is that outlets can be spaced in the centers of 10 ft. to 24 ft. squares if the ceilings are 10 ft. to 14 ft. from the reference plane. With very high ceilings the number of outlets can be considerably reduced. It is simply a question of drawing a cross-section of the room to scale and applying the critical-angle diagram spoken of to determine what the safe procedure may be.

Reflectors, Shades and Glassware.—The reflecting and diffusing equipment of an installation plays an important part in interior illumination. Of the materials used for reflectors and diffusing envelops four classes stand out as the most generally efficient. These are known as silvered glass or mirror reflector, prismatic glassware, opal glassware, and enamel or aluminized metal.

The *silvered glass* reflector is of high efficiency and makes the indirect system a commercial success. It is also well adapted to show window lighting by the direct system where the lamps can be placed very high.

Prismatic glassware has had an extensive use for reflectors and diffusing globes and shades. It offers a number of scientifically designed distributions for different classes of service and is generally efficient. It is much used for the direct systems, and for the semi-direct system either surrounded by an art glass container or having its inner surface finely etched by the acid paste process. This class of reflectors may also be used in opaque fixtures for the indirect system.

Opal glassware, of which the Alba glass is a worthy example, is becoming very popular for both direct and semi-direct lighting. Its diffusing properties are excellent and its efficiency high.

The metal reflectors having an interior finish of either white enamel or aluminum are well suited for rough service where the use of glassware is prohibited.

In addition to those described above there are many other shades, globes and reflectors of various designs. Some of colored or art glass and of artistic design are excellent for giving a subdued well diffused light. Others are scientifically wrong and in direct opposition to any principles of good practice.

The relative efficiencies of some of the preceding reflectors and the relative percentages of light in the 0–60 degree and the 0–90 degree zones as determined by Sweet and Doane are given in the following table. The 0–180 degree flux from the the bare lamp is taken as 100 per cent. It will be seen that the difference between the light in the 0–60 degree and the 0–90 degree zones will be the per cent. in the 60–90 degree zone. Also the difference between the 0–90 degree and the 0–180 degree will give the percentage flux above the horizontal as the unit hangs suspended.

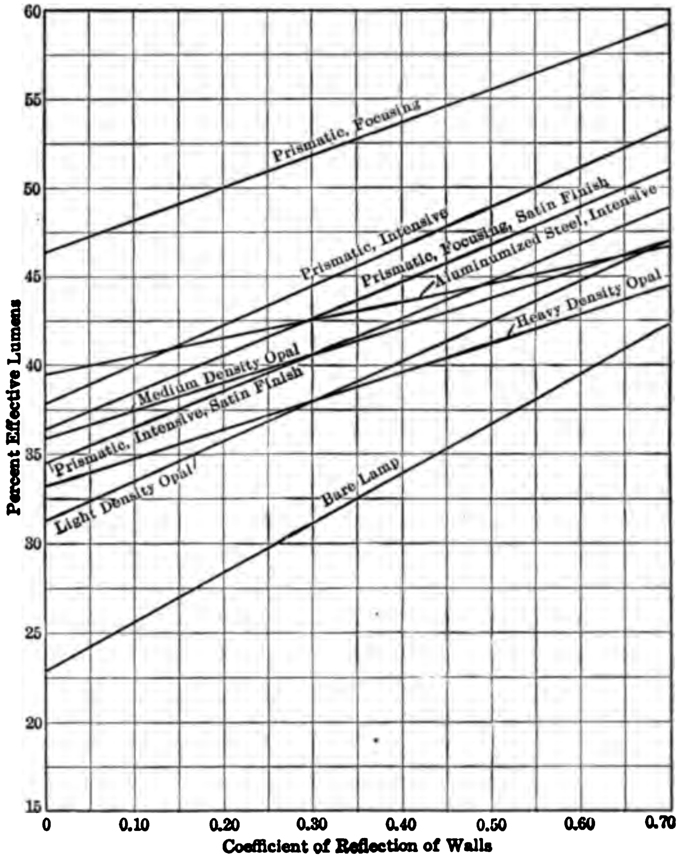

FIG. 168.—Relative illuminating efficiencies for various types of reflectors with light colored (60 per cent. coefficient) ceiling. Small or medium sized room with the distributed unit type of installation.

TABLE 42[1]

The Relative Efficiencies of Various Units in the Various Indicated Zones. The 0–180 degree flux given by the bare lamp is taken as 100 per cent.

Type of lighting unit	Per cent. of the total bare lamp flux in indicated zone		
	0–60 degree	0–90 degree	0–180 degree
	Per cent.	Per cent.	Per cent.
Prismatic focusing	50	68	87
Prismatic intensive	44	65	87
Prismatic focusing, satin finish	40	60	87
Aluminumized steel, intensive	51	58	58
Aluminumized steel, extensive	52	61	61
Prismatic intensive, satin finish	37	57	87
Medium density opal, high glaze	40	54	82
Prismatic extensive	40	62	86
Heavy density opal, depolished	41	59	72
Light density opal	34	57	91
Prismatic extensive, satin finish	33	55	88
Bare lamp	16	47	100
Indirect, mirrored reflector	0	0	82
Semi-direct (8–92 per cent.)	3	5	74

Effect of Reflecting Power of Ceiling and Walls Upon Illumination.—In a valuable series of investigation by Messrs. Sweet and Doane (Ref. Cit.) data were secured for the relative illuminating values of different classes and types of reflectors in small and medium rooms with walls and ceiling having different reflecting coefficient values. The room was rectangular, 13 ft. 6 in. by 23 ft. 6 in. and had a ceiling height of 10 ft. 4 in.

Two types of installation were used and designated as the distributed unit type and the center of the room type. In the former installations six 40-watt tungsten lamps were used giving a total of 1914 lumens. There were six outlets distributed over the ceiling and one lamp at each outlet. In the center of room installation there were two outlets located near the centers of two bays one of which was about two-thirds the size of the other. A 100-watt and a 150-watt tungsten were used in these bays, respectively, and gave a total of 2075 lumens. All the units except the indirect were mounted at a height of 8 ft. 6 in. from the floor and the reference plane at which illumination measurements were made with a Sharp-Millar photometer was 30 in. from the floor.

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[1] Paper read before the Chicago Section of the Illuminating Engineering Society, Feb., 1912.

Obviously the indirect and semi-direct units were not suitable for the distributed system, neither was the focusing type of reflector suitable for the center outlet system. Hence these combinations were omitted.

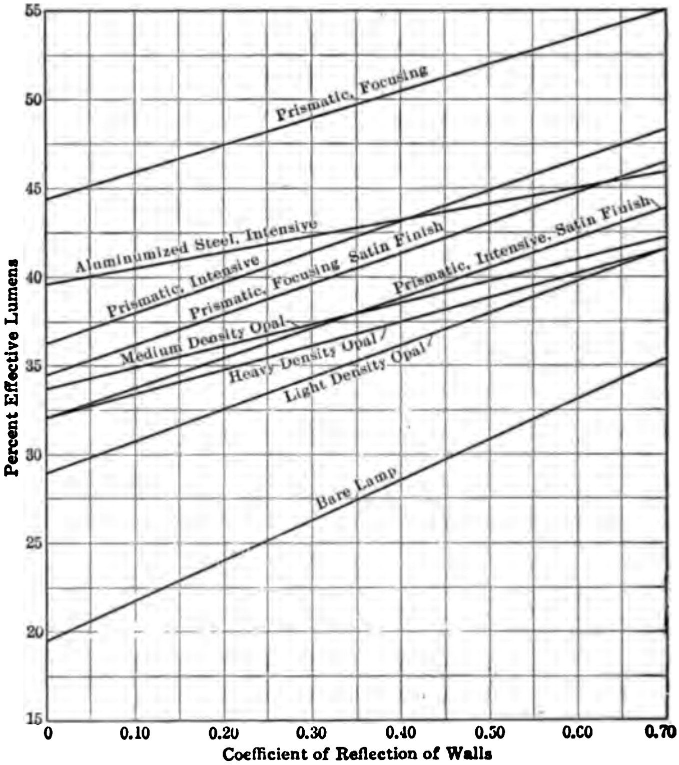

FIG. 169.—Relative illuminating efficiencies for various types of reflectors, with medium colored (40 per cent. coefficient) ceiling. Small or medium sized rooms with the distributed unit type of installation.

Two interiors were used. For the light interior the ceiling and walls were covered with white mat cardboard such as is regularly used for the mat in picture-framing. It had a glazeless surface slightly pebbly in character and had a reflection coefficient of 0.73. For the dark ceiling and walls black canton flannel cloth was used, having a coefficient of diffuse reflection of 0.023.

Tests were made with:

(A) White ceiling and white walls.
(B) Black ceiling and white walls.
(C) White ceiling and black walls.
(D) Black ceiling and black walls.
The floor was of a light oak color.

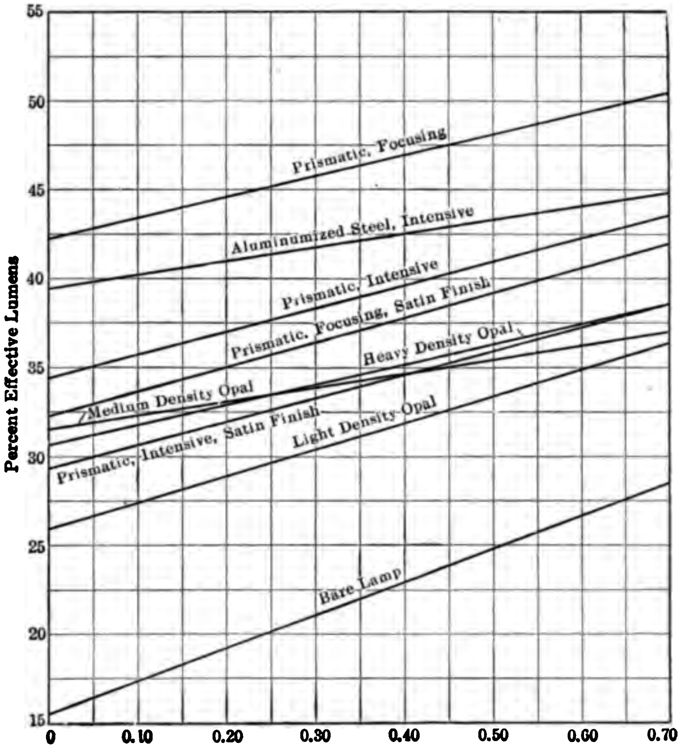

FIG. 170.—Relative illuminating efficiencies for various types of reflectors, with dark colored (20 per cent. coefficient) ceiling. Small or medium sized rooms with the distributed unit type installation.

From this series of four tests the formula constants given in Table 44 were computed and by using these constants and Equation 193 the curves of Figs. 168 to 173, inclusive, were obtained by computation.

The comparatively low influence of color of walls upon illumination efficiency is worthy of very especial note. The influence of color of walls upon illumination efficiency is very generally over-estimated by illuminating engineers. The average wall which would be classed as very light color will have, ater some service, a reflection coefficient of roughly about 40 per cent. The darkest walls which can be employed without involving gloomy or depressing effects will have about a 10 per cent. coefficient of reflection, or a little less.

Table 43 gives for each of the various types of unit, the per cent. of loss in illumination intensity involved in changing from 40 per cent. walls to 10 per cent. walls, the ceiling coefficient being taken in both cases as 60 per cent.

TABLE 43

Loss of Light due to Dark Walls

Type of unit	Per cent. loss distributed unit arrangement	Per cent. loss center of room arrangement
	Per cent.	Per cent.
Prismatic focusing	10	
Prismatic intensive	14	11
Prismatic focusing, satin finish	14	
Aluminumized metal, intensive	7	
Aluminumized metal, extensive	....	5
Prismatic intensive, satin finish	15	12
Medium density opal, high glaze	11	9
Prismatic extensive	....	10
Heavy density opal, depolished	12	8
Light density opal	17	10
Prismatic extensive, satin finish	....	13
Bare lamp	24	20
Indirect, mirrored reflector	....	18
Semi-direct (8–92 per cent.)	....	15
Average, omitting bare lamp	13	11

In this connection, attention is called to the relatively low value of constant K_2 in table 44. This indicates that such light as strikes the walls is generally incident at such an angle as not to be efficiently reflected to the plane of utilization.

It should be noted that all the data here given are for initial conditions with perfectly clean reflectors. The service average of illumination efficiency values will be lower for all types of unit. Reliable data as to the rapidity of deterioration which characterizes the different types of unit are entirely lacking. We can say, however, that the indirect and semi-direct units will drop off in illumination efficiency, on account of dirt deposit, much more rapidly than any of the direct units. Comparisons in illumination efficiency derived from Figs. 171, 172 and 173 are therefore unduly favorable to the indirect and semi-direct systems. On the other hand, it should be said that, if one system of lighting has any appreciable advantage over another, it will almost invariably be decidedly to the light users' interest to adopt such better system, even though more maintenance cost is involved to keep it in clean condition. The dirt argument is, nine cases out of ten, an appeal to the ignorance and prejudice of the man to whom it is addressed. It is the firm belief of Messrs. Sweet and Doane that the ultimate field and extent of use

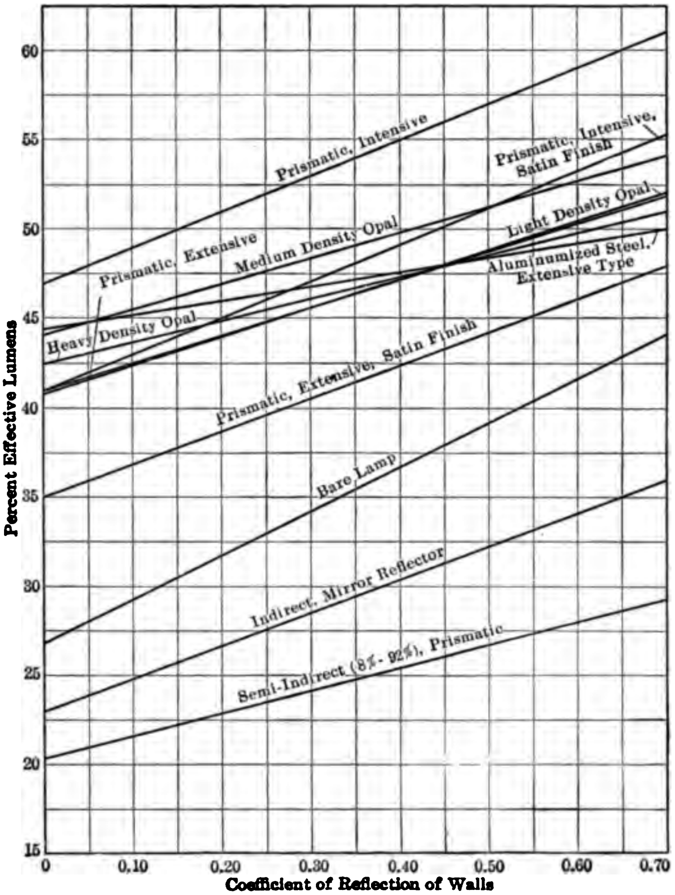

FIG. 171.—Relative illuminating efficiencies for various types of reflectors, with light colored (60 per cent. coefficient) ceiling. Small or medium sized rooms with the center of room type of installation.

of the indirect system will be little affected by the fact that it requires a larger amount of maintenance attention than direct systems.

The attempt was made to derive a formula which would directly give the effective lumens for any type of unit under any reflection value of walls and ceiling, by substituting the correct reflection coefficients and the flux value derived from the photometric curve. The formula which it was attempted to evaluate had the following form:

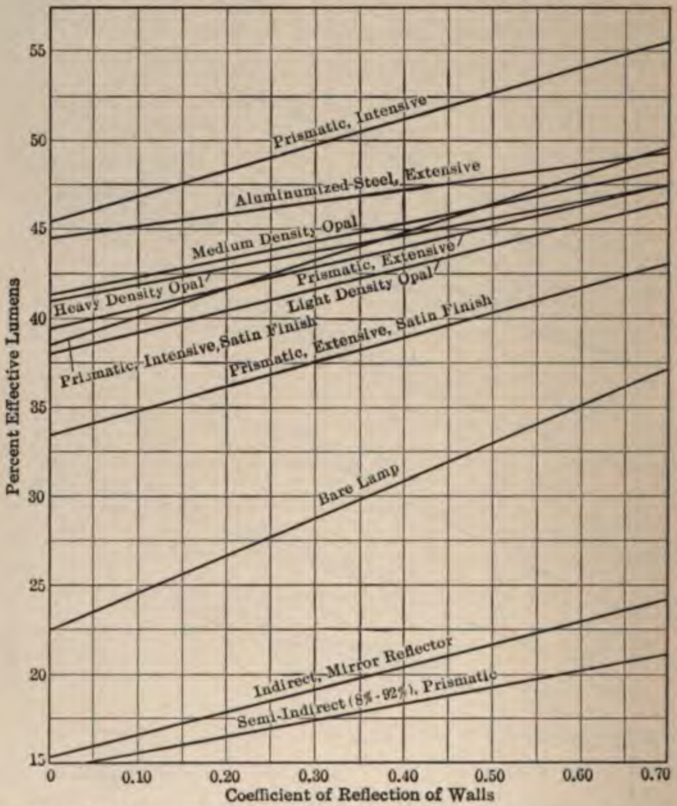

FIG. 172.—Relative illuminating efficiencies for various types of reflectors, with medium colored (40 per cent. coefficient) ceiling. Small or medium sized rooms with center of room type installation.

$$\text{Illumination} = \frac{N(Fo^\circ - x^\circ + K_2CwFx^\circ - 100^\circ + K_3CcF100^\circ - 180^\circ + K_4CwCcFx^\circ - 180^\circ)}{A} \qquad (193)$$

In which N equals number of units.

X equals unknown angle, to be solved for.

$Fo^\circ\text{–}x^\circ$ equals flux 0° to X°.

$Fx^\circ\text{–}100^\circ$ equals flux X° to 100°.

$F100^\circ\text{–}180^\circ$ equals flux 100° to 180°.

$Fx^\circ\text{–}180^\circ$ equals flux X° to 180°.

Cw equals coefficient of reflection of walls.

Cc equals coefficient of reflection of ceiling.

K_2, K_3, and K_4 are constants to be solved for.

Solving the above formula for each type of unit, the following values were obtained:

Table 44

Constants for Equation 193.

Type of unit	Distributed unit arrangement				Center of room arrangement			
	X	K_2	K_3	K_4	X	K_2	K_3	K_4
Prismatic focusing	48½°	.290	.637	.333	....	....	....	....
Prismatic intensive	47½°	.236	.543	.398	58°	.110	.463	.620
Prismatic focusing, satin finish	48½°	.279	.506	.310	....	....	....	....
Aluminumized metal, intensive	48°	.327	.000	.421	....	....	....	....
Aluminumized metal, extensive	....	....	....	....	53½°	.282	.000	.326
Prismatic intensive, satin finish	49½°	.267	.561	.317	56½°	.239	.560	.430
Medium density opal, high glaze	47½°	.139	.461	.441	57°	.082	.550	.409
Prismatic extensive	....	....	....	....	56½°	.115	.472	.381
Heavy density opal, depolished	49°	.312	.590	.338	55½°	.161	.513	.342
Light density opal	49½°	.281	.562	.292	57°	.130	.528	.351
Prismatic extensive, satin finish	....	....	....	....	56°	.160	.366	.371
Bare lamp	53½°	.294	.470	.274	56°	.342	.534	.220
Indirect, mirrored reflector	....	....	....	....	0°	1.000	.462	.345
Semi-indirect (8–92 per cent.)	....	....	....	....	64½°	.416	.440	.236

When an attempt was made to derive a single set of constants applying to all types of unit, it was found that no single set could be obtained which would give results of as great accuracy as is to be desired. A more thorough study of the original data of this research may yet make possible its expression through the medium of such a formula. It is doubtful, however, if the equation form would be more convenient than the graphical form as given in Figs. 168 to 173.

Decrease in Illumination due to the Accumulation of Dirt and to the Deterioration of the Source.—It is difficult to maintain the initial efficiency of any lighting installation. Dust and dirt will collect on the glassware, whether it be bulb, globe, shade or reflector, to a greater or less extent depending on the condition of the surface of the glassware, the cleanliness of the atmosphere where the lamps are installed, and the amount of cleaning which they receive. This factor is obviously a variable. It may reduce the initial illumination by as much as 30 per cent. under unfavorable conditions and where the units are neglected, while it may be negligible if the lighting equipment receives proper attention and cleaning.

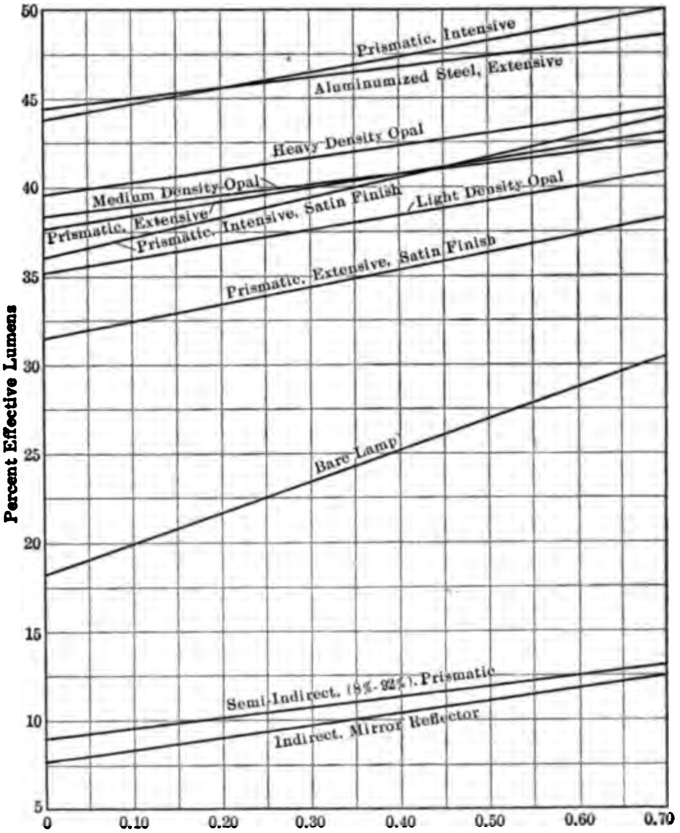

FIG. 173.—Relative illuminating efficiencies for various types of reflectors, with dark colored (20 per cent. coefficient) ceiling. Small or medium rooms with center of room type of installation.

The light source will depreciate and decrease in candle-power after a short period of its life. Incandescent lamps are rated so that their candle-power decreases about 20 per cent. in 800 or 1000 hours. A good grade of gas mantle should show about the same performance. The intensity of an arc lamp decreases during the life of a trimming due to the deposit on the globe.

From the foregoing it is obvious that the questions of dirt and depreciation demand attention in the calculation of an installation, and must be allowed for where conditions are unfavorable. On the other hand, because dirt collects more readily upon one type of unit than another or is more detrimental in one case than another does not necessarily condemn that unit since all lighting equipment should be cleaned occasionally and those making cleaning compulsory are those which will be cleaned the oftenest and are usually those which give the most effective illumination.

The Basis of Comparison of Illumination Installations and Visual Efficiency.—The reader is cautioned against comparing the direct, semi-direct and the indirect systems entirely on the basis of the amount of light received on the working plane. The questions of diffusion, distribution, low intrinsic brightness, absence of glare, and low intensity of walls are of utmost importance, and it is well known that these features are secured only by a loss of light. Moreover, the effectiveness of a system of illumination possessing the qualities mentioned above will be greater than that of another system lacking these qualities, although the intensity of illumination may be much greater in the latter case. Hence a fair and impartial comparison should consist in giving each of the desirable features of an ideal installation a weight comparable with the class of service for which the equipment is to be used.

In line with the foregoing are the researches of Mr. Cravath[1] and Messrs. Sweet and Doane.[3] Mr. Cravath's investigations show that avoidance of glare from glazed paper with resultant depression of visual function, depends upon deriving the illumination upon the paper from a large number of different directions. The relative intensities required for the same degree of visibility with four indirect units, four direct units, one indirect unit, one direct unit and one local unit are shown in Fig. 174. Thus it is shown that, for low intensities at least, illumination measurements without regard to the other features of the system may lead to erroneous conclusions.

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[1] Trans. Illum. Eng. Soc., Vol. 5, p. 782, 1911.

[3] Paper read before the Chicago Section of the Illuminating Engineering Society, Feb., 1912.

Messrs. Sweet and Doane, investigating the influence of reflectors and reflecting power of walls upon depression of the visual function, found that visual efficiency was greatest with dark walls and with the lighting system which cast least light upon the walls. The results of these investigations are shown in Figs. 175 and 176. In Fig. 175 are shown the visual efficiencies for four common types of lighting units with different colored walls, the incident wall illuminations being 0.802, 0.86, 1.19 and 1.49, as indicated on the curves. These results refer to effects on the subject when facing the wall. The average of these four curves

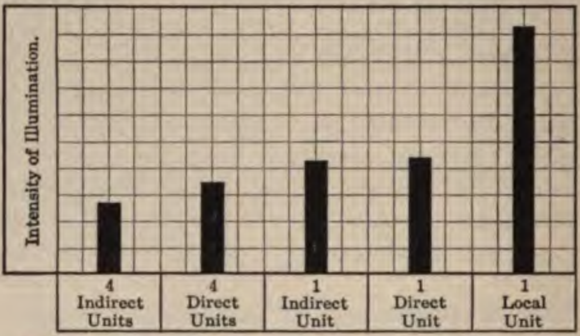

FIG. 174.—Relative average illumination intensities considered the minimum for comfortable reading.

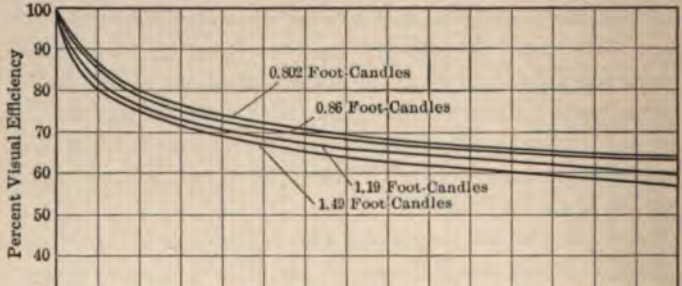

FIG. 175.—Typical curves showing relative influence on visual efficiency of types of lighting units giving different wall illuminations.

and also the average for the four systems with the subject facing the room are shown in Fig. 176.

Some of the conclusions drawn from the investigation of Messrs. Sweet and Doane are given below:

1. Except where aesthetic considerations outweigh the consideration of illumination efficiency, reflectors or shades which allow much light to pass through to the walls of a room should not be used in illuminating interiors having walls of a high coefficient of reflection, where long-continued, close visual work is to be performed.

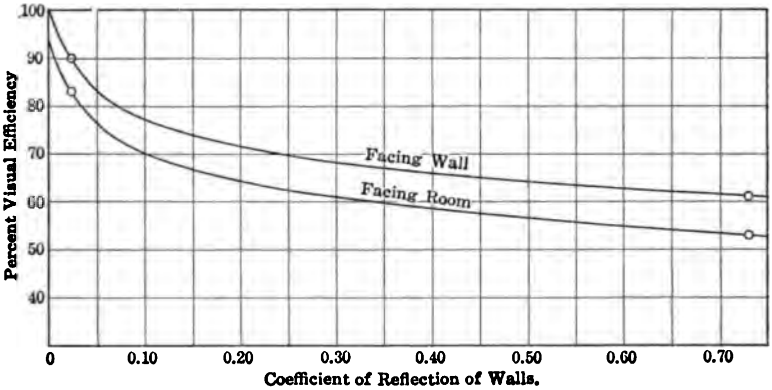

FIG. 176.—Variation in visual efficiency with coefficient of reflection of walls, average for four representative lighting systems.

2. Dark walls exercise much less influence in cutting down the efficiency of illumination than has been previously supposed.

3. Where wall coefficients are low, relative depression in visual function affords no highly important reason for choosing one type of reflector over another. On the whole, however, if long continued close visual work is to be performed under the lighting installation, it will be wiser to choose the type of unit giving minimum depression in visual function even when wall coefficients are low. If the wall coefficients are of high value, it is of very considerable importance to select units giving minimum depression in visual function, especially for service where long continued, close work is to be performed.

4. The relative influence of different types of lighting unit in causing depression in visual function may correctly be judged by the relative light flux within, approximately, the angles of

50 degrees with the vertical and 80 degrees with the vertical. The flux emitted by the light unit in the 60–70 degree zone is especially fruitful in causing depression in visual function. In this connection, it should be noted that, in the case of indirect lighting, the distribution curve of the reflected light from the ceiling should be taken as the distribution curve of the light unit. In the case of semi-direct lighting, the net curve of the ceiling reflection and of the direct light from the unit should be taken as the distribution curve of the unit.

5. The ideal type of lighting unit, from the joint standpoints of illumination efficiency and of visual efficiency, will be one which meets with the following specifications as to light distribution.

(a) 0–50 degrees: Sufficient flux in the 0–50 degree zone to give satisfactorily high illumination efficiency. The candlepower values from 30 to 50 degrees should be sufficiently high to give satisfactory uniformity of illumination under the spacing conditions for which the units are intended to be employed.

(b) 50–120 degrees: The minimum possible amount of light in the 50–120 degree zone.

(c) 120–180 degrees: The maximum amount of light in the 120–180 degree zone, except as limited by the requirements expressed above under (a).

6. Walls, up to the height normally coming within the field of vision, should have as low coefficients of reflection as is consistent with the avoidance of gloomy and depressing effects. In this connection, it will be noted that the loss in illumination efficiency, incident to the use of dark walls, is fully compensated for by the gain in visual efficiency.

It will be noted that conclusion 6 applies with maximum force primarily to interiors where close visual work is to be performed, such as offices and factories. It does not necessarily apply to every conceivable type of service. In corridor lighting, for instance, there is no reason why light walls should be avoided. No close visual work is performed in such corridors; nor is the eye long exposed to the illumination conditions. Indeed, light walls are to be recommended for corridor lighting, permitting as they do a brilliancy of illumination, partially real and partially apparent, at a comparatively low expenditure of energy.

CHAPTER XII

THE PRINCIPLES OF STREET ILLUMINATION

Exterior lighting pertains principally to the lighting of streets, with occasionally a public square or park. The fundamental principles are essentially the same for each.

In analyzing the principles which go to make good street illumination it seems logical to consider first those features of natural illumination at night when moonlight makes conditions almost ideal. The effectiveness of moonlight illumination may be attributed to three causes. First, and exerting by far the greatest influence is the uniformity of illumination; second, the moon is high in the sky out of the line of vision when moonlight is most effective; and third, the brightness of the source itself is low.

In order to approximate these conditions which make moonlight so effective, the artificial source of light must have a desirable distribution and be placed high above the street in order to give uniformity and be out of the range of vision. It should also be equipped with diffusing glassware to reduce the intrinsic brightness.

The moon was the only source of light for street illumination in the early settlements of man and at the present time some lighting companies follow moonlight schedules and extinguish the street lamps when the moon is visible and near full.

Street lighting received a great impetus with the introduction of the arc lamp in 1880 and Welsbach mantle about 15 years later. During the past 20 years there has been a great and rapid improvement in the efficiency and in the operating characteristics of street illuminants.

While much attention has been given to the development of the lamps and to the methods of energy generation and supply, little thought and rational study has been given to the location of the lamp in service and the distribution and effect of the light resulting therefrom.

An analysis of the principles of street illumination by Mr. Sweet[1] forms a classic on this phase of artificial lighting and is reproduced (slightly abstracted) on this and the following pages:

"There are two results to be achieved in adequate street illumination which are both of such paramount importance that it is properly a matter for individual opinion as to which should be ranked first. These are the avoidance of glare effect and the obtaining of an approximately uniform degree of illumination at all points along the course of the street, with higher intensities at street corners. These two results can only be obtained, the one by the correct solution of the allied problems of intrinsic brilliancy, total light flux, and shadow contrasts, the other by the correct solution of the problem of distribution.

"Let us separately take up in preliminary fashion each of the results sought and define it a little more exactly.

"**Glare.**—Glare effect results whenever, throughout any infinitesimal period of time, an amount of light continues to fall upon the actively visualizing portion of the retina of the eye sufficient in quantity to cause chemical changes more rapidly than the regenerative functions of the eye can keep pace with. Thus when an eye, which has been exposed for some time to conditions of dark with resultant pupillary expansion, is suddenly exposed to even a moderate light, glare effect results. An excessive amount of light reaches the retina through the expanded pupil, and the regenerative functions cannot keep pace with the chemical changes induced. Rapidly, however, the pupil contracts, the regenerative functions regain their normal control, and the glare effect passes away. If now the eye be exposed to a still greater intensity of light, beyond the very limited capacity of pupillary contraction to compensate for, or if an even moderately bright light source is introduced near the center of the field of vision, glare effect is again experienced; and this glare effect, after a brief period, reaches a certain state of equilibrium characterized by a quite definite decrease in efficiency of vision, which decrease thereafter increases in magnitude only rather slowly. We may, therefore, for convenience designate a glare condition as casual, when it results from the inability of the pupil to adjust itself instantly to a changed condition of light intensity, and as fundamental when it results from a light intensity exceeding the contractive power of the pupil to compensate for, or when it results from a light source near the center of the field of vision.

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[1] *Jour. Franklin Institute*, Vol., 169, p. 359, May, 1910.

"The presence of strong shadow contrasts tends to produce a casual condition of glare. The evil effect of the shadow contrast lies in the demand it introduces for instantaneous pupillary changes of considerable magnitude when the eye is turned from the shaded to the unshaded area, or *vice versa.* In street lighting, this casual condition of glare is much less objectionable to the slowly moving pedestrian than to the more rapidly moving automobilist or carriage driver. It is usually sufficient, therefore, to throw the shadows toward the sidewalk and to provide against too deep shadows. The avoidance of objectionably located shadow contrasts can only be taken care of by the proper placement of the light sources, while the intensity of the shadow contrast can be decreased by using smaller light sources placed nearer together. The problem of shadow contrasts, while of considerable importance, is one that must be solved individually for each individual case. It needs no further independent consideration in this general study of our subject.

"Brilliant, high candle-power light sources in the field of vision, the usual practice in contemporary street lighting, produce a fundamental condition of glare, with resultant heavy decrease in efficiency of vision—with probably also, be it noted, a permanently injurious effect on the faculty of vision. This fundamental condition of glare arises from the intrinsic brilliancy and total light flux of the light units employed and can only be avoided by the proper limitation of these. The study of the relations existing between intrinsic brilliancy and total light flux on the one hand, and glare effect on the other, becomes, therefore, one of the two chief purposes of an analysis of the illumination requirements of street lighting. The end sought should be the elimination of glare effect, or at least its reduction to such a permissible minimum as will not seriously decrease efficiency of vision nor be sufficient to produce eye discomfort.

"**Proper Distribution of Light and Illumination.**—The derivation of the polar curves of light distribution which will give the proper relative intensity of illumination for points along the course of the street is, as has already been noted, the second of the two chief purposes of an analysis of illumination requirements in street lighting. Here the end sought is the uniform illumination of all points along the course of the street. There is no reason either in scientific fact or in common sense why one point along the course of the street should be more brightly lighted than another, with the exception of street intersections and similar crossings later to be noted. The present almost universal condition of brighter illumination under and near the light unit is simply a concession to our present ignorance of how to obtain something better. There is, on the other hand, abundant reason to condemn this spot-lighting practice.

"At street intersections and similar crossings a brighter illumination is needed, for obvious reasons. This should be of from four to eight times the intensity of the illumination along the main course of the street. Such brighter illumination is properly provided for by employing from four to eight light units each having the same light distribution as the units used along the course of the street, or by one or two light units of similar light distribution and larger size. The street intersection, therefore, presents no complication to our problem, and its proper treatment becomes merely a matter for adequate consideration at the time of installation.

"The statement is often heard that in street lighting absolute uniformity of illumination is not required nor of any particular importance. Taken in a very strict sense, this statement contains much of truth. But this statement has been so very generally used as a refuge for loose thinking, by the scientific man as well as the layman, that it is worth a moment's careful consideration. It is true that a variation of two to one, taking the minimum intensity as unity, is only barely noticeable and is entirely permissible. But let us invariably ask our friend who says uniformity is not required, what numerical value he will assign as ratio of maximum to minimum intensity. If he gives us four to one, or a less ratio, we will agree with him and ask him to oblige us by keeping this ratio in mind in our further discussion. If he gives us eight to one or twenty to one or some such figure, we will disagree with him; and we will propose a hopeless task to him if we ask him to justify, by properly established scientific fact, such a ratio as permissible from any other standpoint than that of our possible inability to obtain a greater uniformity. At the same time we may profitably point out to him that the ratio in present practice most frequently falls between one hundred to one and five hundred to one.

" **Measurement of Street Illumination.**—Another much-mooted question concerns the way in which illumination should be measured, whether horizontally or perpendicular to the light ray. The latter method has been largely adopted in American practice chiefly through the influence of the commercial interests concerned, who desire to make the minimum value look as large as possible. There is little to be said in favor of it except that it may help us to remain in a fool's Paradise concerning the satisfactoriness of the illumination we are getting. It is true that in street service, illumination is desired at different times on almost every conceivable plane. But *the best single measure of how well these many diverse requirements are met is the intensity of illumination on the horizontal plane.* The horizontal plane, therefore, which is the accepted reference plane in interior lighting, is the logical and most satisfactory basis for street illumination measurements.

"The ideal to be aimed at, therefore, in street lighting, as far as intensity is concerned, is uniform horizontal illumination. On this basis we can now proceed to derive the ideal curves of light distribution. These curves once derived, we may properly take into account the fact that absolute uniformity is not of importance; and we will only ask that the actual light distribution sought for as a practical ideal shows no greater variation from our theoretical ideal than four to one in ratio of maximum positive variation to maximum negative variation as read in percentage values of the theoretical curve along the corresponding angles.

"The nature of a street makes most natural a placement of light units as a line of single units extending along the street, the units being hung over the center of the roadway or located alternately on either side. For all practical purposes, these two cases can be treated as one, the separation of adjacent units being taken as the *straight line* distance between the units, and the distribution curve derived being the correct distribution in a vertical plane passing through the unit concerned and the adjacent unit.

"There are an infinite number of light distributions which will give uniform illumination from a line of light units along a line extending immediately under those units. Other considerations than the obtaining of uniformity must therefore serve to enable us to select the most desirable distribution curve.

"Two such considerations are of especial importance and may therefore be applied with especial propriety. First, we should seek such a distribution as will be best suited for actual, practical attainment. This consideration is of the more importance because, as we shall find, even when maximum weight is given this consideration, our derived curve will present considerable though by no means insurmountable difficulties of practical realization. Second, the illumination at any point should be derived from a source as near as possible in order to avoid both excessively long shadows, and the glare effect which results from the high candle-power made necessary by illumination at great distances. . . . . .

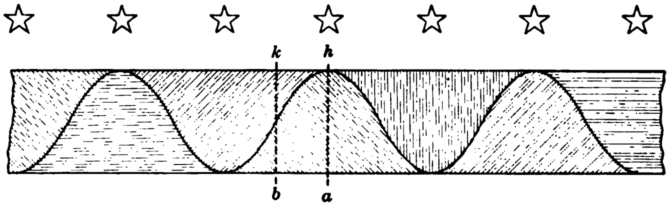

FIG. 177.—Distribution of illumination (lamps staggered).

"Fig. 177 shows a sine wave type of illumination curve. This distribution curve has a relatively low maximum and is as well adaptable to practical realization as any curve obtainable. It has, however, one defect. The maximum value occurs at an angle considerably greater than that subtended by half the separation of the units. To obtain minimum glare effect, the angle of maximum candle-power should be as small as possible in order that the light source may be removed as far as possible from the center of the field of vision when the eye reaches the point toward which the maximum candle-power is directed.

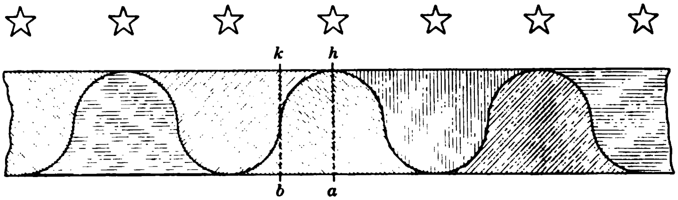

FIG. 178.—Distribution of illumination (lamps staggered).

"Many types of illumination curve could be given and the corresponding distribution curves shown to be open to objection. Suffice it to say that it can be established by a mathematical line of reasoning, which space limitations do not permit to be here given in detail, that the curve which best meets the conditions imposed is such as shown in Fig. 178. This curve is formed by two circle quadrants combined as shown. The corresponding distribution curve shows a low maximum which occurs at a slightly smaller angle than that subtended by half the separation of the units. It is also as well adaptable to practical realization as any curve obtainable.

"We have, therefore, obtained the ideal illumination curve from which the desired distribution curve can be easily derived. Our next task is to put these results into such form as to be easily applicable to practical conditions.

"**Location of Units.**—The relation between any desired illumination results and the distribution required to produce those results depends, in the type of problem under consideration, upon:

d. The distance between adjacent units.

h. The height of the light unit above the street.

That is to say, for any given separation of units and height above street, we can easily calculate the distribution curve which will produce the desired illumination result.

"It is obvious that any given distribution curve will give the same proportionate illumination results so long as the relation between the separation of units and the height above street is kept constant, since so long as this relation remains constant, the angular relations are unchanged. We can combine, therefore, the factors (d) and (h) (see above), and say that the distribution required to produce any given illumination result depends upon the relation between (d) and (h). Let us call this relation $M = d/h$.

$$M = \frac{\text{distance between adjacent light units}}{\text{height of light unit above street}} = d/h$$

For any value of M there is a given distribution curve which will produce the desired illumination result.

"Present-day street lighting practice shows a wide variation in the value of M. The writer has recently addressed the managements of the central stations and gas interests in most of the large cities of America and through their kind coöperation obtained an official statement in each case as to average, maximum and minimum separations, and mounting heights above street. The values of M thus obtained (based on average separations) vary from $M=5$ to $M=31$. The smaller cities would probably show in general a higher value for M than the large cities. All things considered, a value $M=15$ can probably be taken as the most representative value of present-day American practice, with a marked tendency toward smaller values in the more recent installations.

"**Distribution Curves for Uniform Illumination.**—Fig. 179 shows the distribution curve required to produce uniform illumination when $M=12$. Here we are at once brought face to face with the nature of the problem that confronts us. It needs no specialist to point out that such a curve as shown is impractical for actual realization, and would be extremely undesirable, even if it were practical to obtain it, on account of the excessive glare effect resulting from the very high candle-power maximum at a large angle. It will be observed that the maximum candle-power is about 130 times the minimum, or zero degree, candle-power.

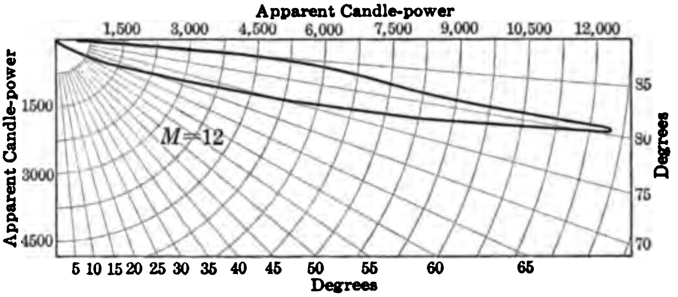

FIG. 179.—Candle-power curve, $d/h=12$.

"Fig. 179 serves one excellent purpose. It shows concisely and beyond argument that, with a value $M=12$ or greater, any attempt at adequate street lighting is absurd. If ignorance and a false economy compel an installation value of $M=12$ or greater, the pretense of obtaining "street illumination" should frankly be abandoned. Instead, the lights should be installed on the basis of a system of street markers, and located where they will best serve as markers. In such case, maximum usefulness as markers will be obtained when the candle-power values at the different angles are as close as possible to, yet not exceeding, the limits imposed by the avoidance of glare effect. Be it noted that in sparsely settled districts, where the very slight use of the streets by night may not warrant adequate street illumination, such a system of markers has a very real field of usefulness. But it should avowedly be a system of markers, not an abortive attempt at street illumination.

"Fig. 180 shows the required distribution curve for uniform illumination when $M = 8$. Here we are beginning to approach a form of curve practical for actual attainment; but the still very high candle-power maximum is prohibited by the limitations imposed by avoidance of glare effect.

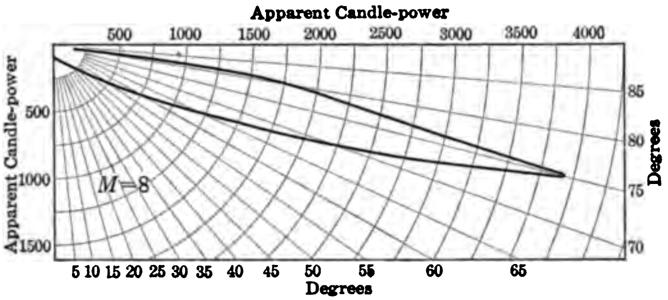

FIG. 180.—Candle-power curves, $d/h = 8$.

"Figs. 181 and 182 give the required distribution curves when $M = 6$ and $M = 4$, respectively. These distributions are entirely practical for actual attainment. For instance, referring to Fig. 183 the full-line curve shows the actual distribution of one make of arc lamp to-day on the market, the curve being one obtained by test by the Electrical Testing Laboratories. The dotted curve is the ideal curve for $M = 4$. It will be seen that the agreement of the two curves is admirable up to an angle of

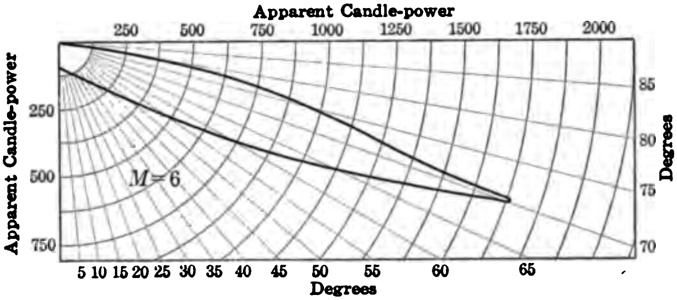

FIG. 181.—Candle-power curves, $d/h = 6$.

about 62 degrees. The curves $M=4$ and $M=6$ would also be comparatively easy of attainment with the incandescent lamp or with the inverted gas mantle. This is of especial importance since, with these lower values of M, such smaller units give amply sufficient intensity of illumination except in those business centers where night traffic is considerable.

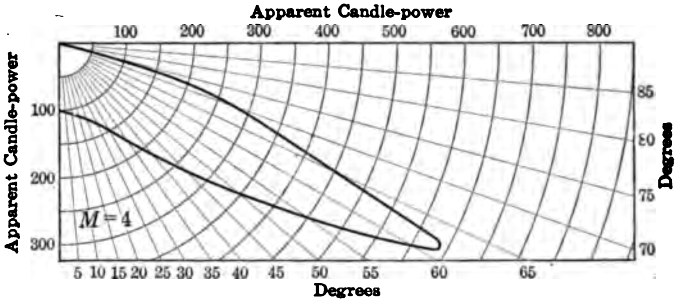

FIG. 182.—Candle-power curves $d/h=4$.

"Smaller values for M than 4 would obviously fulfill all the conditions imposed by the requirements of good illumination. It will later be shown by our study of glare effect that a value $M=3$ is necessary to obtain best results whenever, on account of shaded conditions of street or other considerations, it is

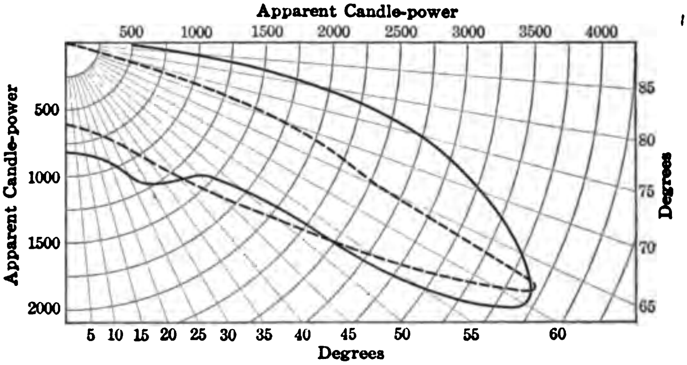

FIG. 183.—Theoretical and actual candle-power curves, $d/h=4$.

desirable to mount the units at a height of 15 ft. or less above the street. Closer spacing of units than $M=3$ entails a corresponding increase in installation and maintenance costs, and is not demanded by the illumination limitations of the problem. . . .

"Summing up the previous paragraphs, the requirements of good street illumination, as determined by distribution considerations, are as follows: Uniform horizontal illumination is the ideal to be aimed at, and the approximation to this ideal in actual practice to within a four to one ratio of maximum to minimum may be taken as sufficiently close for all practical purposes. The most desirable type of curve for the illumination delivered by the individual unit is such as shown in Fig. 178. When the light sources are separated by a distance greater than six times the mounting height above street, it becomes impractical to design a unit giving such light distribution as would be required for a sufficiently close approximation to uniform illumination. When the light sources are separated by twelve times the mounting height or greater, the pretense of adequate street illumination becomes an absurdity; and when ignorance of the factors involved has led to such separations, best results can be attained by abandoning all pretense of street illumination and by installing the light units as a system of street markers. Separations of three to six times the mounting height are required by distribution considerations.

"**Glare Effect.**—The second great division of the subject, the study of glare effect, has already been introduced in general terms. It now remains to analyze glare effect closely, to determine the laws of operation of the conditions that produce it, and to learn the limitations which it imposes on our problem.

"As will at once be recognized, the consideration of avoidance of glare effect will operate by imposing a definite candle-power limit at each distribution angle, this limit varying only with the mounting height. In proportion as this candle-power limit is exceeded a greater or less glare effect will result. . . . . .

"Subject to confirmation by research results, glare effect may be assumed to vary with the following factors:

(*a*) Distance of the eye from the light source.
(*b*) Total light flux in the direction of the eye, this being measured in apparent candle-power at the distance of the eye.
(*c*) Intrinsic brilliancy of the light source.
(*d*) Angular position of the light source in the field of vision, the visualized object being assumed to occupy the center of the field of vision.

(*e*) Distance of the light source from the eye relative to the distance of the visualized object upon which the eye is focused.

"In the research to be reported upon in the following paragraphs, the factor (*a*) was separately studied and determined with sufficient exactness to reliably indicate its character and approximate magnitude for distances up to 100 ft. Factors (*b*) and (*c*) were studied unseparated, as a single factor, the light flux varying directly as the intrinsic brilliancy. Factor (*d*) was separately studied and evaluted. Factor (*e*) was not investigated, the distance of the light source from the eye being in all cases taken as equal to the distance of the visualized object. This condition corresponds to the most usual condition of actual street vision.

"Heretofore no satisfactory measure of glare effect has been proposed. Since we are dealing with a psychological phenomenon, the basis of measure must obviously be a psychological one. On the other hand, the measure must be expressible in definite numerical units. A basis of measure fulfilling these requirements was conceived of for this investigation and proved a very satisfactory one. Preliminary to a more detailed description, such a measure may be described as the ratio of the two minimum light intensities on the visualized object, of magnitude just sufficient to enable its discernment, when the eye was successively exposed to the two conditions under comparison.

"A careful preliminary investigation established the fact that, under widely varying daylight conditions, visual acuity was wholly independent of distance, within the limits investigated; that is to say, with any given type of visualized object, the minimum size of object which can be just barely distinguished as a separate unit is exactly proportional to the visual distance. . .

"The ability to visualize objects was wholly determined by the visual efficiency of the eye and by the intensity of illumination on the object. Expressed mathematically, we are here dealing with a constant which is a function of two variables. One variable, therefore, the minimum intensity of illumination on the objects required to produce just-visibility, may be correctly employed as a measure of the other variable, the efficiency of the eye under any given condition.

"The basis of measurement being established, tests were first made to determine the relation between distance of the light source from the eye and the magnitude of the decrease in visual efficiency due to glare. The area of the light source throughout all these tests was kept constant, at about 50 sq. in., the intrinsic brilliancy thus varying directly with the candle-power, as has already been stated.

"Fig. 184, derived from the test data obtained as just described, expresses the relation between glare effect and distance, the light unit being situated beside the visualized object in the center of the field of vision. The ordinates express visual efficiency in terms of the per cent. intensity of illumination on the visualized object which would be required for equal visual acuity were the light source entirely removed from the field of vision. It will be noted that at 12 1/2 candle-power and an intrinsic brilliancy of 0.25 (British units), the glare effect drops off at a moderate rate with increase in distance. At higher candle-powers and higher intrinsic brilliancy, the glare effect decreases more slowly with increase in distance; until finally, at 400 candle-power and an intrinsic brilliancy of 8, there is no measurable decrease in glare effect with increase in distance within the limits investigated.

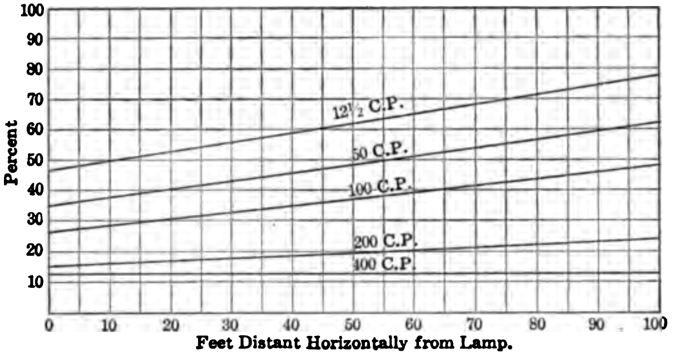

FIG. 184.—Relation between glare effect and distance.

"Fig. 185, derived from the same test data, expresses the relation between glare effect and candle-power (or, as it may be instead, between glare effect and intrinsic brilliancy). These curves are especially significant. It will be seen that, for all ordinary distances, the presence of a light source of even a very low candle-power in the center of the field of vision causes a tremendous drop in visual efficiency. It will also be noted that at 300 candle-power the glare effect has nearly reached its maximum, and that beyond 300 candle-power a large increase in candle-power produces but a small decrease in visual efficiency. Finally one should not fail to observe that at any given candle-power a considerable increase in distance produces only a small decrease in glare effect.

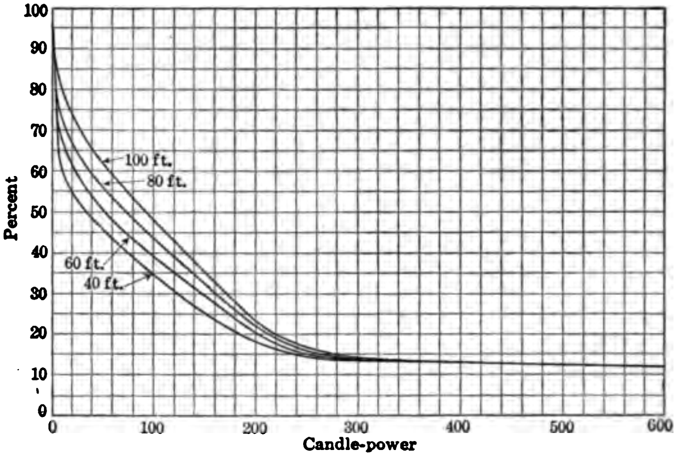

FIG. 185.—Relation between glare effect and candle-power.

"The relation between glare effect and the angular position of the light source in the field of vision was determined by a series of tests similar in general method to the tests already described. The amount of illumination required to see the test object when

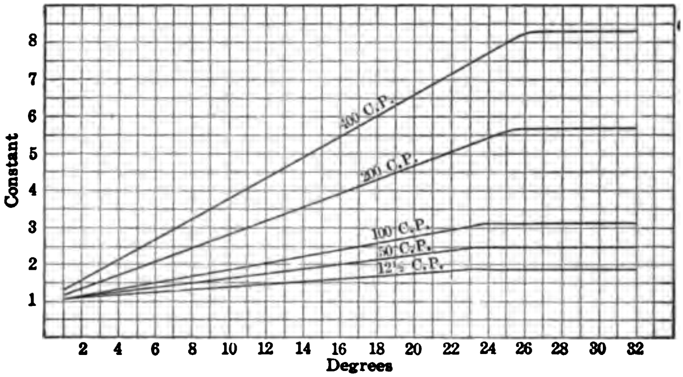

FIG. 186.—Relation between glare effect and angular position of the source.

the light source was in the angular position was compared with the amount required when the light source was in the center of the field of vision. The results obtained are shown in Fig. 186. The ordinates here express the intensity of illumination required with the light source in the center of the field of vision as measured in terms of the required intensity with the light source in the indicated angular position. This basis of plotting was chosen because of its conveninece in enabling the curves of Figs. 184 and 186 to be combined, thus giving the net glare effect for any desired angle and distance. The abcissæ in Fig. 186 represent the angles which the line from the light source to the eye makes with the line of vision.

''The curves of Fig. 186 express a fact of the very highest importance in the study of glare, namely, that, at an angle varying, depending on the candle-power (intrinsic brilliancy?), between approximately 22 degrees and approximately 26 degrees the glare effect reaches zero, although the light source is still well within the field of vision. This would seem to indicate that on its physiological side, glare effect is a local retinal disturbance which, when sufficiently separated on the retina from the disturbance produced by the visualized object, does not extend to or affect the latter area of disturbance. This apparent fact seems to hold true even when a considerable time factor is introduced. Apparently pupillary contraction acts merely as a function protecting, within its very limited capacity, from a glare effect more or less suffused over the whole retina. The writer has previously believed that pupillary contraction was in itself a cause of decreased visual efficiency in that, on account of pupillary contraction, a lesser amount of light would be admitted to the retina from the visualized object. He now believes that pupillary contraction should be regarded as a purely protective function and not, through its operation, in any sense itself a cause of decreased visual efficiency.

"The practical importance attaching to these facts is of no less vital interest than the scientific. Fig. 186 means that the light instrumental in causing glare comes entirely from the polar angles above 60 degrees. In the average actual case, glare is almost wholly caused by the light between the angles of 65 degrees and 80 degrees with the nadir. The light within these angles must be partially or wholly suppressed, as the further facts of the case shall show to be required, if glare is to be avoided.

"Fig. 187, derived from Figs. 184 and 186, gives the curves of zero glare effect for various mounting heights. Comparing these curves with the curve of ideal distribution when $M=6$ (Fig. 181), we see at once that this curve, together with curves of larger values of M, will be eliminated on account of the serious glare effect which would result from the application of such distributions to actual practice. The curve $M=4$, however, comes within the limitations imposed by the curve of zero glare effect, providing the scale of the $M=4$ curve does not exceed that amount which would make the zero-degree candle-power equal to 100 candle-power. The distribution curve corresponding to $M=3$ would come well within the limitations imposed by zero glare.

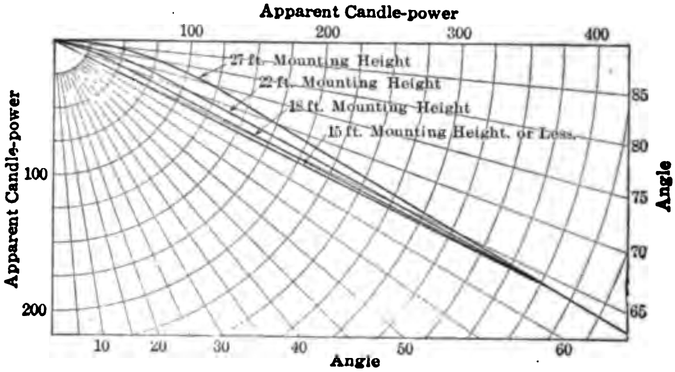

FIG. 187.—Curves for zero glare for various mounting heights.

"Reference to Fig. 187 shows that for all practical purposes, 60 degrees is an absolute limit for maximum candle-power if serious glare effect is to be avoided. It will also be observed that an increase in mounting height, while it does not change the limiting angle for maximum candle-power, does considerably lighten the severity of the limitations at 65 degrees and above. Mounting heights of the larger values, when made possible by actual conditions, are also desirable as permitting wider separation of units with resultant decreased installation and maintenance cost.

"For mounting heights of 15 ft. or less, the curve $M=4$ will produce considerable glare, beyond, indeed, what proper limitations can permit. For such lesser mounting heights, as has already been pointed out, a closer spacing of units and a distribution corresponding to the curve $M=3$ should be used.

"In designing a light unit to give the desired distribution which has largest field of application, namely the distribution from $M=4$, it will be preferable by far that the candle-power values at 60 degrees and above, be less than those made proper by the ideal curve, rather than that these ideal values be exceeded. With an ideal curve of such candle-power magnitude as would be most useful, the candle-power values at the angles under consideration come perilously near to the glare limit, or may even exceed the glare limit. A per cent. variation from the ideal curve of four to one is, as has been pointed out, permissible. Even a moderate condition of glare, on the other hand, is very undesirable. Practical considerations, therefore, will lead us, in the design of the actual unit, to intentionally deviate from the $M=4$ curve at the angles of 60 degrees and above, this deviation being in the direction of lesser candle-power values.

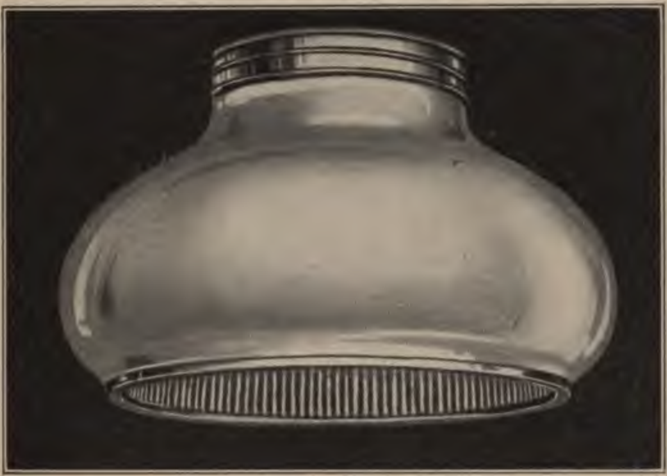

FIG. 188.—Holophane reflector for street lighting.

"It is of the utmost importance to note that if the glare limit is exceeded at all, it will almost of necessity be exceeded excessively. The curve of zero glare effect runs almost instantly back to zero near the angle of 65 degrees. The glare effect would therefore increase with great rapidity as an angle of

65 degrees was exceeded, even in the case of a distribution whose curve approximated to one of the circular coördinates. The curves required for uniform illumination, on the other hand, increase with great rapidity in candle-power values to their point of turning. It is obvious that the curve of zero glare effect must be taken as a line of absolute prohibition. If it is to be exceeded at all, it may as well be entirely ignored, for an apparently small violation of the limit will produce a decrease in visual efficiency of approximately the same order of magnitude as the maximum possible effect.

"It is also of great importance to note that the curve of ideal distribution for $M = 4$ remains, though not an ideal, yet none the less the best possible curve when applied to actual conditions where M has a larger value than 4. The curve $M = 4$ gives the best spot-lighting effect and the best marker effect which are possible without engendering a condition of glare. This research has indicated that, for ordinary mounting heights, moderate glare is a mere name; the reality is always excessive glare. It is the opinion of the writer, therefore, that glare must be avoided, even if so important a consideration as proper distribution has to be sacrificed. And indeed, as has been shown, other condi-

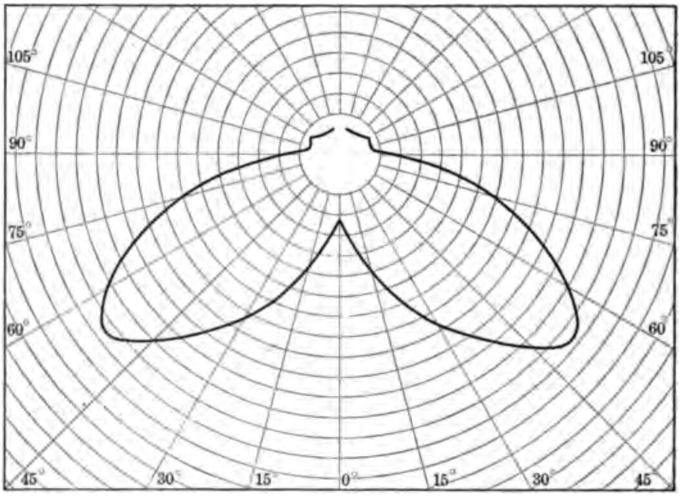

FIG. 189.—Distribution of light for an 80 c.p. series street lamp and Holophane street lighting reflector.

tions than glare make a proper distribution impractical or even impossible with the larger values of M. The curve $M=4$ is to be regarded, therefore, not merely as the ideal curve for a certain relatively close spacing of light units which the public is not as yet educated up to, but as the best possible distribution which can be applied to any of our present-day spacings."

That it is possible to closely approximate the ideal conditions of light distribution with a minimum of glare is shown by the curves of Figs. 189 and 190 which may be compared with Figs. 182 and 187.

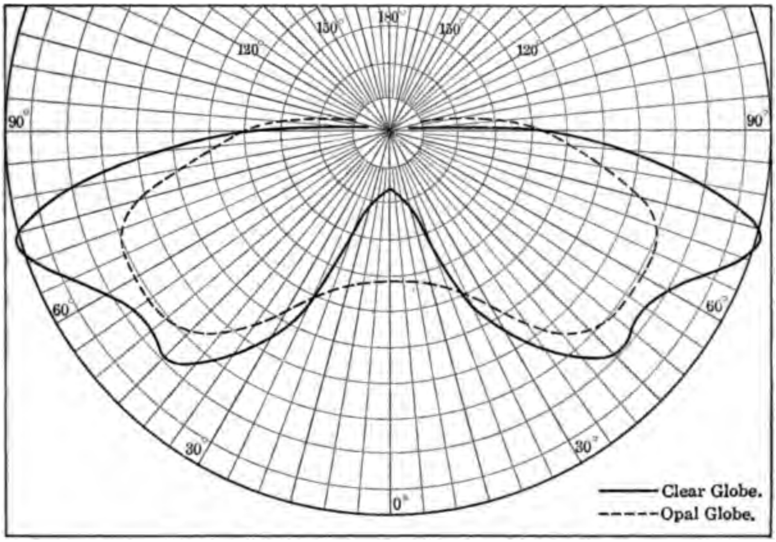

FIG. 190.—Distribution of light from a flame arc lamp equipped with co-axial electrodes.

Fig. 189 shows the distribution of light from an 80 candle-power series street lamp equipped with a prismatic reflector in a milk-glass envelope as shown in Fig. 188.[1]

Such a unit is well adapted to street lighting where small units placed low and comparatively near together ($d/h=4$) are desirable or in outlying districts where low intensities are sufficient. The unit gives good results when used for sidewalk illumination in connection with the common street lighting methods when lamps are placed so low and far apart that the walks receive but little light.

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[1] *Bul.* 16, Nat. Elect. Lamp Assoc.

In Fig. 190 are shown the distributions of light from a high candle-power flame arc lamp of the long-burning Stave type. The full line refers to the clear-glass enclosing globe while the broken line was obtained with opal enclosing glassware. The latter equipment is obviously most desirable because of the lower brightness. This is the class of lamps now being installed in many of our cities at a distance of 200 to 250 ft. between posts. A glance at the polar curves will show that *d/h* for uniform illumination is equal to approximately 4. Hence with $d/h = 4$ and the lamps spaced 200 ft. there is required a height of suspension of 50 ft. This is scarcely practical for obvious reasons but a height of 30 or 35 ft. is not unreasonable nor impractical. On the other hand it is a very proper height at which to place such high candle-power units. The practice of placing such lamps at 15 or 20 ft. above the street should be severely condemned.

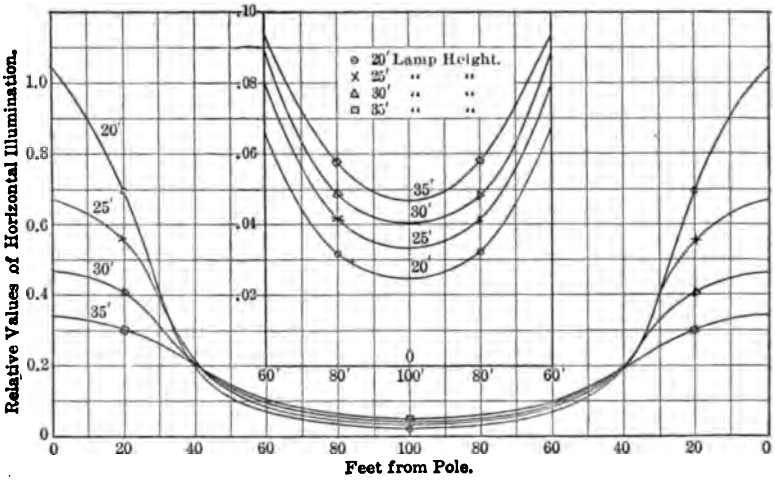

Fig. 191.—Illumination between lamps placed 200 feet apart. (Stave lamps, opal globes).

Height for Suspending Lamps.—It has already been shown that a uniform illumination is desirable and that a maximum ten times the minimum is the greatest difference which should be allowed. Having given the type of unit this result can be obtained in two ways, either by placing the lamps nearer one another than is the common practice or by placing the lamps higher. To illustrate this the author has derived the curves shown in Fig. 191 which show the relative values of the horizontal illumination with two lamps placed 200 ft. apart, at heights of 20, 25, 30 and 35 ft., and emitting light as represented by the broken line curve of Fig. 190.

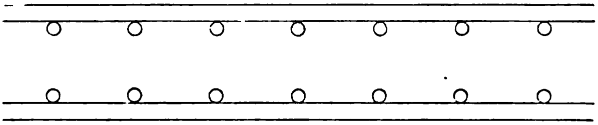

Fig. 192.—"Parallel" arrangement of street lighting units.

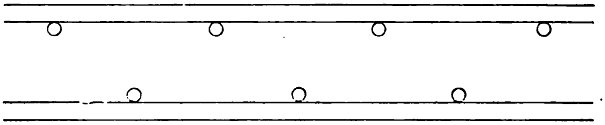

Fig. 193.—"Staggered" street lighting installation.

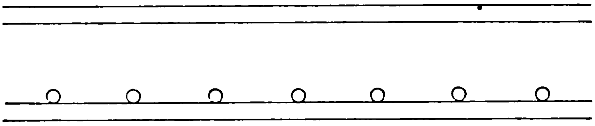

Fig. 194.—"Single line" installation of street lighting units.

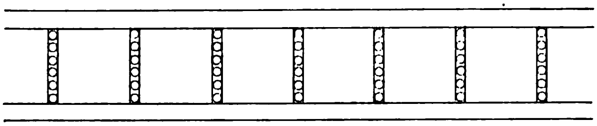

Fig. 195.—"Arch" installation of street lighting units.

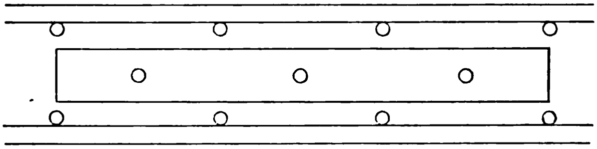

Fig. 196.—The "Parkway" arrangement of lighting units.

Referring a moment to these curves it will be seen that about three times the illumination intensity is obtained directly beneath the lamp at 20 ft. as when at 35 ft. Notice the values midway between lamps as shown to a larger scale in the middle of the figure. Here the conditions are reversed and with the lamp at 35 ft. above the street the illumination is nearly twice as much as when located at 20 ft. It should be remembered at this point that if values of illumination normal to the rays were here considered the difference would be only 8 per cent.

The ratios of maximum to minimum for the lamps placed at the four heights are as follows:

Height of lamp	Ratio of max. to min.
20	43–1
25	21–1
30	11.5–1
35	7.5–1

According to these data and our previous study it is shown that high candle-power lamps of the flaming arc class should not be placed less than 30 or 35 ft. above the street. Moreover, at this height the unit is more out of the range of vision and less glare, due to the relative values of illumination intensities, will be experienced.

Location of Lamps along the Street.—There are several methods of placing street lighting units relative to the street. A number of these are shown in Figs. 192–196. These figures are self-explanatory, the circles indicating the location of the lamps. Small units may be placed according to any of these methods. Large units would obviously not be placed in arches. They would be spaced at greater distances, and are commonly staggered as shown in Fig. 193. They are also often placed as shown in Figs. 192, 194, 196 and sometimes over the middle of the street supported by a cable stretched across the street. This latter is not desirable unless the unit is placed very high.

APPENDIX

Tables of Trigonometric Functions, Constants for Point-by-point Calculations, and Wiring Data and Symbols.

Table of Trigonometric Functions[1]

In the table on page 314 are given the values of sin a, cos a, tan a, $\sin^2 a$, $\cos^2 a$, $\sin^3 a$, and $\cos^3 a$, from 0 to 90 degrees. The cosine and tangent values are for those values of a which increase from top to bottom, while the sine values are for those values of a increasing from the bottom toward the top.

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[1] Tables 1 and 2 taken from the author's lecture at Baltimore, "Lectures on Illuminating Engineering," Vol. 2, p. 605, 1910.

TABLE I

Cos *a*	Tan *a*	*a*	Cos^2 *a*	Cos^3 *a*		Cos *a*	Tan *a*	*a*	Cos^2 *a*	Cos^3 *a*	
1.00	.000	0	1.00	1.000	90	.694	1.04	46	.482	.335	44
.99985	.0175	1	.999	.999	89	.682	1.07	47	.465	.317	43
.9994	.0349	2	.998	.998	88	.669	1.11	48	.447	.299	42
.998	.0524	3	.997	.996	87	.656	1.15	49	.430	.282	41
.997	.0699	4	.995	.993	86	.643	1.19	50	.413	.265	40
.996	.0875	5	.992	.988	85	.629	1.23	51	.395	.249	39
.994	.105	6	.989	.983	84	.615	1.28	52	.379	.233	38
.992	.123	7	.985	.978	83	.602	1.33	53	.362	.218	37
.990	.1405	8	.981	.971	82	.588	1.38	54	.345	.203	36
.988	.1584	9	.975	.963	81	.573	1.43	55	.329	.189	35
.985	.176	10	.970	.955	80	.559	1.48	56	.312	.175	34
.982	.194	11	.963	.946	79	.544	1.54	57	.296	.161	33
.978	.212	12	.957	.936	78	.530	1.60	58	.280	.149	32
.974	.230	13	.949	.925	77	.515	1.66	59	.265	.137	31
970	.249	14	.941	.913	76	.500	1.73	60	.250	.125	30
.966	.268	15	.933	.901	75	.485	1.80	61	.235	.113	29
.961	.287	16	.924	.888	74	.469	1.88	62	.220	.103	28
.956	.306	17	.914	.874	73	.454	1.96	63	.206	.0936	27
.951	.325	18	.904	.860	72	.438	2.05	64	.192	.0843	26
.945	.344	19	.894	.845	71	.423	2.14	65	.178	.0755	25
.939	.364	20	.883	.830	70	.407	2.25	66	.165	.0673	24
.933	.384	21	.872	.814	69	.391	2.35	67	.152	.0596	23
.927	.404	22	.859	.797	68	.375	2.47	68	.140	.0526	22
.920	.424	23	.847	.780	67	.358	2.60	69	.128	.0460	21
.913	.445	24	.834	.762	66	.342	2.75	70	.117	.0400	20
.906	.466	25	.821	.744	65	.325	2.90	71	.106	.0345	19
.899	.488	26	.808	.725	64	.309	3.08	72	.0955	.0295	18
.891	.509	27	.794	.707	63	.292	3.27	73	.0855	.0250	17
.882	.532	28	.779	.688	62	.275	3.48	74	.0759	.0209	16
.874	.554	29	.764	.669	61	.259	3.73	75	.0670	.0173	15
.866	.577	30	.750	.649	60	.242	4.01	76	.0586	.0142	14
.857	.601	31	.735	.630	59	.225	4.33	77	.0506	.0114	13
.848	.625	32	.719	.610	58	.208	4.70	78	.0432	.00899	12
.838	.649	33	.703	.590	57	.191	5.14	79	.0363	.00686	11
.829	.675	34	.687	.570	56	.173	5.67	80	.0301	.00520	10
.819	.700	35	.671	.550	55	.156	6.31	81	.0244	.00379	9
.809	.726	36	.655	.529	54	.139	7.11	82	.0193	.00268	8
.798	.753	37	.637	.509	53	.122	8.14	83	.0148	.00181	7
.788	.781	38	.621	.489	52	.1045	9.51	84	.0109	.00115	6
.777	.810	39	.604	.469	51	.0872	11.43	85	.00760	.000661	5
.766	.839	40	.587	.449	50	.0697	14.3	86	.00486	.000339	4
.754	.869	41	.569	.430	49	.0523	19.08	87	.00274	.000144	3
.743	.900	42	.552	.410	48	.0349	28.64	88	.00122	.0000425	2
.731	.932	43	.534	.391	47	.0174	57.29	89	.000306	.0000053	1
.719	.966	44	.517	.372	46	.00	∞	90	.0000	.0000	0
.707	1.00	45	.500	.353	45						
Sin *a*			Sin^2 *a*	Sin^3 *a*	*a*	Sin *a*			Sin^2 *a*	Sin^3 *a*	*a*

Tables for Point-by-point Calculations

It will be seen that in the equation $E_h = I_a \frac{\cos^3 a}{h^2}$ the values of $\frac{\cos^3 a}{h^2}$ can be determined for various values of a and h, and designated by K, then $E_h = KI_a$ and $I_a = \frac{E_h}{K}$.

In Table 2 the values of $\frac{\cos^3 a}{h^2} = K$ are tabulated for heights from 1 to 50 ft. and for every 5 degrees from 0 to 85 degrees from the vertical. It will also be noticed that the values of the horizontal distance, d, are also given for each value of K.

These tables can be extended by multiples of 10, the values of d being multiplied by 10 and the values of K divided by 100, as may be noted from the relations of the values of d and K for heights of 1 and 10 ft.

To make clear the manipulation of these tables we will find the illumination on a horizontal surface 10 ft. below a luminous source and at a point where the light rays make an angle of 60 degrees with the vertical. From the tables it will be seen that for 10 ft. in height and 60 degrees from the vertical, $K = .00125$ and $d = 17.3$. If the candle-power of the source at 60 degrees from the vertical is 200 candle-power then the horizontal illumination at a point 17.3 ft. from a vertical through the lamp will be

$$200 \times .00125 = 0.25 \text{ foot-candle.}$$

The candle-power of the source in that direction necessary to give an illumination of 0.25 foot-candle at that point will be

$$0.25/.00125 = 200 \text{ candle-power.}$$

Table 2

h		0°	5°	10°	15°	20°	25°	30°	35°	40°
1	*d*	.00	.09	.18	.27	.36	.47	.58	.70	.84
	K	1.00	.989	.955	.901	.830	.744	.650	.550	.450
2	*d*	.00	.18	.35	.54	.73	.93	1.15	1.40	1.68
	K	.25	.247	.239	.225	.207	.186	.162	.137	.112
3	*d*	.00	.26	.53	.80	1.09	1.40	1.73	2.10	2.52
	K	.111	.110	.106	.100	.0922	.0827	.0722	.0611	.0500
4	*d*	.00	.35	.71	1.07	1.46	1.87	2.31	2.80	3.36
	K	.0625	.0618	.0597	.0563	.0519	.0465	.0406	.0344	.0281
5	*d*	.00	.44	.88	1.34	1.82	2.33	2.89	3.50	4.20
	K	.0400	.0395	.0382	.0361	.0332	.0298	.0260	.0220	.0180
6	*d*	.00	.53	1.06	1.61	2.18	2.80	3.46	4.20	5.03
	K	.0278	.0275	.0265	.0250	.0231	.0207	.0180	.0153	.0125
7	*d*	.00	.61	1.23	1.88	2.55	3.26	4.04	4.90	5.87
	K	.0204	.0202	.0195	.0184	.0169	.0152	.0133	.0112	.00918
8	*d*	.00	.70	1.41	2.14	2.91	3.73	4.62	5.60	6.71
	K	.0156	.0154	.0149	.0141	.0130	.0116	.0102	.00859	.00702
9	*d*	.00	.79	1.59	2.41	3.28	4.20	5.20	6.30	7.55
	K	.0123	.0122	.0118	.0111	.0102	.00919	.00802	.00679	.00555
10	*d*	.00	.88	1.76	2.68	3.64	4.66	5.77	7.00	8.39
	K	.0100	.00989	.00955	.00901	.00830	.00744	.00650	.00550	.00450
11	*d*	.00	.96	1.94	2.95	4.00	5.13	6.35	7.70	9.23
	K	.00826	.00817	.00789	.00745	.00686	.00615	.00537	.00454	.00372
12	*d*	.00	1.05	2.12	3.21	4.37	5.60	6.93	8.40	10.07
	K	.00694	.00686	.00663	.00626	.00576	.00517	.00451	.00382	.00312
13	*d*	.00	1.14	2.29	3.48	4.73	6.06	7.51	9.10	10.91
	K	.00592	.00585	.00565	.00533	.00491	.00440	.00384	.00325	.00266
14	*d*	.00	1.23	2.47	3.75	5.10	6.53	8.08	9.80	11.75
	K	.00510	.00504	.00487	.00460	.00423	.00380	.00331	.00280	.00229
15	*d*	.00	1.31	2.64	4.02	5.46	7.00	8.66	10.5	12.6
	K	.00444	.00439	.00425	.00400	.00369	.00331	.00289	.00244	.00200
16	*d*	.00	1.40	2.82	4.29	5.82	7.46	9.24	11.2	13.4
	K	.00391	.00386	.00373	.00352	.00324	.00291	.00254	.00215	.00176
17	*d*	.00	1.49	3.00	4.55	6.19	7.93	9.82	11.9	14.3
	K	.00346	.00342	.00331	.00312	.00287	.00258	.00225	.00190	.00156
18	*d*	.00	1.58	3.17	4.82	6.55	8.40	10.4	12.6	15.1
	K	.00309	.00305	.00295	.00278	.00256	.00230	.00201	.00170	.00139
19	*d*	.00	1.66	3.35	5.09	6.92	8.86	11.0	13.3	15.9
	K	.00277	.00274	.00265	.00250	.00230	.00206	.00180	.00152	.00125
20	*d*	.00	1.75	3.53	5.36	7.28	9.33	11.55	14.0	16.8
	K	.00250	.00247	.00239	.00225	.00207	.00186	.00162	.00137	.00112
21	*d*	.00	1.85	3.70	5.63	7.64	9.60	12.1	14.7	17.6
	K	.00227	.00224	.00217	.00204	.00188	.00189	.00147	.00125	.00102

Table 2

45°	50°	55°	60°	65°	70°	75°	80°	85°
1.00	1.19	1.43	1.73	2.14	2.75	3.73	5.67	11.4
.354	.266	.189	.125	.0755	.0400	.0173	.00524	.000661
2.00	2.38	2.86	3.46	4.29	5.50	7.46	11.34	22.8
.0884	.0664	.0472	.0313	.0189	.0100	.00433	.00131	.000165
3.00	3.58	4.28	5.20	6.43	8.24	11.20	17.0	34.3
.0393	.0295	.0210	.0139	.00839	.00445	.00193	.000582	.0000735
4.00	4.77	5.71	6.93	8.58	10.99	14.93	22.69	45.7
.0221	.0166	.0118	.00781	.00472	.00250	.00108	.000327	.0000413
5.00	5.96	7.14	8.66	10.72	13.74	18.66	28.36	57.2
.0141	.0106	.00755	.00500	.00302	.00160	.000694	.000209	.0000264
6.00	7.15	8.57	10.4	12.9	16.5	22.4	34.0	68.6
.00982	.00738	.00524	.00347	.00210	.00111	.000482	.000145	.0000184
7.00	8.34	10.0	12.1	15.0	19.2	26.1	39.7	80.1
.00722	.00542	.00385	.00255	.00154	.000817	.000354	.000107	.0000135
8.00	9.53	11.4	13.8	17.2	22.0	30.0	45.4	91.5
.00553	.00415	.00295	.00195	.00118	.000625	.000271	.0000818	.0000103
9.00	10.7	12.8	15.6	19.3	24.7	33.6	51.0	103.
.00437	.00328	.00233	.00154	.000932	.000494	.000214	.0000646	.00000816
10.0	11.9	14.3	17.3	21.5	27.5	37.3	56.7	114.
.00354	.00266	.00189	.00125	.000755	.000400	.000173	.0000524	.00000661
11.0	13.1	15.7	19.0	23.6	30.2	41.0	62.4	126.
.00292	.00220	.00156	.00103	.000624	.000330	.000143	.0000433	.00000546
12.0	14.3	17.1	20.8	25.7	33.0	44.8	68.0	137.
.00246	.00184	.00131	.000868	.000524	.000278	.000120	.0000364	.00000459
13.0	15.5	18.6	22.5	27.9	35.7	48.5	73.7	149.
.00209	.00157	.00112	.000740	.000447	.000237	.000103	.0000310	.00000391
14.0	16.7	20.0	24.2	30.0	38.5	52.2	79.4	160.
.00180	.00136	.000963	.000638	.000385	.000204	.0000885	.0000267	.00000337
15.0	17.9	21.4	26.0	32.2	41.2	56.0	85.1	171.
.00157	.00112	.000839	.000556	.000336	.000178	.0000771	.0000233	.00000294
16.0	19.1	22.8	27.7	34.3	44.0	59.7	90.7	183.
.00138	.00104	.000737	.000488	.000295	.000156	.0000677	.0000205	.00000258
17.0	20.3	24.3	29.5	36.5	46.7	63.5	96.4	194.
.00122	.000919	.000653	.000433	.000261	.000138	.0000600	.0000181	.00000194
18.0	21.5	25.7	31.2	38.6	49.5	67.2	102.	206.
.00109	.000820	.000583	.000386	.000233	.000124	.0000535	.0000162	.00000206
19.0	22.6	27.1	32.9	40.7	52.2	70.9	108.	217.
.000980	.000736	.000523	.000346	.000209	.000111	.0000480	.0000145	.00000183
20.0	23.8	28.5	34.6	42.9	55.0	74.6	113.	229.
.000884	.000664	.000472	.000313	.000189	.000100	.0000433	.0000131	.00000229
21.0	25.1	30.0	36.4	45.0	57.7	78.4	119.	240.
.000802	.000602	.000428	.000284	.000171	.0000907	.0000393	.0000119	.00000150

TABLE 2—*Continued*

h		0°	5°	10°	15°	20°	25°	30°	35°	40°
22	*d*	.00	1.93	3.88	5.89	8.01	10.3	12.7	15.4	18.5
	K	.00207	.00204	.00197	.00186	.00171	.00154	.00134	.00114	.000929
23	*d*	.00	2.01	4.05	6.16	8.37	10.7	13.3	16.1	19.3
	K	.00189	.00187	.00181	.00170	.00157	.00141	.00123	.00104	.00085
24	*d*	.00	2.10	4.23	6.43	8.74	11.2	13.9	16.8	20.1
	K	.00174	.00172	.00166	.00156	.00144	.00129	.00113	.000954	.000781
25	*d*	.00	2.19	4.41	6.70	9.10	11.7	14.4	17.5	21.0
	K	.00160	.00158	.00153	.00144	.00133	.00119	.00104	.000879	.000719
26	*d*	.00	2.28	4.58	6.97	9.46	12.1	15.0	18.2	21.8
	K	.00148	.00146	.00141	.00133	.00123	.00110	.000961	.000813	.000665
27	*d*	.00	2.36	4.76	7.23	9.83	12.6	15.6	18.9	22.7
	K	.00137	.00136	.00131	.00124	.00114	.00102	.000891	.000754	.000617
28	*d*	.00	2.45	4.94	7.50	10.2	13.1	16.2	19.6	23.5
	K	.00128	.00126	.00122	.00115	.00106	.000950	.000829	.000701	.000573
29	*d*	.00	2.54	5.11	7.77	10.6	13.5	16.7	20.3	24.3
	K	.00119	.00118	.00114	.00107	.000987	.000885	.000772	.000654	.000535
30	*d*	.00	2.63	5.29	8.04	10.9	14.0	17.3	21.0	25.2
	K	.00111	.00110	.00106	.00100	.000922	.000827	.000722	.000611	.000500
32	*d*	.00	2.80	5.64	8.57	11.3	14.9	18.5	22.4	26.8
	K	.000976	.000964	.000932	.000880	.000811	.000726	.000633	.000537	.000438
34	*d*	.00	2.97	6.00	9.12	12.4	15.9	19.6	23.8	28.5
	K	.000866	.000855	.000826	.000780	.000718	.000644	.000561	.000476	.000388
35	*d*	.00	3.06	6.17	9.37	12.4	16.3	20.2	24.5	29.4
	K	.000816	.000806	.000779	.000736	.000677	.000606	.000530	.000449	.000366
36	*d*	.00	3.15	6.35	9.63	13.1	16.8	20.8	25.2	30.2
	K	.000772	.000762	.000737	.000695	.000640	.000574	.000500	.000424	.000346
38	*d*	.00	3.33	6.71	10.2	13.8	17.7	21.9	26.6	31.9
	K	.000692	.000684	.000661	.000624	.000575	.000515	.000449	.000381	.000311
40	*d*	.00	3.50	7.06	10.7	14.6	18.6	23.1	28.0	33.5
	K	.000625	.000617	.000597	.000562	.000518	.000465	.000405	.000344	.000280
42	*d*	.00	3.67	7.41	11.2	15.3	19.6	24.2	29.4	35.2
	K	.000567	.000560	.000541	.000511	.000470	.000421	.000368	.000312	.000255
44	*d*	.00	3.85	7.76	11.8	16.0	20.5	25.4	30.8	36.9
	K	.000516	.000510	.000493	.000465	.000429	.000384	.000335	.000284	.000232
45	*d*	.00	3.94	7.94	12.1	16.4	21.0	26.0	31.5	37.7
	K	.000493	.000487	.000471	.000445	.000410	.000367	.000320	.000272	.000222
46	*d*	.00	4.03	8.12	12.3	16.8	21.5	26.5	32.2	38.6
	K	.000473	.000467	.000451	.000425	.000392	.000352	.000306	.000260	.000212
48	*d*	.00	4.2	8.47	12.8	17.5	22.4	27.2	33.6	40.3
	K	.000434	.000428	.000414	.000391	.000361	.000323	.000282	.000239	.000195
50	*d*	.00	4.38	8.82	13.4	18.2	23.3	28.8	35.0	41.9
	K	.000400	.000395	.000382	.000360	.000332	.000297	.000260	.000220	.000180

TABLE 2—*Continued*

45°	50°	55°	60°	65°	70°	75°	80°	85°
22.0 .000731	26.2 .000549	31.4 .000390	38.1 .000258	47.2 .000156	60.5 .0000827	82.1 .0000358	125. .0000108	251. .00000137
23.0 .000669	27.4 .000502	32.8 .000357	39.8 .000236	49.3 .000143	63.2 .0000757	85.8 .0000328	130. .00000990	263. .00000125
24.0 .000614	28.6 .000461	34.3 .000328	41.6 .000217	51.5 .000131	65.9 .0000695	89.6 .0000301	136. .00000909	275. .00000115
25.0 .000566	29.8 .000425	35.7 .000302	43.3 .000200	53.6 .000121	68.7 .000064	93.3 .0000277	142. .00000838	286. .00000106
26.0 .000523	31.0 .000393	37.1 .000279	45.0 .000185	55.8 .000112	71.4 .0000592	97.0 .0000257	147. .00000775	297. .000000977
27.0 .000485	32.2 .000364	38.6 .000259	46.8 .000172	57.9 .000104	74.2 .0000549	101. .0000238	153. .00000718	309. .000000907
28.0 .000451	33.4 .000339	40.0 .000241	48.5 .000159	60.0 .0000963	76.9 .0000510	105. .0000221	159. .00000668	320. .000000844
29.0 .000420	34.5 .000316	41.4 .000224	50.2 .000149	62.2 .0000898	79.7 .0000476	108.2 .0000206	164.5 .00000622	332. .000000786
30.0 .000393	35.7 .000295	42.8 .000210	52.0 .000139	64.3 .0000839	82.4 .0000445	112. .0000193	170. .00000582	343. .000000735
32.0 000345	38.1 .000249	45.6 .000185	55.4 .000122	68.6 .0000737	88.0 .0000390	119. .0000169	181. .00000507	366. .000000645
34.0 .000305	40.5 .000229	48.5 .000164	58.8 .000108	73.0 .0000653	93.5 .0000347	127. .0000150	193. .00000450	389. .000000571
35.0 .000288	41.8 .000216	50.0 .000154	60.6 .000102	75.0 .0000616	96.3 .0000326	131. .0000141	199. .00000425	400. .000000540
36.0 .000272	42.9 .000205	51.4 .000146	62.3 .0000965	77.2 .0000582	99.0 0000309	134. .0000133	204. .00000401	412. .000000510
38.0 .000245	45.3 .000184	54.2 .000131	65.9 .0000866	81.5 .0000522	104. .0000277	142. .0000120	215. .00000360	435. .000000457
40.0 .000220	47.7 .000166	57.2 .000118	69.4 .0000781	85.9 .0000471	110. .0000250	149. .0000108	227. .00000325	457. .000000413
42.0 .000200	50.0 .000150	60.0 .000107	72.7 .0000709	90.0 .0000428	115.5 .0000227	157. .00000981	238. .00000295	480. .000000375
44.0 .000182	52.5 .000137	62.8 .0000976	76.3 .0000645	94.5 .0000390	121. .0000207	164. .00000894	250. .00000269	503. .000000341
45.0 .000174	53.6 .000131	64.3 .0000933	78.0 .0000617	96.5 .0000373	124. .0000198	168. .00000854	255. .00000257	515. .000000326
46.0 .000167	54.9 .000125	65.7 .0000893	79.8 .0000591	98.8 .0000357	126. .0000189	172. .00000818	261. .00000246	526. .000000312
48.0 .000153	57.3 .000115	68.6 .0000821	83.2 .0000543	103. .0000328	132. .0000174	179. .00000752	272. .00000226	549. .000000287
50.0 .000141	59.6 .000106	71.5 .0000755	86.6 .0000500	107. .0000302	137. .0000160	186. .00000692	284. .00000208	572. .000000265

When, as is often the case in interior lighting, it is desirable to obtain the values of illumination at definite distances along the horizontal, tables like the following set will be found more convenient. These tables give values of $K\left(=\frac{\cos^3 a}{h^2}\right)$ and a (degrees from the vertical) for various heights and horizontal distances. The constants in these tables are used in the same manner as illustrated in the foregoing problem.

TABLE 3

h	d	0′	2′	4′	6′	8′	10′	12′	14′	16′	18′	20′
1	α	00°	63° 25′	75° 55′	80° 30′	82° 50′	84° 20′	85° 15′	85° 55′	68° 25′	....	....
	K	1.00	.893	.0144	.00441	.00190	.000961	.000567	.000361	.000244	....	....
2	α	00°	45° 0′	63° 25′	71° 35′	76° 0′	78° 40′	80° 35′	81° 50′	82° 55′	83° 40′	84° 20′
	K	.250	.0883	.0224	.0079	.00355	.00191	.00111	.000722	.000473	.000336	.000246
3	α	00°	33° 40′	53° 5′	63° 25′	69° 25′	73° 20′	75° 55′	77° 55′	79° 25′	80° 30′	81° 30′
	K	.111	.0640	.0241	.00992	.00480	.00262	.00159	.00102	000693	.000493	.000362
4	α	00°	26° 35′	45° 0′	56° 20′	63° 25′	68° 10′	71° 35′	74° 5′	76° 0′	77° 30′	78° 45′
	K	.0625	.0447	.0221	.0106	.00560	.00322	.00197	.00130	.000887	.00064	.00047
5	α	00°	21° 50′	38° 40′	50° 10′	58° 00′	63° 25′	67° 20′	70° 20′	72° 40′	74° 40′	76° 0′
	K	.0400	.0320	.0191	.0105	.00596	.00357	.00228	.00152	.00106	.00077	.00057
6	α	00°	18° 25′	33° 40′	45° 0′	53° 5′	59° 0′	63° 25′	66° 50′	69° 25′	71° 35′	73° 25′
	K	.0278	.0236	.0160	.00982	.00602	.00380	.00248	.00169	.00121	.00088	.00066
7	α	00°	15° 55′	29° 45′	40° 35′	48° 50′	55° 0′	59° 45′	63° 25′	66° 20′	68° 45′	70° 40′
	K	.0204	.0182	.0134	.00892	.00583	.00385	.00261	.00182	.00131	.00097	.00074
8	α	00°	14° 0′	26° 35′	36° 50′	45° 0′	51° 20′	56° 20′	60° 15′	63° 25′	66° 0′	68° 10′
	K	.0156	.0143	.0112	.00801	.00552	.00382	.00266	.00191	.00140	.00105	.00080
9	α	00°	12° 30′	24° 0′	33° 40′	41° 40′	48° 0′	53° 5′	57° 15′	60° 40′	63° 25′	65° 45′
	K	.0123	.0115	.00941	.00711	.00515	.00369	.00268	.00195	.00146	.00110	.00085
10	α	00°	11° 20′	21° 50′	31° 0′	38° 40′	45° 0′	50° 10′	54° 30′	58° 0′	61° 0′	63° 25′
	K	.0100	.00942	.00800	.00630	.00476	.00353	.00262	.00196	.00149	.00115	.00089

TABLE 3—*Continued*

h	d	0′	4′	8′	12′	16′	20	24′	28′	32′	36′	40′
11	α	00°	20° 0′	36° 0′	47° 30′	55° 30′	61° 10′	65° 25′	68° 35′	71° 0′	73° 0′	74° 40′
	K	.00826	.00685	.00441	.00254	.00150	.000925	.000597	.000407	.000284	.000207	.000154
12	α	00°	18° 25′	33° 40′	45° 0′	53° 5′	59° 0′	63° 25′	66° 50′	69° 25′	71° 35′	73° 20′
	K	.00694	.00592	.00400	.00245	.00151	.000950	.000620	.000425	.000300	.000219	.000165
13	α	00°	17° 5′	31° 35′	42° 40′	50° 55′	56° 55′	61° 35′	65° 5′	67° 55′	70° 10′	72° 0′
	K	.00592	.00517	.00366	.00235	.00148	.000965	.000640	.000440	.000316	.000232	.000174
14	α	00°	16° 0′	29° 45′	40° 40′	48° 50′	55° 0′	59° 45′	63° 25′	66° 20′	68° 45′	70° 40′
	K	.00510	.00453	.00333	.00222	.00145	.000965	.000653	.000455	.000329	.000243	.000184
15	α	00°	14° 55′	28° 5′	38° 40′	46° 50′	53° 5′	58° 0′	61° 50′	64° 55′	67° 20′	69° 25′
	K	.00444	.00401	.00305	.00212	.00142	.000964	.000661	.000469	.000340	.000253	.000192
16	α	00°	14°0′	26° 35′	36° 50′	45° 0′	51° 20′	56° 20′	60° 15′	63° 25′	66° 0′	68° 10′
	K	.00391	.00357	.00279	.00200	.00138	.000954	.000668	.000476	.000349	.000262	.000200
17	α	00°	13° 15′	25° 10′	35° 15′	43° 15′	49° 40′	54° 40′	58° 45′	62° 0′	64° 40′	67° 0′
	K	.00346	.00319	.00256	.00189	.00134	.000937	.000668	.000485	.000358	.000269	.000207
18	α	00°	12° 30′	24° 0′	33° 40′	41° 40′	48° 0′	53° 10′	57° 15′	60° 40′	63° 25′	65° 45′
	K	.00309	.00287	.00235	.00178	.00129	.000927	.000667	.000488	.000364	.000276	.000213
19	α	00°	11° 55′	22° 50′	32° 20′	40° 5′	46° 25′	51° 40′	55° 50′	59° 20′	62° 10′	64° 30′
	K	.00277	.00260	.00217	.00167	.00124	.000906	.000662	.000490	.000368	.000282	.000219
20	α	00°	11° 20′	21° 50′	31° 0′	38° 40′	45° 0′	50° 10′	54° 30′	58° 0′	61° 0′	63° 25′
	K	.0025	.00235	.00200	.00157	.00119	.000883	.000655	.000490	.000373	.000287	.000224

Table 3—*Continued*

h	*d*	0′	20′	40′	60′	80′	100′	120′	140′	160′	180′	200′
20	*α*	00°	45° 0′	63° 25′	71° 35′	76° 0′	78° 40′	80° 30′	81° 50′	82° 50′	83° 40′	84° 20′
	K	.0025	.000883	.0002236	.000078	.0000357	.0000188	.0000111	.00000707	.00000477	.00000336	.00000246
25	*α*	00°	38° 40′	58° 0′	67° 20′	72° 40′	76° 0′	78° 15′	79° 50′	81° 10′	82° 5′	82° 50′
	K	.0016	.000761	.000238	.0000911	.0000422	.0000228	.0000135	.00000868	.00000587	.00000416	.00000305
30	*α*	00°	33° 40′	53° 10′	63° 25′	69° 25′	73° 20′	75° 55′	77° 55′	79° 20′	80° 30′	81° 30′
	K	.00111	.00064	.00024	.0000994	.000048	.0000264	.0000159	.0000102	.00000694	.00000493	.00000362
35	*α*	00°	29° 45′	48° 50′	59° 45′	66° 0′	70° 40′	73° 45′	76° 0′	77° 40′	79° 0′	80° 5′
	K	.000816	.00534	.000233	.000104	.0000535	.0000294	.0000179	.00000116	.00000799	.00000569	.00000419
40	*α*	00°	26° 35′	45° 0′	56° 20′	63° 25′	68° 10′	71° 35′	74° 5′	76° 0′	77° 30′	78° 40′
	K	.000625	.000447	.000221	.000106	.0000560	.0000322	.0000197	.0000130	.00000887	.00000638	.0000047
45	*α*	00°	24° 0′	41° 40′	53° 0′	60° 35′	65° 45′	69° 30′	72° 10′	74° 20′	76° 0′	77° 20′
	K	.000494	.000376	.000206	.000107	.0000583	.0000342	.0000213	.0000142	.0000098	.00000705	.00000522
50	*α*	00°	21° 50′	38° 40′	50° 10′	58° 00′	63° 25′	67° 20′	70° 20′	72° 40′	74° 30′	76° 0′
	K	.00040	.000320	.000191	.000105	.0000596	.0000357	.0000228	.0000152	.0000108	.00000766	.0000057
55	*α*	00°	20° 0′	36° 0′	47° 30′	55° 30′	61° 15′	65° 20′	68° 30′	71° 0′	73° 0′	74° 35′
	K	.00033	.000274	.000175	.000102	.0000601	.0000371	.000024	.0000162	.00001135	.00000825	.00000586
60	*α*	00°	18° 25′	33° 40′	45° 0′	53° 5′	59° 0′	63° 25′	66° 50′	69° 25′	71° 35′	73° 20′
	K	.000278	.000236	.000160	.0000982	.0000602	.0000380	.0000248	.0000169	.0000121	.00000878	.00000659

On the following pages are given data and equations for wiring calculations together with the set of standard symbols for wiring plans as adopted and recommended by the National Electrical Contractors' Association of the United States and the American Institute of Architects.

TABLE 4[1]

Dimensions, Weights and Resistances of Pure Copper Wire.

B. & S. gauge	Diameter inches	Area circular mils	Weight lbs. per 1000 ft.	Length		Resistance at 20° C. (68° F.)	
				Ft. per lb.	Ft. per ohm.	ohms per 1000 ft.	ohms per lb.
0000	0.4600	211600	640.5	1.56	20440.0	0.04893	0.00007639
000	0.4096	167800	508.0	1.97	16210.0	0.06170	0.0001215
00	0.3648	133100	402.8	2.48	12850.0	0.07780	0.0001931
0	0.3249	105500	319.5	3.13	10190.0	0.09811	0.0003071
1	0.2893	83690	253.3	3.95	8083.0	0.1237	0.0004883
2	0.2576	66370	200.9	4.98	6410.0	0.1560	0.0007765
3	0.2294	52630	159.3	6.28	5084.0	0.1967	0.001235
4	0.2043	41740	126.4	7.91	4031.0	0.2480	0.001963
5	0.1819	33100	100.2	9.98	3197.0	0.3128	0.003122
6	0.1620	26250	79.5	12.58	2535.0	0.3944	0.004963
7	0.1443	20820	63.0	15.87	2011.0	0.4973	0.007892
8	0.1285	16510	50.0	20.01	1595.0	0.6271	0.01255
9	0.1144	13090	39.6	25.23	1265.0	0.7908	0.01995
10	0.1019	10380	31.4	31.82	1003.0	0.9972	0.03173
11	0.09074	8234	24.9	40.12	795.3	1.257	0.05045
12	0.08081	6530	19.8	50.59	630.7	1.586	0.08022
13	0.07196	5178	15.7	63.79	500.1	1.999	0.1276
14	0.06408	4107	12.4	80.44	396.6	2.521	0.2028
15	0.05707	3257	9.9	101.44	314.5	3.179	0.3225
16	0.05082	2583	7.8	127.91	249.4	4.009	0.5128
17	0.04526	2048	6.2	161.29	197.8	5.055	0.8153
18	0.04030	1624	4.9	203.37	156.9	6.374	1.296

The following table gives the current-carrying capacities of the respective sizes of copper wire with rubber and weatherproof insulation as approved by the National Board of Fire Underwriters.

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[1] *Bull.* 7B, Nat. Elect. Lamp Assoc.

TABLE 5

B. & S. gauge	Circular mils	Amperes rubber-covered	Amperes weather-proof
18	1,624	3	5
16	2,583	6	8
14	4,107	12	16
12	6,530	17	23
10	10,380	24	32
8	16,510	33	46
6	26,250	46	65
5	33,100	54	77
4	41,740	65	92
3	52,630	76	110
2	66,370	90	131
1	63,690	107	156
0	105,500	127	185
00	133,100	150	220
000	167,800	177	262
0000	211,600	210	312

The question of potential drop is not considered in the above tables.

No wire smaller than No. 14 is used except for fixture work and flexible cords.

In computing the size of wire to be used for lighting circuits, the following formula will be of service:

$$A = \frac{I \times l \times K}{e}$$

where A equals the area of the wire in circular mils,
e equals the permissible drop in volts,
I equals the current on the line in amperes,
l equals the distance from the mains to the center of distribution or outlet in feet, and
K equals 21.62 for direct-current circuits,
K equals 24.00 for lighting load on single-phase alternating-current circuits,
K equals 26.60 for lighting and power loads on single-phase alternating-current circuits,
K equals 12.00 for lighting load on two-phase, four-wire, and on three-phase, three-wire, alternating-current circuits,
K equals 13.30 for lighting and power load on two-phase four-wire, and three-phase, three-wire alternating-current circuits.

TABLE 6[1]

Table Showing Size of Wire to Use for Drop of One Volt for Various Currents and Distances.

Size	Distance in feet to center of distribution															
No. of Wire B & S.	20′	25′	30′	35′	40′	45′	50′	60′	70′	80′	90′	100′	120′	140′	160′	180′	200′
14	9.5	7.60	6.34	5.12	4.75	4.22	3.80	3.17	2.62	2.37	2.11	1.90	1.58	1.35	1.18	1.05	.95
12	15.1	12.08	10.06	8.64	7.55	6.61	6.04	5.03	4.31	3.78	3.36	3.02	2.51	2.15	1.88	1.68	1.51
10	24.0	19.20	16.00	13.60	12.00	10.66	9.60	8.00	6.86	6.00	5.33	4.80	4.00	3.43	3.00	2.66	2.40
8	33	30.55	25.45	21.80	19.10	16.90	15.28	12.72	10.90	9.55	8.49	7.64	6.36	5.45	4.77	4.24	3.82
6	46	46	40.4	34.60	30.30	26.95	24.25	20.20	17.30	15.15	13.48	12.12	10.10	8.66	7.58	6.24	6.06
5	54	54	51.0	43.75	38.25	34.00	30.60	25.50	21.85	19.12	17.00	15.30	12.75	10.92	9.56	8.50	7.65
4	65	65	64.5	55.15	48.25	42.90	38.60	32.17	27.58	24.13	21.44	19.30	16.08	13.79	12.06	10.72	9.65
3	76	76	76	69.6	60.9	54.1	48.7	40.60	34.80	30.45	27.05	24.35	20.30	17.40	15.22	13.52	12.17
2	90	90	90	87.6	76.6	68.1	61.4	51.1	43.8	38.30	34.05	30.66	25.55	21.90	19.15	17.02	15.33
1	107	107	107	107	95.5	84.9	76.4	69.5	54.6	47.75	42.45	38.20	31.85	27.30	23.86	21.22	19.10
0	127	127	127	127	122.0	108.4	97.6	81.3	69.7	61.00	54.20	48.80	40.65	34.85	30.50	27.10	24.40
00	150	150	150	150	150	136.6	123.0	102.5	87.9	76.80	68.30	61.50	51.25	43.90	38.40	34.15	30.75
000	177	177	177	177	177	172.2	155.0	129.2	110.7	96.9	86.1	77.5	64.6	55.35	48.45	43.05	38.75
0000	210	210	210	210	210	210	195.6	163.0	139.6	122.3	109.6	97.8	81.5	69.8	61.15	54.30	48.90

Formula used $A = \frac{ID\ 21.62}{V}$

Currents found under the broken line are the maximum allowable by National Board of Underwriters for the size specified (rubber-covered wire). The drop for these currents is less than one volt. This table is for continuous current only. If A. C. single phase, for lighting load multiply values given in the table by .9, for lighting and power load multiply the values given in the table by .82. If A. C. two phase (4 wire) or three phase (3 wire), for lighting load multiply values given in the table by 1.8, for lighting and power load, multiply the values given in the table by 1.64.

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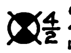
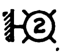
[1] *Bull.* 7 B, Nat. Elect. Lamp Assoc

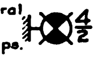
STANDARD SYMBOLS FOR WIRING PLANS

as adopted and recommended by
THE NATIONAL ELECTRICAL CONTRACTORS ASSOCIATION of the UNITED STATES and THE AMERICAN INSTITUTE of ARCHITECTS

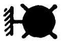
Ceiling Outlet; Electric only. Numeral in center indicates number of Standard 16 C.P. Incandescent Lamps.

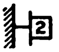
 Ceiling Outlet, Combination. $\frac{4}{2}$ indicates 4-16 C P. Standard Incandescent Lamps and 2 Gas Burners.

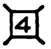
 Bracket Outlet; Electric only. Numeral in center indicates number of Standard 16 C.P. Incandescent Lamps.

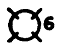
 Bracket Outlet; Combination. $\frac{4}{2}$ indicates 4-16 C P. Standard Incandescent Lamps and 2 Gas Burners.

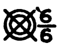
Ceiling Outlet; Gas only.

 Bracket Outlet; Gas only.

 Wall or Baseboard Receptacle. Numeral in center indicates number of Standard 16 C.P. Incandescent Lamps.

 Floor Outlet Numeral in center indicates number of Standard 16 C P Incandescent Lamps.

 Outlet for Outdoor Standard or Pedestal; Electric only. Numeral indicates number of Stand. 16 C.P. Incan. Lamps.

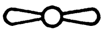
 Outlet for Outdoor Standard or Pedestal. Combination $\frac{6}{6}$ indicates 6-16 C P Stand. Incan. Lamps and 6 Gas Burners.

Drop Cord Outlet.

One Light Outlet for Lamp Receptacle.

Arc Lamp Outlet.

Special Outlet, for Lighting, Heating and Power Current, as described in Spec.

 Ceiling Fan Outlet.

S^1 S.P. Switch Outlet.

S^2 D.P. Switch Outlet.

S^3 3-Way Switch Outlet.

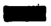
S^4 4-Way Switch Outlet

S^D Automatic Door Switch Outlet.

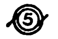
S^E Electrolier Switch Outlet.

Show as many symbols as there are switches, or, in case of a very large group of switches, indicate number of switches by a Roman Numeral, thus: S^1 XII, meaning 12 Single Pole switches. Describe type of switch in specifications, that is, Flush or Surface, Push Button or Snap.

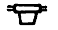
Meter Outlet.

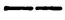
 Distribution Panel.

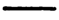
Junction or Pull Box.

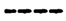
 Motor Outlet: Numeral in center indicates H P.

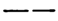
Motor Control Outlet.

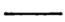
 Transformer

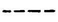
 Main or Feeder run concealed under floor.

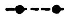
 Main or Feeder run concealed under floor above.

 Main or Feeder run exposed.

 Branch Circuit run concealed under floor.

 Branch Circuit run concealed under floor above

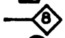
 Branch Circuit Run Exposed

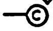
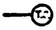
 Pole Line.

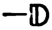
● Riser

Telephone Outlet, Private Service Telephone Outlet; Public Service Bell Outlet.

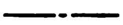
Buzzer Outlet Push Button Outlet; Numeral Indicates number of Pushes

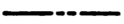
 Annunciator: Numeral indicates number of Points Speaking Tube

 Watchman Clock Outlet. Watchman Station Outlet.  Master Time Clock Outlet.

 Secondary Time Clock Outlet. Door Opener

Special Outlet; for Signal Systems, as described in Specifications. Battery Outlet.

 Circuit for Clock, Telephone, Bell or other Service, run under floor, concealed. Kind of Service wanted ascertained by Symbol to which line connects

 Circuit for Clock, Telephone, Bell or other Service, run under floor above concealed. Kind of Service wanted ascertained by Symbol to which line connects.

Note — If other than Standard 16 C P. Incandescent Lamps are desired, Specifications should describe capacity of Lamp to be used

SUGGESTIONS IN CONNECTION WITH STANDARD SYMBOLS FOR WIRING PLANS

It is important that ample space be allowed for the installation of mains, feeders, branches and distribution panels. It is desirable that a key to the symbols used accompany all plans.
If mains, feeders, branches and distribution panels are shown on the plans, it is desirable that they be designated by letters or numbers.

Heights of center of Wall Outlets (unless otherwise specified)	Living Rooms	5'-6"
	Chambers	5'-0"
	Offices	6'-0"
	Corridors	6'-3"
Height of switches (unless otherwise specified)		4'-0"

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